Emergence and equilibration of jets in planetary turbulence

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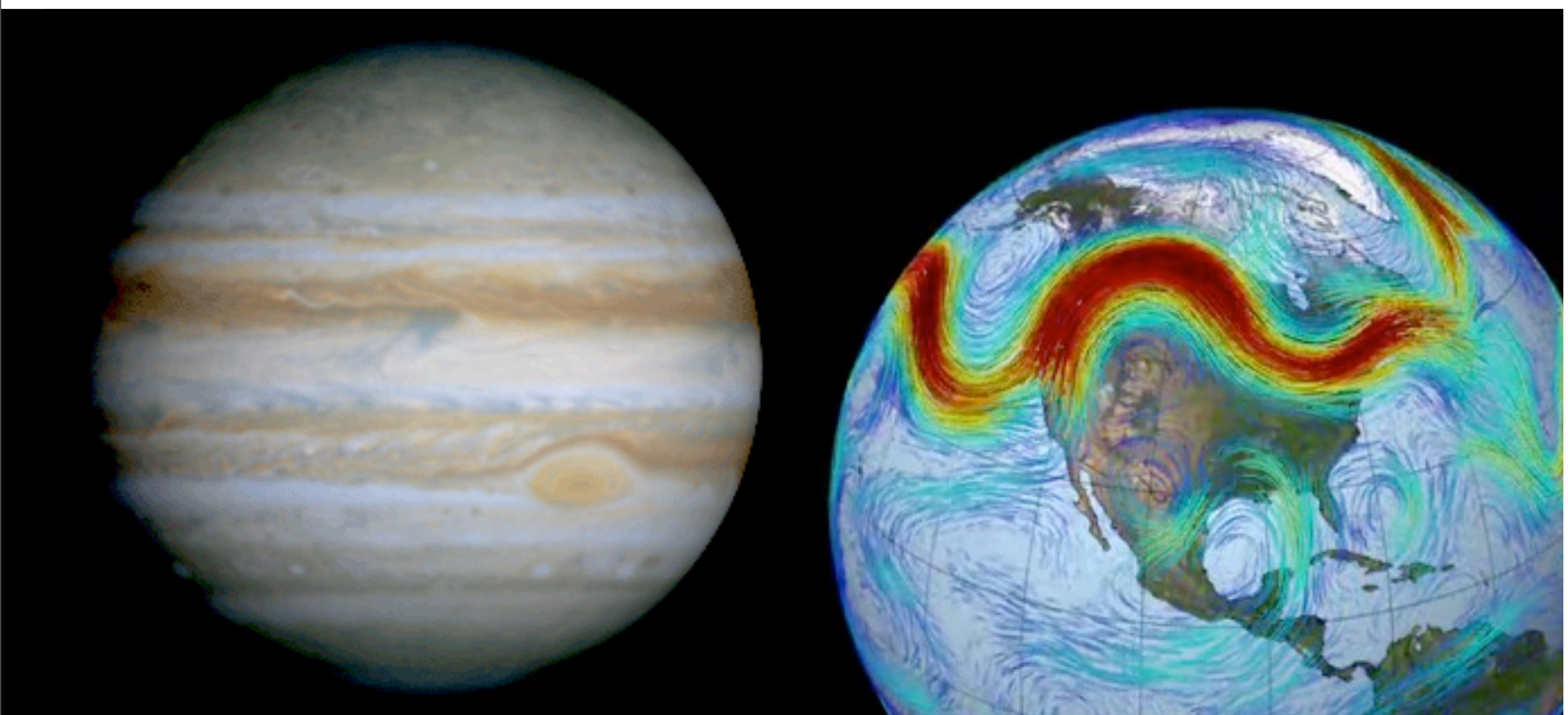
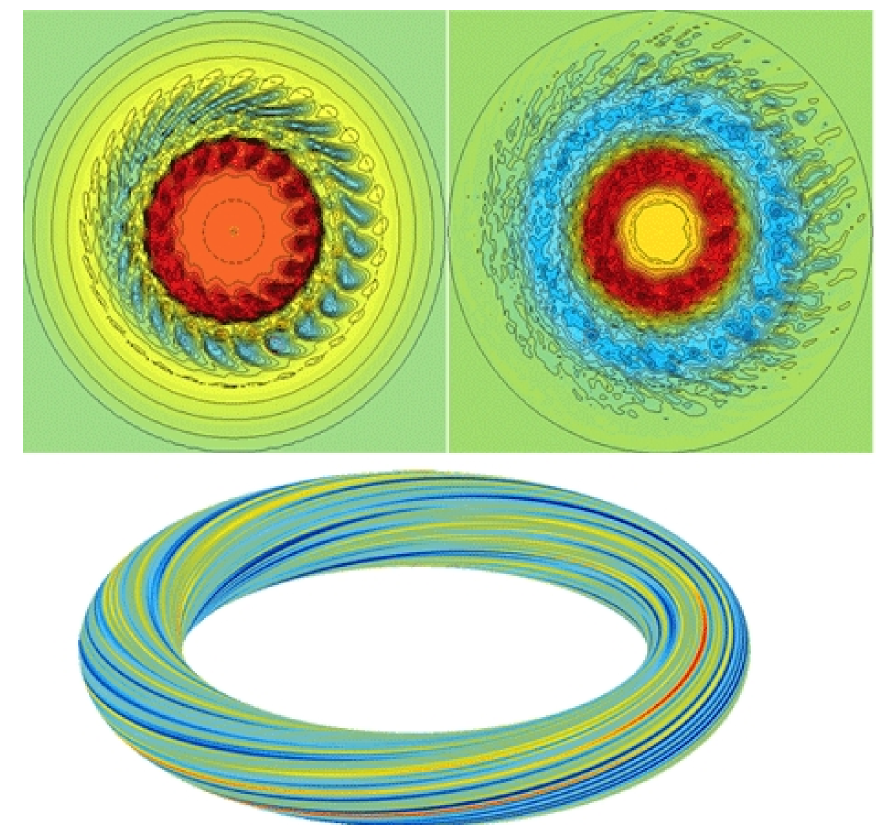
zonal flows coexist with turbulence





