Emergence and equilibration of jets in planetary turbulence

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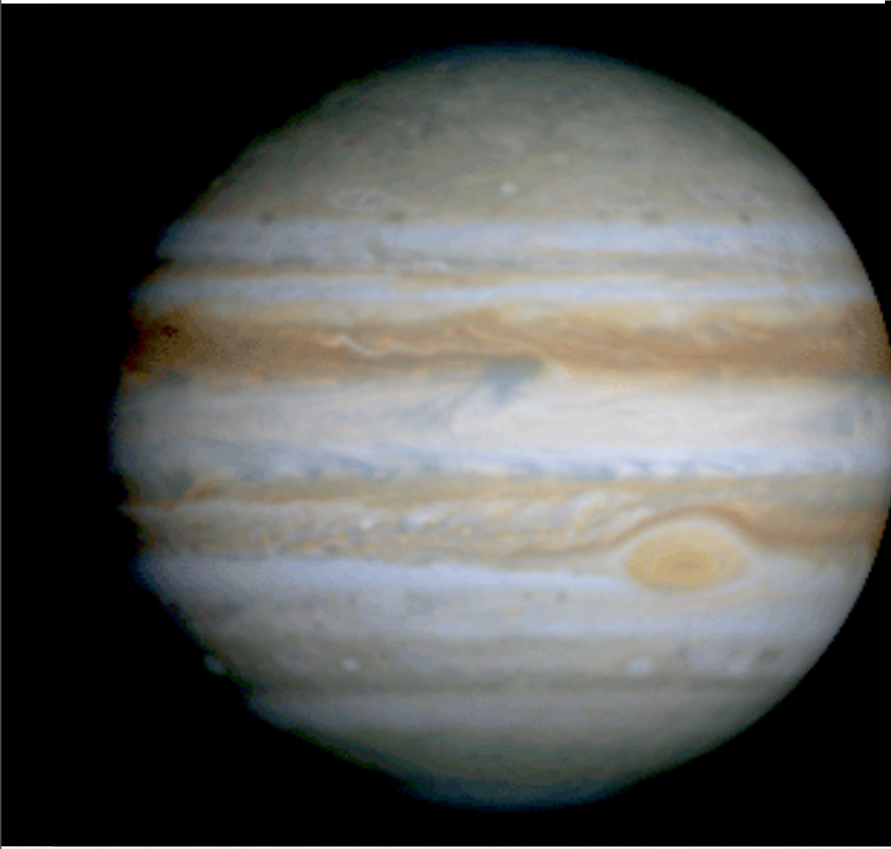
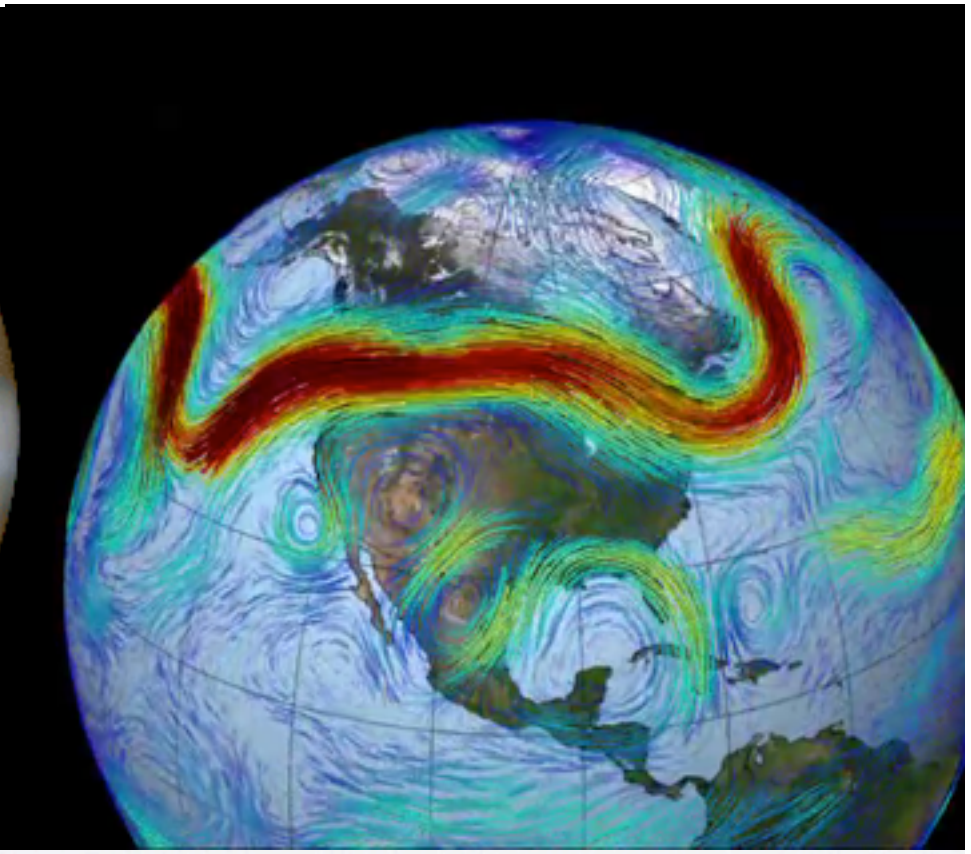
Zonal flows coexist with turbulence



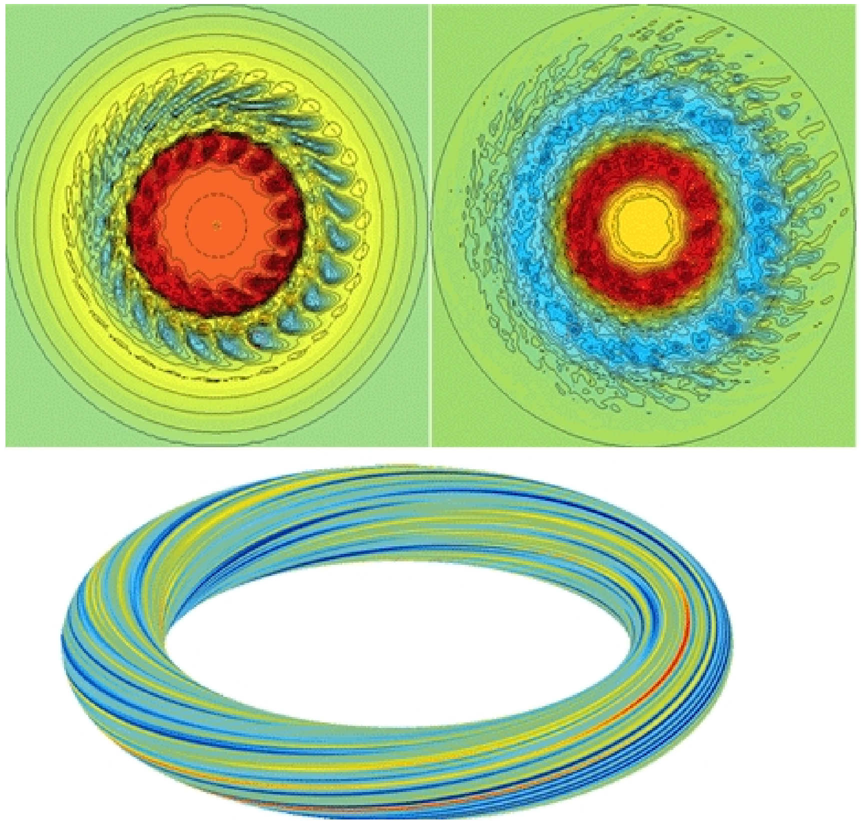
banded Jovian jets

NASA/Cassini Jupiter Images



polar front jet

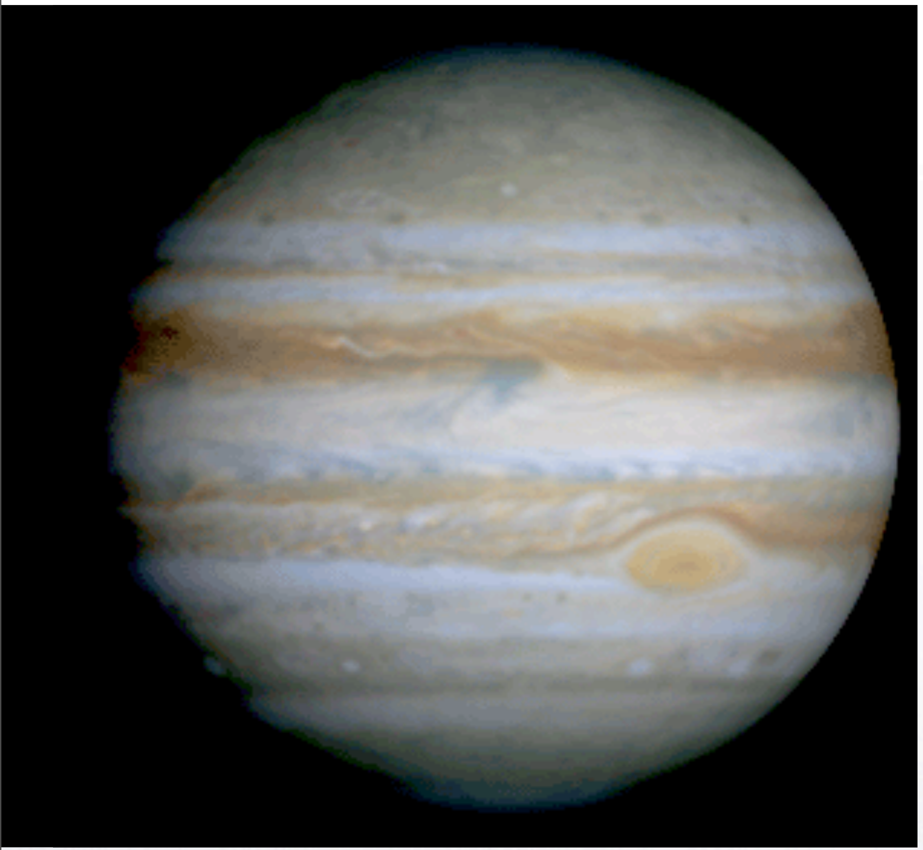
NASA/Goddard Space Flight Center



jets in tokamaks

courtesy: L. Villard

Zonal flows coexist with turbulence



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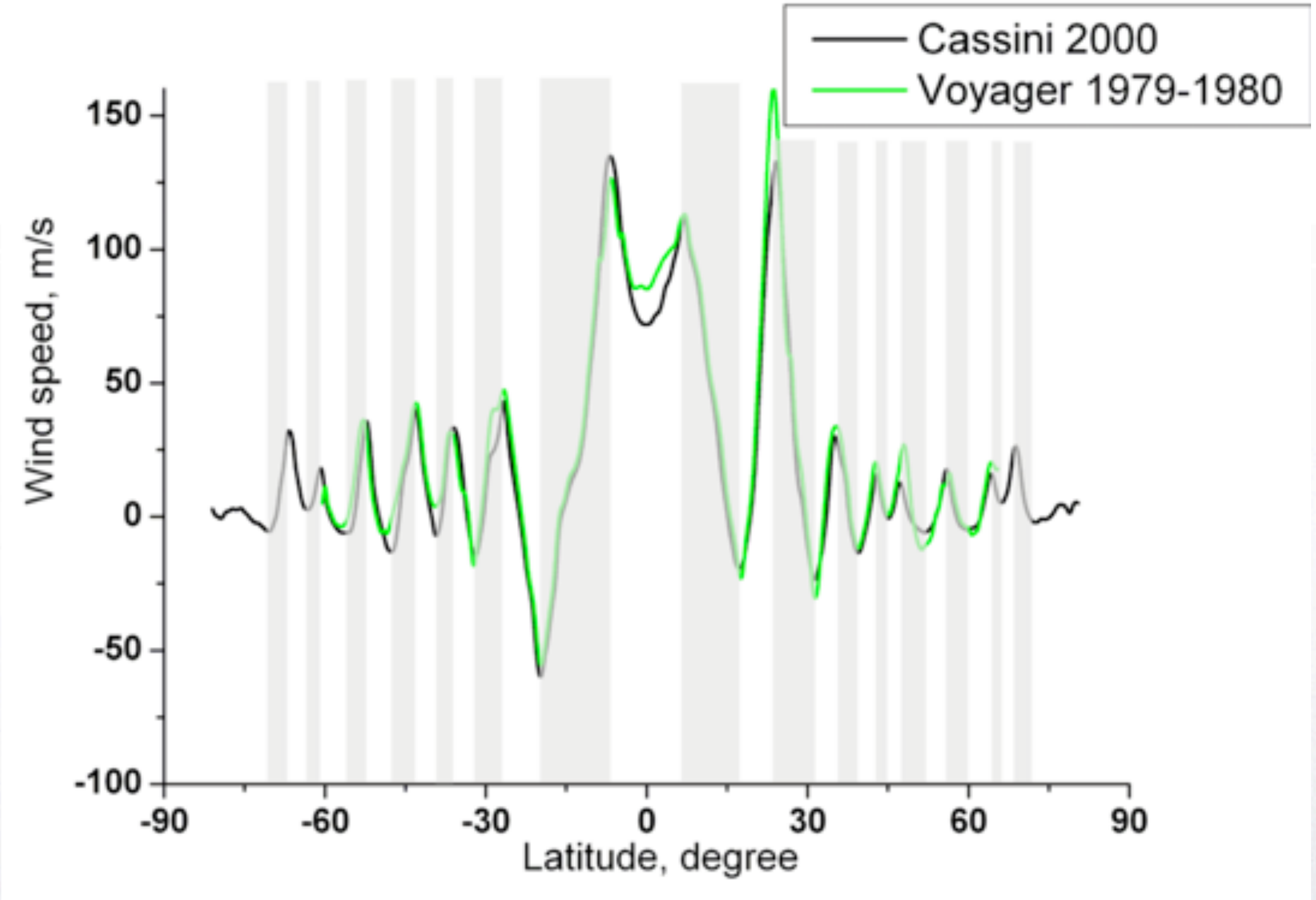
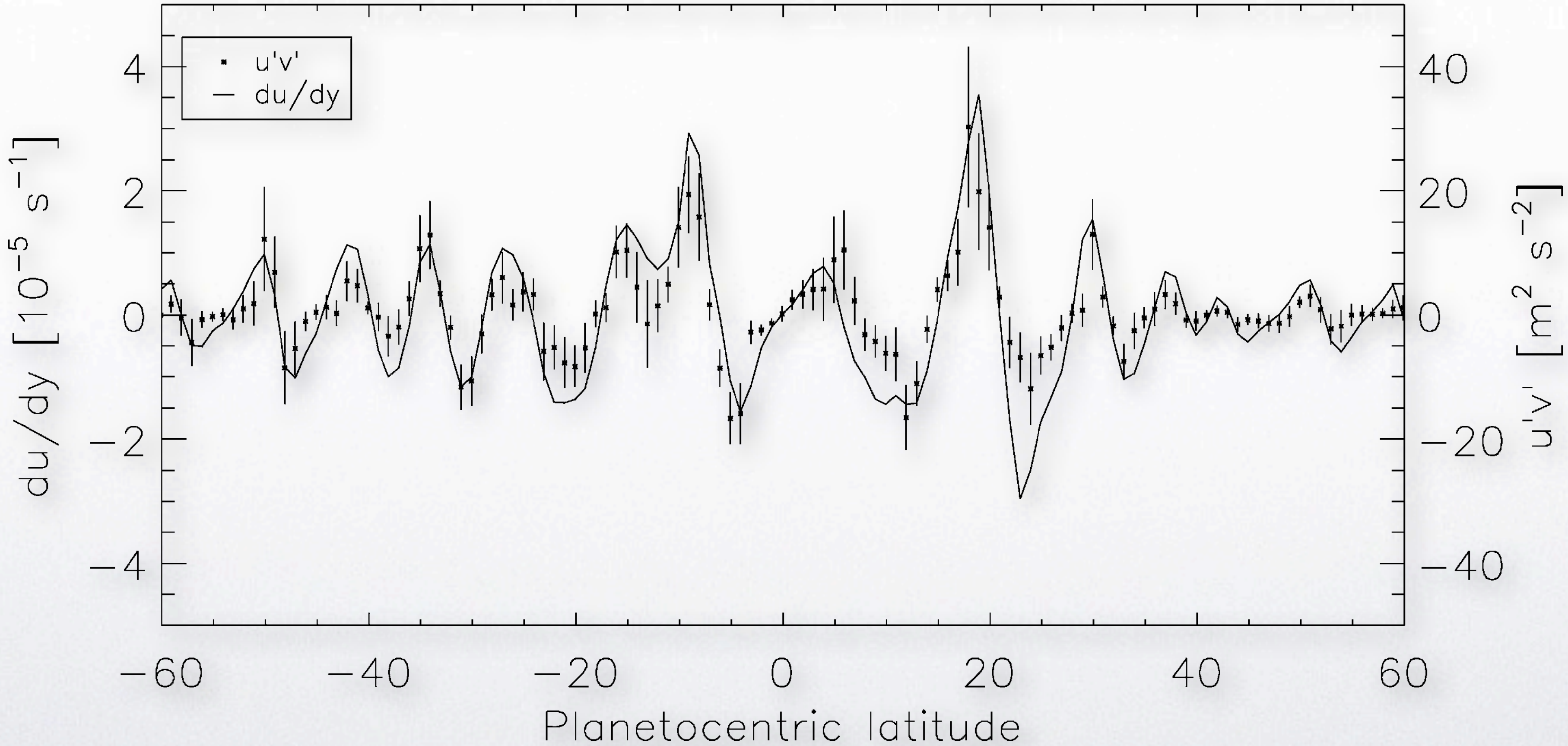