banded Jovian jets

NASA/Cassini Jupiter Images

polar front jet

NASA/Goddard Space Flight Center

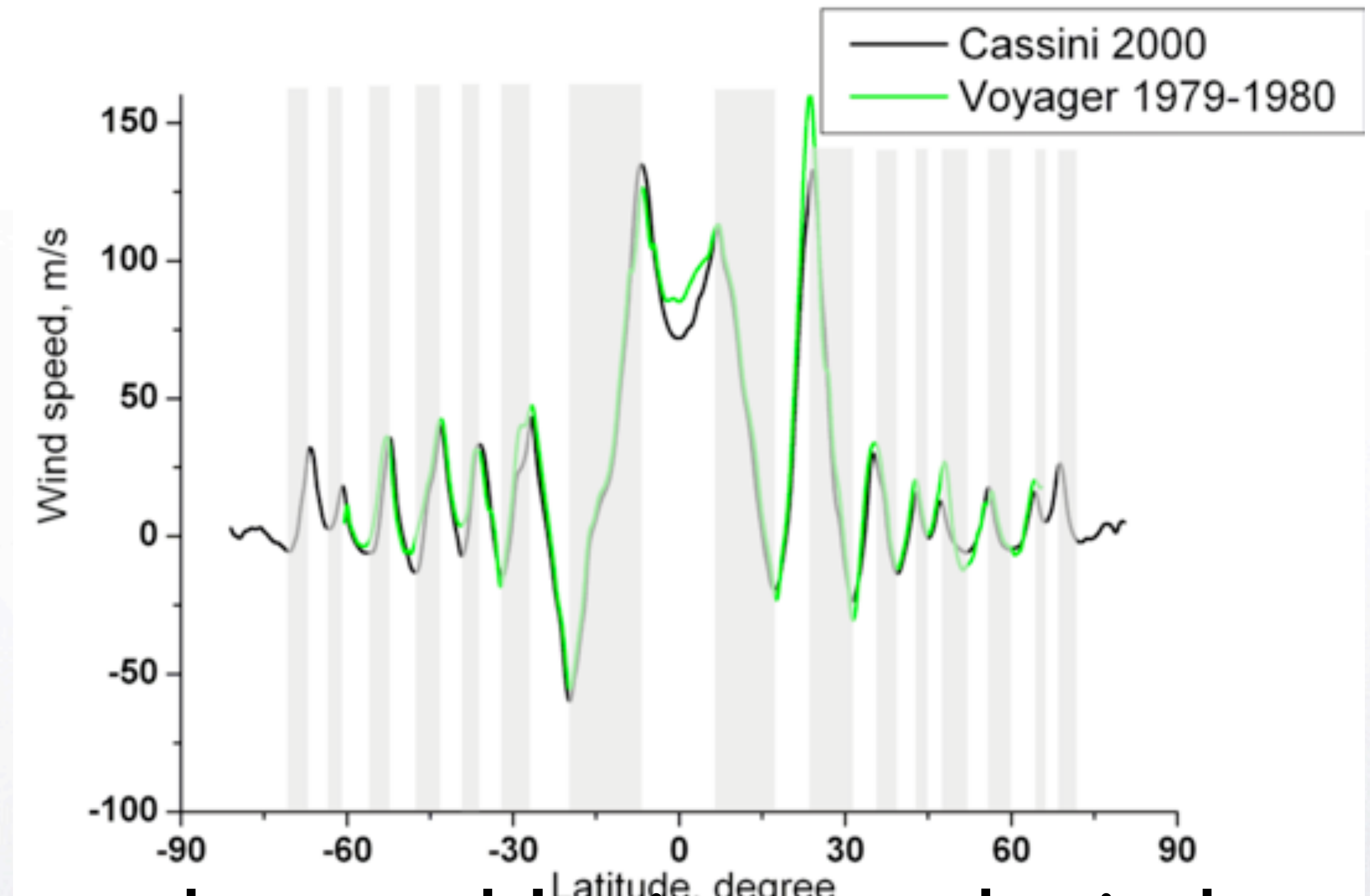
jets in tokamaks

courtesy: L. Villard

zonal flows coexist with turbulence

banded Jovian jets

NASA/Cassini Jupiter Images

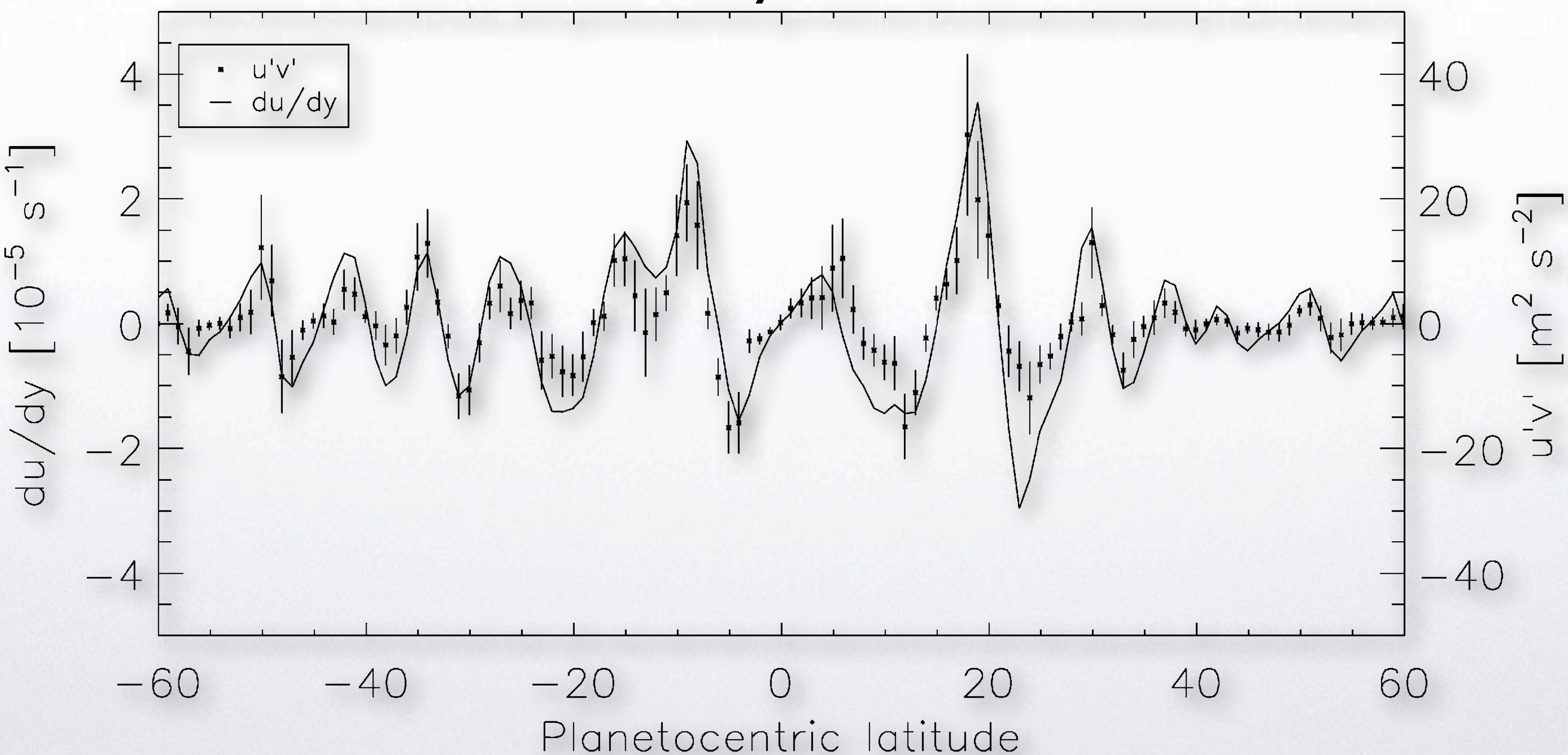