observed Jovian zonal winds
at cloud level

Vasadava & Showman, 2005

Zonal flows are maintained by eddies

$$\frac{\mathrm{d}}{\mathrm{d}t}\int \frac{U^2}{2}\,\mathrm{d}y = \int \frac{\mathrm{d}U}{\mathrm{d}y}\overline{u'v'}\,\mathrm{d}y - \text{Dissipation}$$



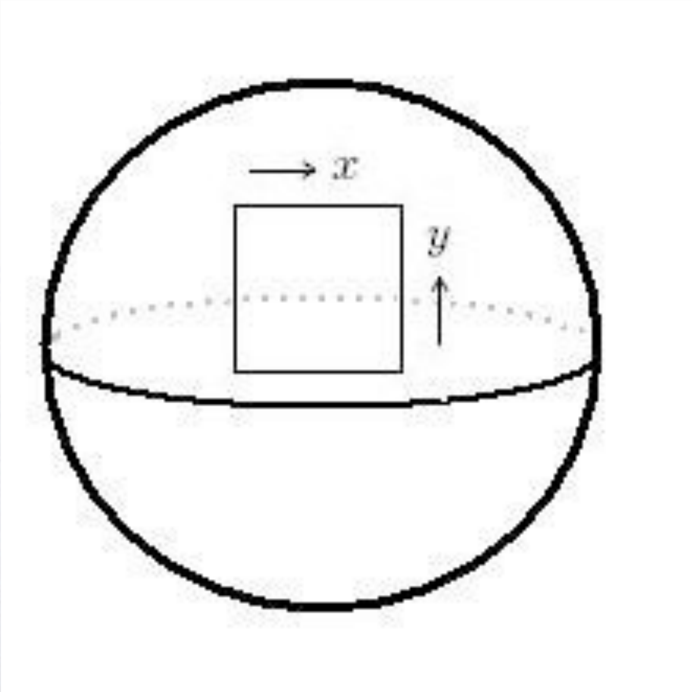
(Salyk et. al. 2006)

Barotropic vorticity equation on a beta-plane

$$\partial_t q + u\partial_x q + v\partial_y q + \beta v = \sqrt{\epsilon}\, f - rq$$

stochastic forcing

dissipation



$$q = v_x - u_y$$

relative vorticity

Zonal - Eddy field decomposition

$$\varphi(x,y,t) = \underset{\substack{\uparrow \\ \text{zonal mean}}}{\Phi(y,t)} + \underset{\substack{\uparrow \\ \text{eddy}}}{\varphi'(x,y,t)}$$

where $\Phi(y,t) = \overline{\varphi}(y,t) = \dfrac{1}{L_x}\displaystyle\int_0^{L_x} \varphi(x',y,t)\,\mathrm{d}x'$