observed Jovian zonal winds at cloud level

Vasadava & Showman, 2005

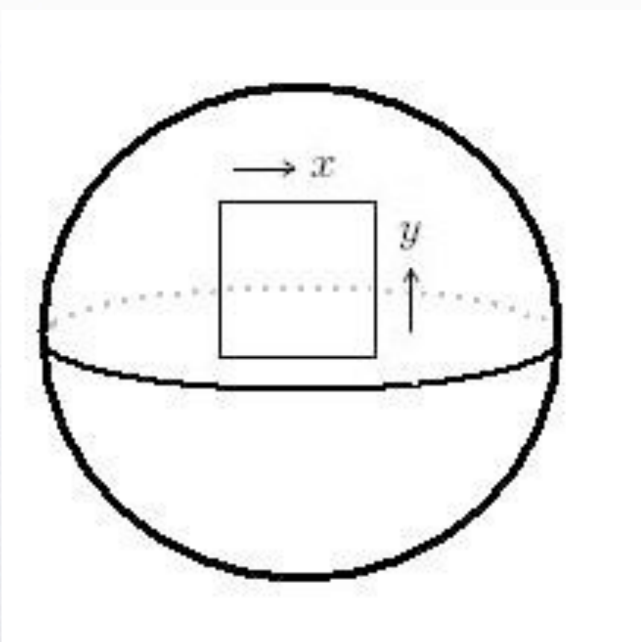
zonal flows are maintained by eddies

$$\frac{\mathrm{d}}{\mathrm{d}t}\int \frac{U^2}{2}\,\mathrm{d}y = \int \frac{\mathrm{d}U}{\mathrm{d}y}\overline{u'v'}\,\mathrm{d}y - \text{Dissipation}$$



Barotropic vorticity equation on a beta-plane (or Charney-Hasegawa-Mima equation)

$$\partial_t q + u\partial_x q + v\partial_y q + \beta v = \sqrt{\epsilon}\, F - rq - \nu_4\, \Delta^2 q$$



$q = v_x - u_y$

vorticity

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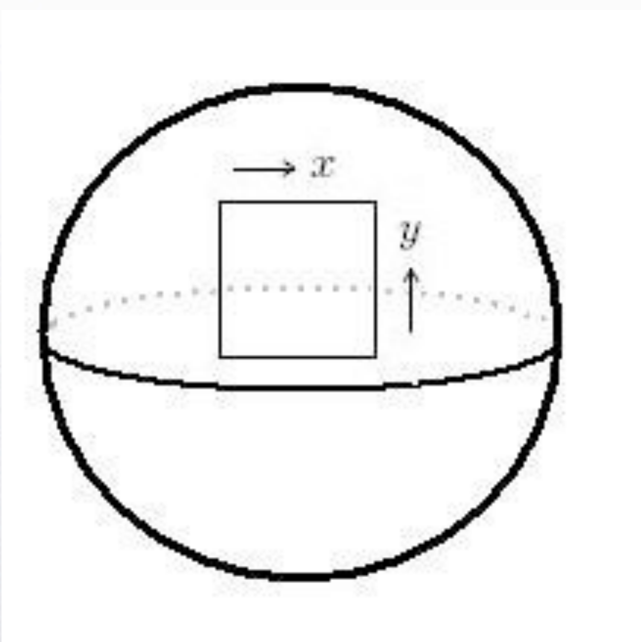
stochastic forcing

dissipation



$q = v_x - u_y$
vorticity

Zonal - Eddy field decomposition

$$\varphi(x,y,t) = \Phi(y,t) + \varphi'(x,y,t)$$

where $$\Phi(y,t) = \overline{\varphi}(y,t) = \frac{1}{L_x}\int_0^{L_x} \varphi(x',y,t)\,\mathrm{d}x'$$

Zonal - Eddy field decomposition

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zonal mean ($\Phi(y,t)$) eddy ($\varphi'(x,y,t)$)

where $$\Phi(y,t) = \overline{\varphi}(y,t) = \frac{1}{L_x}\int_0^{L_x} \varphi(x',y,t)\,\mathrm{d}x'$$

NL (nonlinear) System

$$\partial_t U = \overline{v'q'} - r_m U$$

$$\partial_t q' = -U\partial_x q' + \left(U_{yy} - \beta\right) v' - r q' - \nu_4 \Delta^2 q' + F_e + \sqrt{\epsilon} F$$

where

$$F_e = \left(\partial_y \left(\overline{v'q'}\right) - \partial_y (v'q')\right) - \partial_x (u'q')$$

eddy-eddy interaction term

QL (quasi-linear) System

$$\partial_t U = \overline{v'q'} - r_m U$$

$$\partial_t q' = -U\partial_x q' + (U_{yy} - \beta)v' - rq' - \nu_4 \Delta^2 q' + F_e + \sqrt{\epsilon} F$$

where

$$F_e = \left(\partial_y \left(\overline{v'q'}\right) - \partial_y (v'q')\right) - \partial_x (u'q')$$

eddy-eddy interaction term

Two-point eddy vorticity covariance

$\langle \cdot \rangle =$ ensemble average

$$C(x_a, y_a, x_b, y_b, t) = \left\langle q'(x_a, y_a, t) q'(x_b, y_b, t) \right\rangle$$

$$= \frac{1}{2} \mathrm{Re} \left[\sum_{k=1}^{N_k} \left\langle \hat{q}_k(y_a, t) \hat{q}_k^*(y_b, t) \right\rangle \mathrm{e}^{\mathrm{i}k(x_a - x_b)} \right]$$

$$= \frac{1}{2} \mathrm{Re} \left[\sum_{k=1}^{N_k} \hat{C}_k(y_a, y_b, t) \, \mathrm{e}^{\mathrm{i}k(x_a - x_b)} \right]$$