NL (nonlinear) System

$$\partial_t U = \overline{v' q'} - r_{\mathrm{m}} U$$

$$\partial_t q' = -U \partial_x q' + \left(U_{yy} - \beta\right) v' - r q' + F_{\mathrm{e}} + \sqrt{\epsilon}\, f$$

where

$$F_{\mathrm{e}} = \left(\partial_y \left(\overline{v' q'}\right) - \partial_y (v' q')\right) - \partial_x (u' q')$$

eddy-eddy
interaction term

QL (quasi-linear) System

$$\partial_t U = \overline{v'q'} - r_{\mathrm{m}} U$$

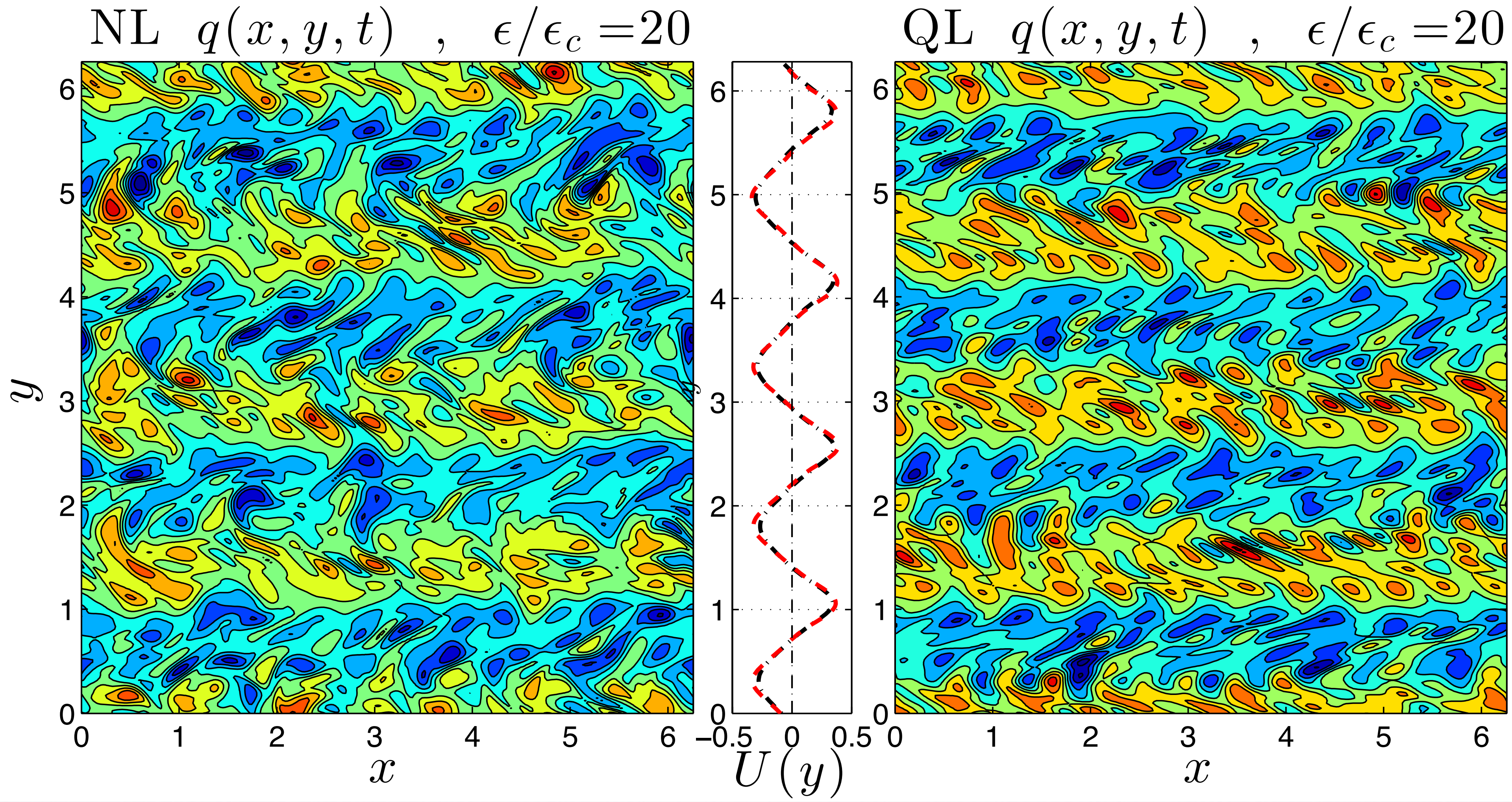
$$\partial_t q' = -U\partial_x q' + \left(U_{yy} - \beta\right) v' - r q' + \cancel{F_{\mathrm{e}}} + \sqrt{\epsilon} f$$

where

$$F_{\mathrm{e}} = \xcancel{\left(\partial_y\left(\overline{v'q'}\right) - \partial_y\left(v'q'\right)\right) - \partial_x\left(u'q'\right)}$$

eddy-eddy interaction term

QL captures the NL dynamics



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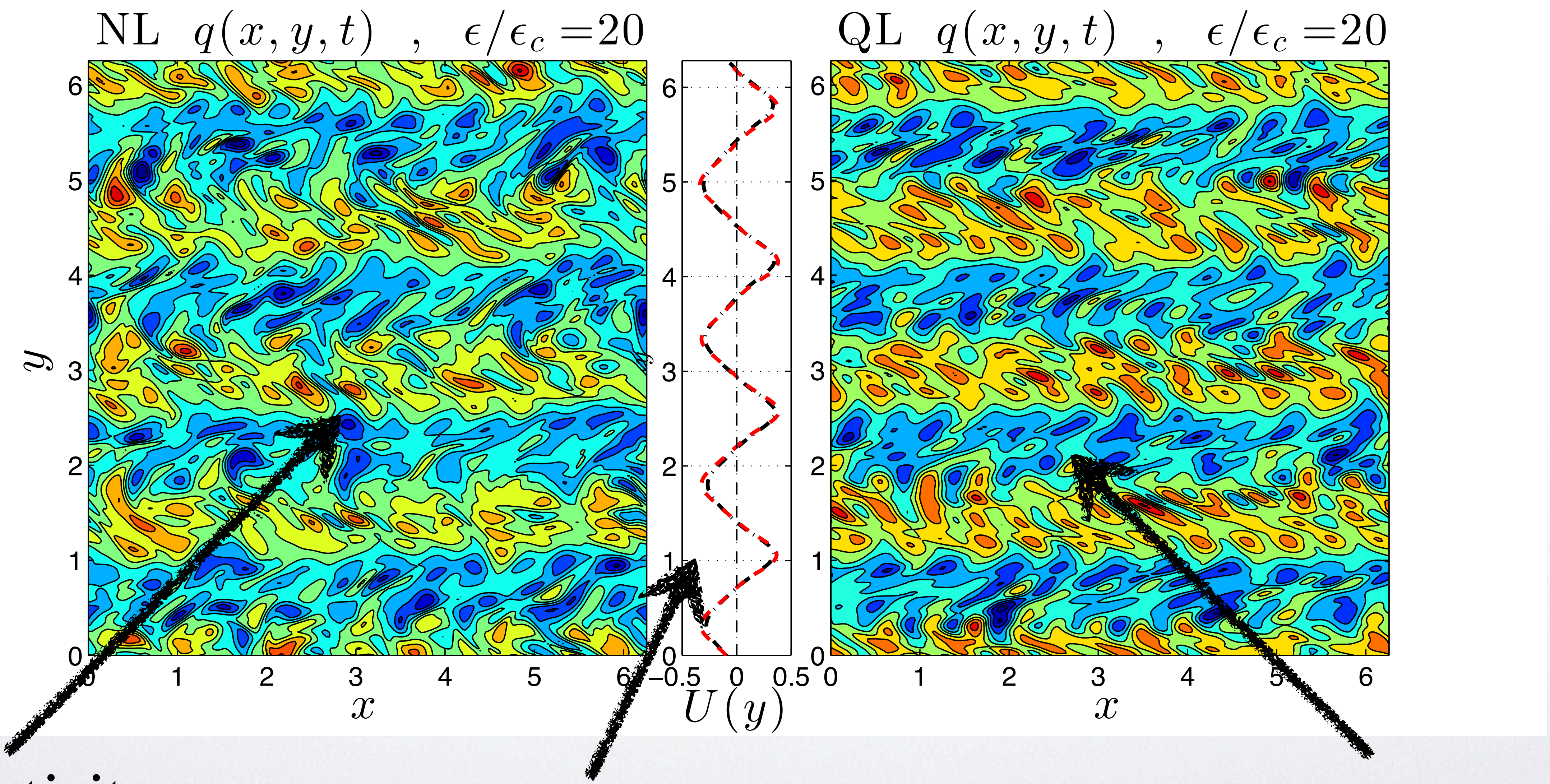