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$$= \frac{1}{2}\,\mathrm{Re}\left[\sum_{k=1}^{N_k} \hat{C}_k(y_a, y_b, t)\, e^{ik(x_a - x_b)}\right]$$

QL system → SSST system = ensemble average dynamics of the QL system

$U\,,\ q'$ → $U\,,\ \mathrm{C}_k$

SSST System

$$\partial_t U = -\sum_{k=1}^{N_k} \frac{k}{2} \text{vecd}\left[\text{imag}(\mathbf{\Delta}_k^{-1}\mathbf{C}_k)\right] - r_\text{m} U$$

$$\partial_t \mathbf{C}_k = \mathbf{A}_k(U)\mathbf{C}_k + \mathbf{C}_k\,\mathbf{A}_k(U)^\dagger + \epsilon\,\mathbf{Q}_k$$

linear operator that evolves the eddy vorticity in QL

spatial covariance of the stochastic forcing

where $\mathbf{A}_k(U) = -\text{i}k\left[\mathbf{U} - \left(\mathbf{U}_{yy} - \beta\mathbf{I}\right)\mathbf{\Delta}_k^{-1}\right] - r\mathbf{I} - \nu_4 \mathbf{\Delta}_k^2$

SSST System

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$$\partial_t \mathbf{C}_k = \mathbf{A}_k(U)\mathbf{C}_k + \mathbf{C}_k\,\mathbf{A}_k(U)^\dagger + \epsilon\,\mathbf{Q}_k$$

autonomous

no fluctuations
deterministic

SSST equilibria - Stability

for $\nu_4 = 0$ → $U^E = 0\ ,\ C_k^E = \epsilon \frac{Q_k}{2r}$ homogeneous turbulent equilibrium

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for $\epsilon = 0$ **STABLE**