NL vorticity snapshot

mean flow comparison

QL vorticity snapshot

Our goal

While QL captures and elucidates the jet-eddy dynamics
it does not provide a predictive theory

Can we construct a theory that predicts
a) When organized flows emerge?
b) What is structure and the stability of the emergent zonal flows?

Such a theory can be constructed. It is based on the statistical dynamics associated with the QL equations

The theory:

Stochastic Structural Stability Theory (SSST)

QL system → SSST system = ensemble average dynamics of the QL system

$U \,,\, q'$ $\qquad$ $U \,,\, C$

$$\frac{\mathrm{d}U}{\mathrm{d}t} = \langle v' q' \rangle - r_{\mathrm{m}} U$$
$$\frac{\mathrm{d}C}{\mathrm{d}t} = (A_1 + A_2) C + \epsilon Q$$

$$C = \langle q'(x_1, y_1, t) q'(x_2, y_2, t) \rangle$$
$$Q = \langle f(x_1, y_1, t) f(x_2, y_2, t) \rangle$$
$$A_j = -U(y_j)\partial_{x_j} + (\beta - U''(y_j))\partial_{x_j}\Delta_j^{-1} - r$$
$$\overline{v'q'} = \langle v' q' \rangle = R(C) \qquad (j = 1, 2)$$

SSST equilibria

$$\boxed{\begin{aligned} \frac{\mathrm{d}U}{\mathrm{d}t} &= R(C) - r_{\mathrm{m}} U \\ \frac{\mathrm{d}C}{\mathrm{d}t} &= (A_1 + A_2)C + \epsilon\, Q \end{aligned}}$$

SSST system admits equilibria (U^E, C^E)

For example, when we have homogeneity then

$$U^E = 0 \;\text{ and }\; C^E = \frac{\epsilon Q}{2r}$$

is an equilibrium for all β, dissipation values $r > 0$ and energy input rates $\epsilon > 0$.

SSST stability

perturbing the SSST equilibrium: $(U^E + \delta U, C^E + \delta C)$

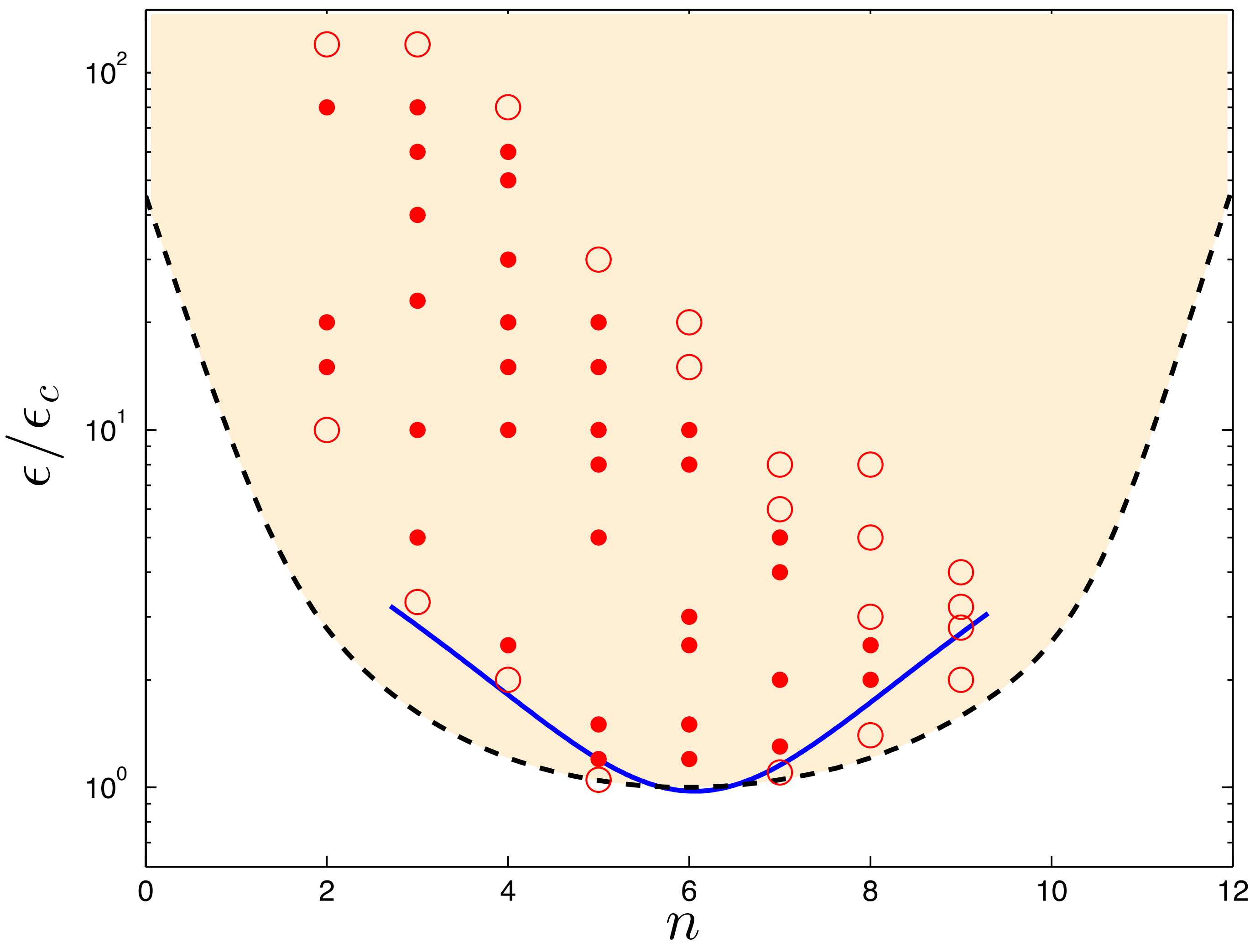
$$\frac{d}{dt}\begin{pmatrix}\delta U\\ \delta C\end{pmatrix} = \mathbb{L}\begin{pmatrix}\delta U\\ \delta C\end{pmatrix} \qquad \left(\mathbb{L} = \mathbb{L}(U^E, C^E)\right)$$