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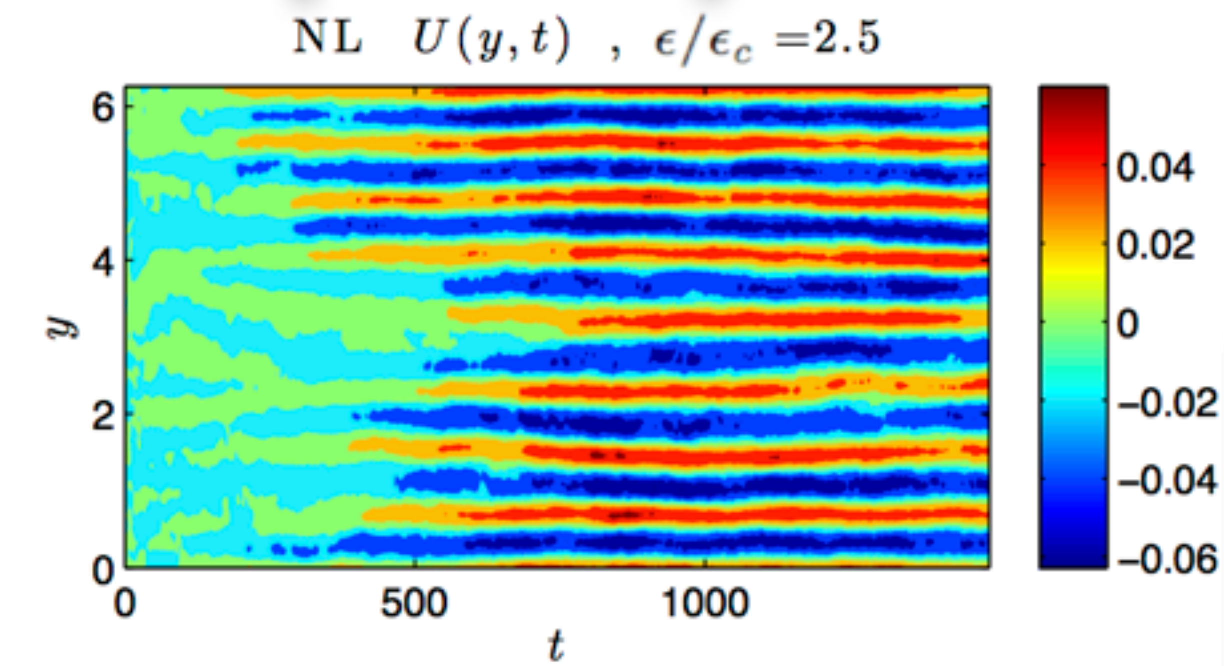
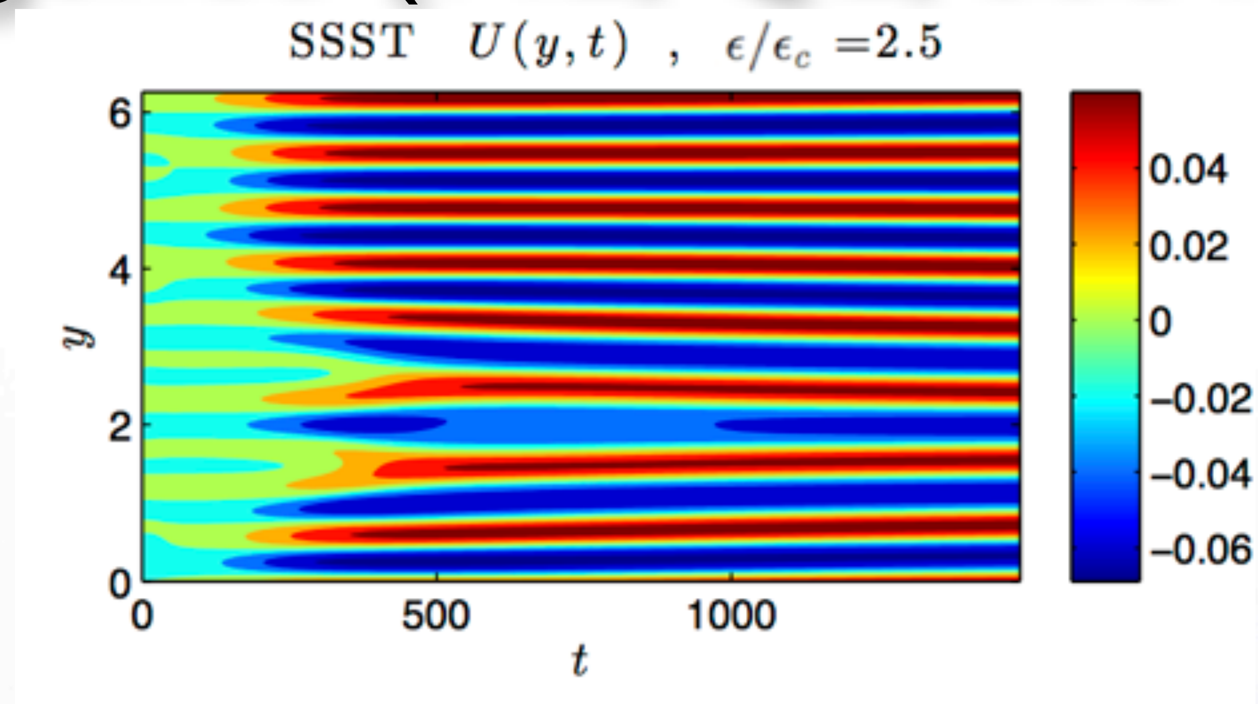
stability ?