In this way we can check the stability of these ideal equilibrium states

SSST stability

Stability analysis of the ideal states predicts:

- formation of jets
- existence of multiple equilibria and their domain of attraction
- merging of jets



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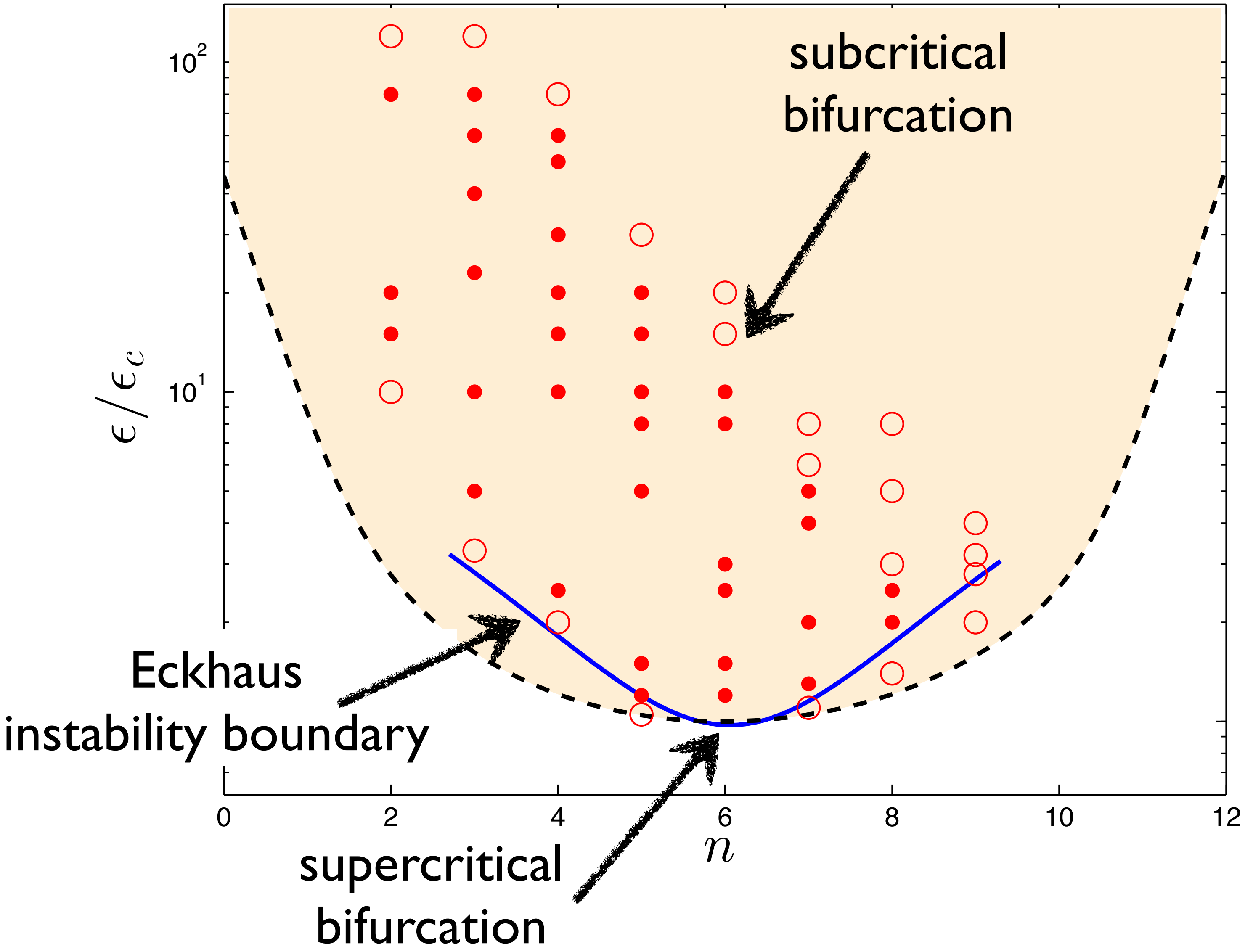
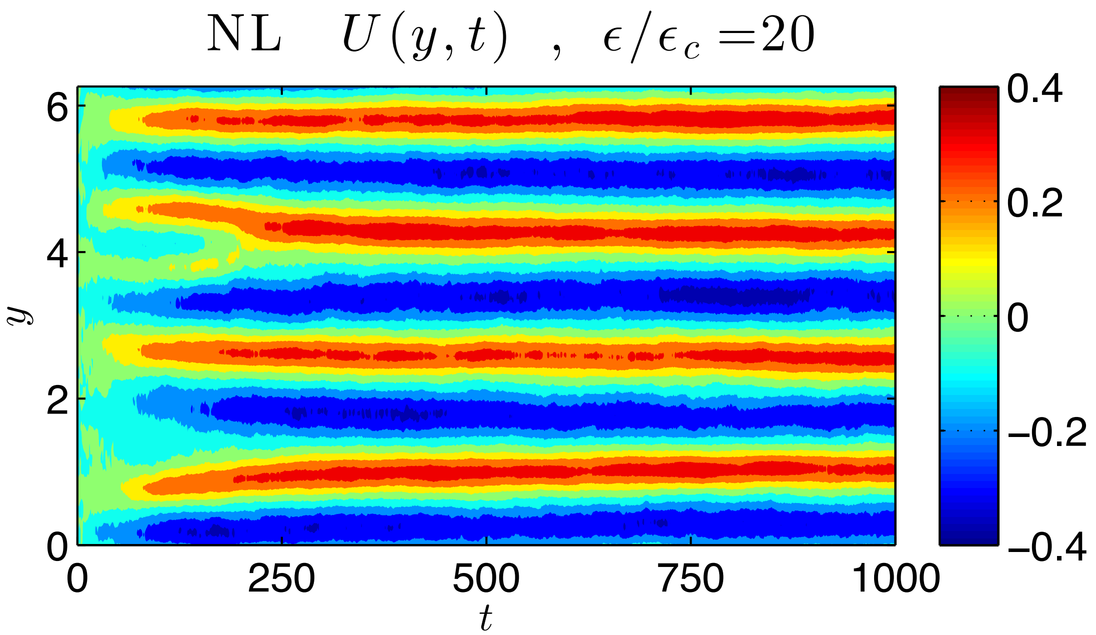

subcritical bifurcation

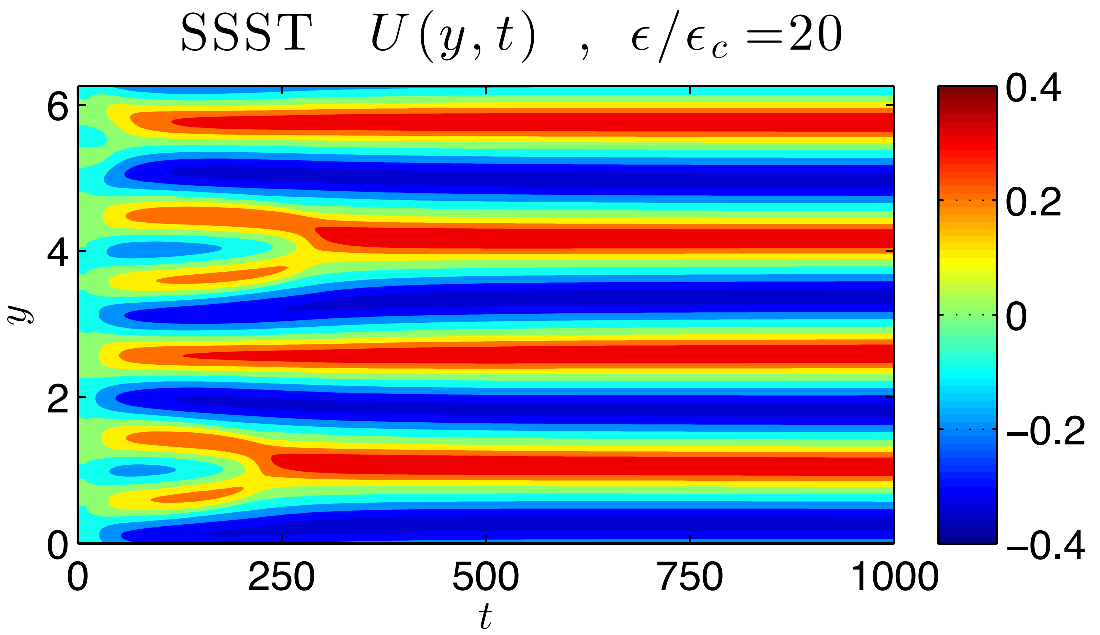
Eckhaus instability boundary

supercritical bifurcation

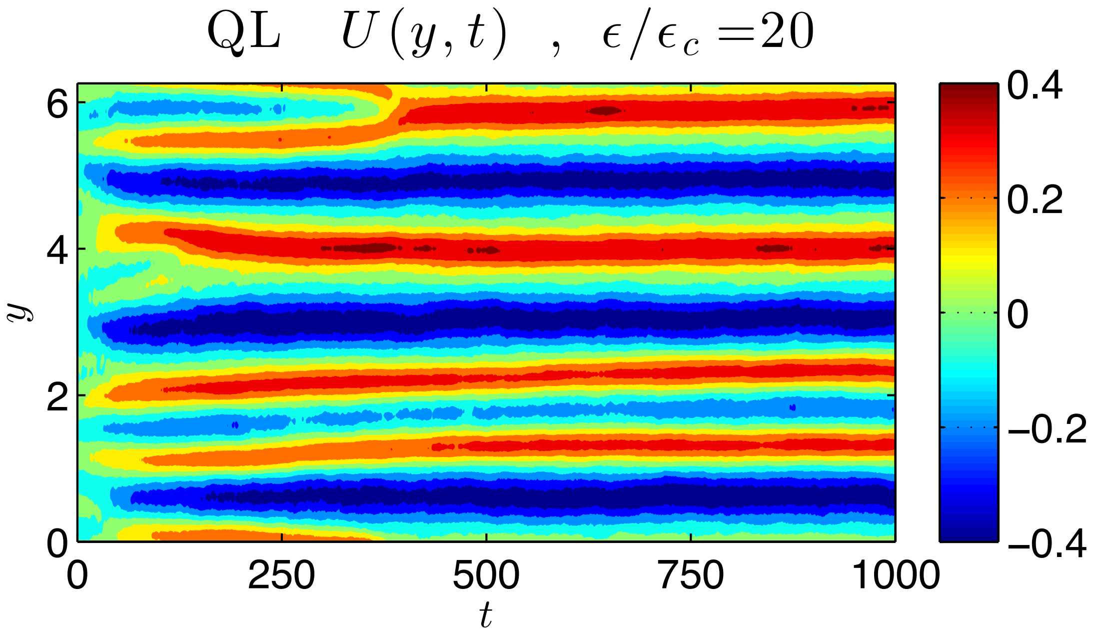
A comparison of NL, QL and SSST



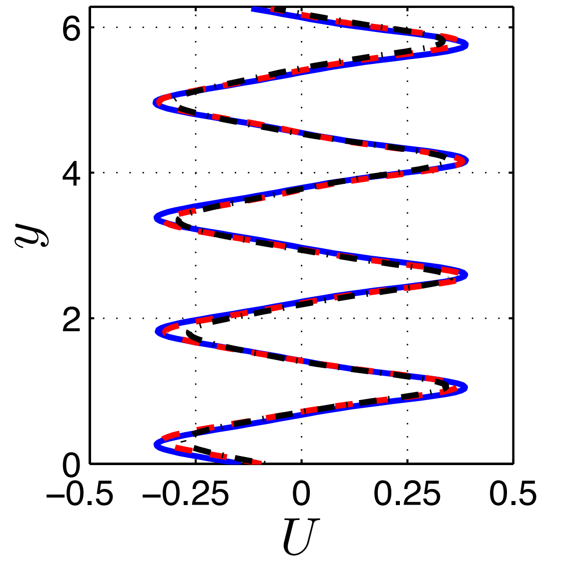
NL $U(y,t)$, $\epsilon/\epsilon_c = 20$



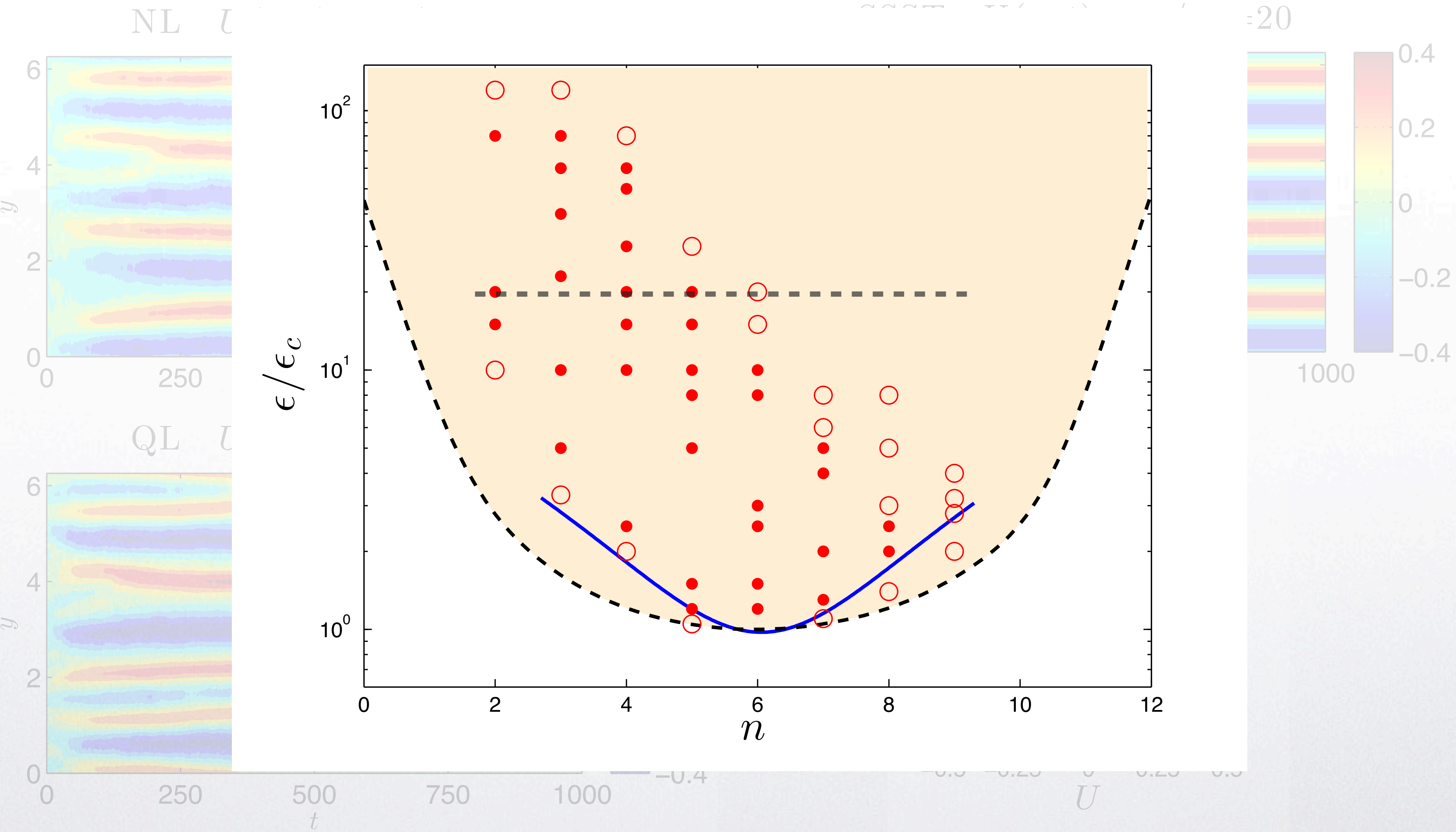
SSST $U(y,t)$, $\epsilon/\epsilon_c = 20$



QL $U(y,t)$, $\epsilon/\epsilon_c = 20$



A comparison of NL, QL and SSST



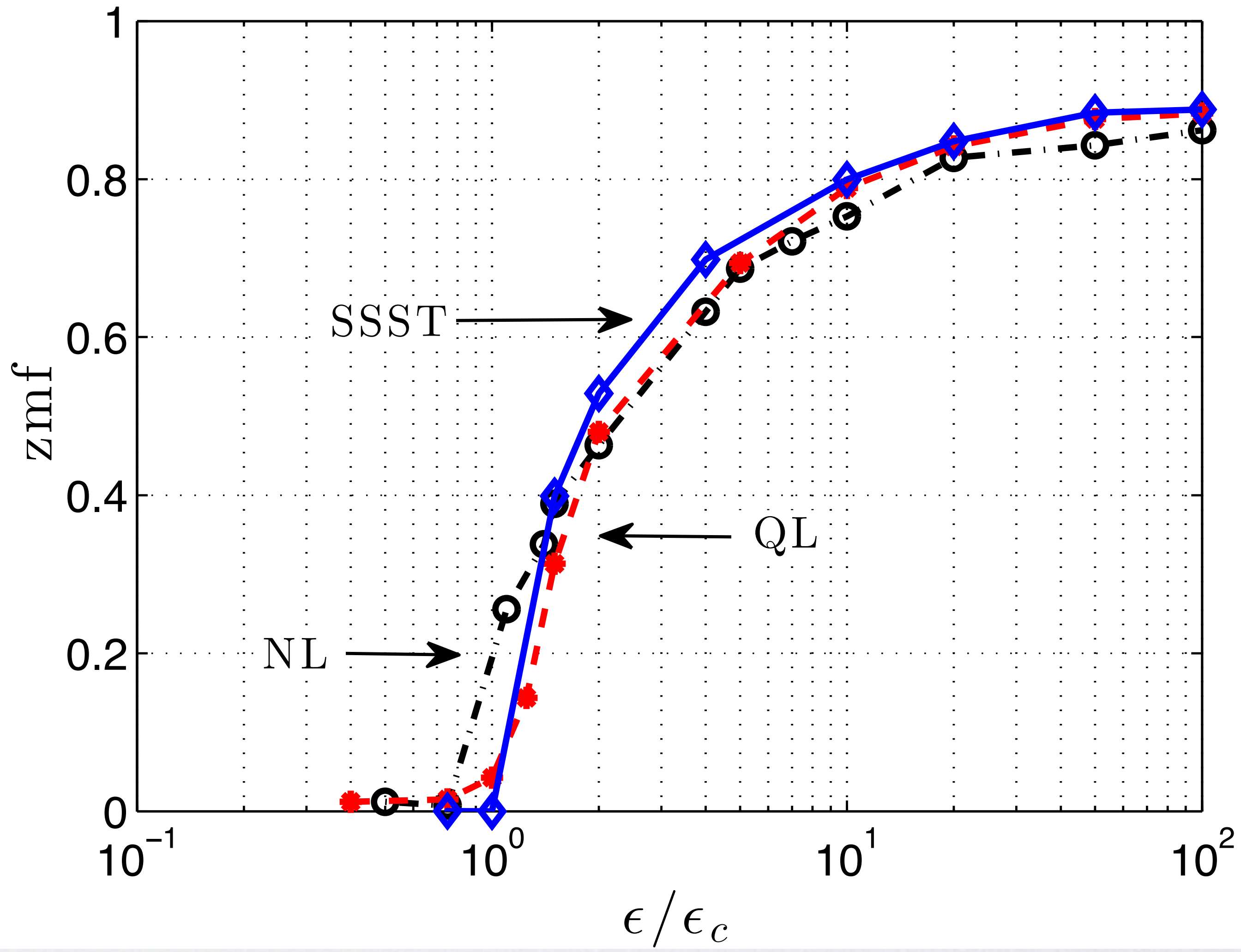
Agreement between NL, QL and SSST

bifurcation diagram of

$$\text{zmf} = \frac{E_{\text{mean}}}{E_{\text{mean}} + E_{\text{pert}}}$$

as a function of
energy input rate, ϵ/ϵ_c

(ϵ_c is the critical energy
input rate for SSST instability of
the homogeneous ideal state)



Conclusions

- QL dynamics captures the jet formation process - The turbulent state is essentially determined by a wave/mean flow interaction
- SSST provides a closure of this turbulent system and a theory for the emergence, equilibration and the structural stability of the associated turbulent equilibria
- SSST introduces a new concept of instability arising from the interaction between turbulence with the large scale flow
- SSST predicts:
  - the formation of jets as an eddy/mean flow SSST instability
  - the existence of multiple equilibria as climate states and their stability
  - jet merger dynamics

Thank you

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Constantinou, N.C, Ioannou, P.J. and Farrell, B.F., 2012:
Emergence and equilibration of jets in beta-plane turbulence.
(submitted to J. Atmos. Sci., arXiv:1208.5665 [physics.flu-dyn])