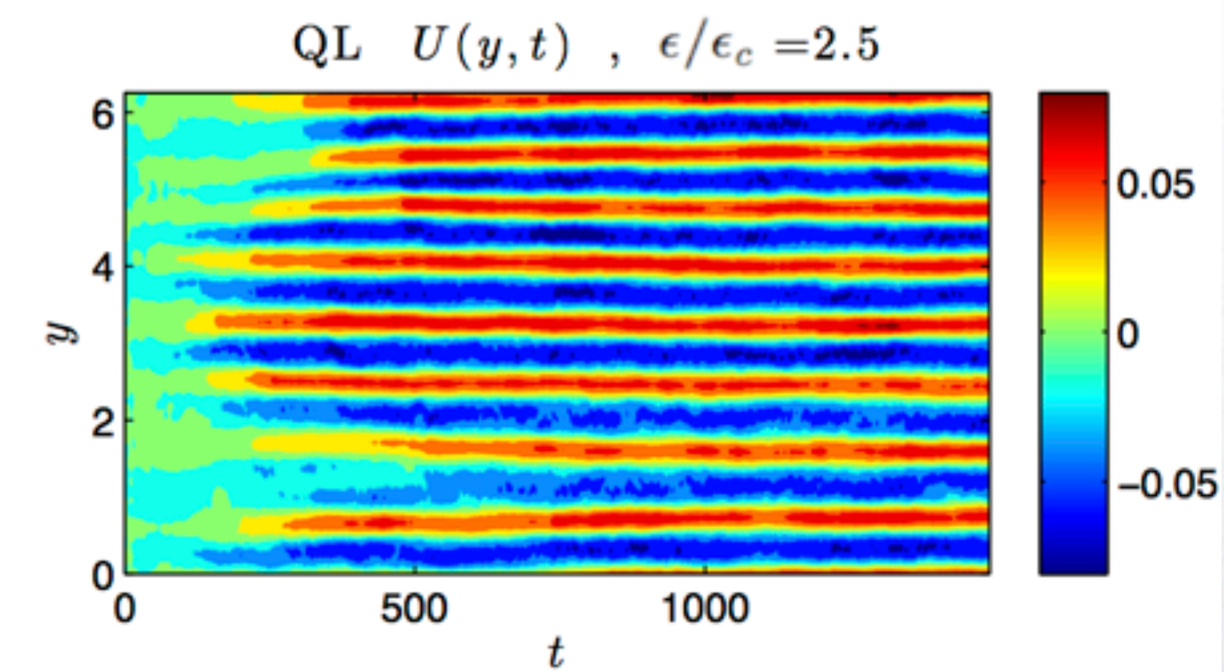
for $\epsilon = 0$ **STABLE**

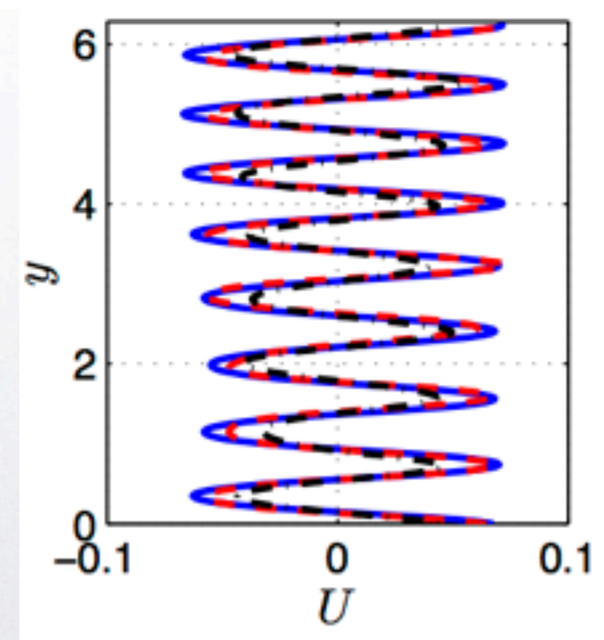
for $\epsilon > \epsilon_c$ **UNSTABLE** → U increases and equilibrates to a finite amplitude mean flow

Example of jet emergence (NL, QL, SSST)



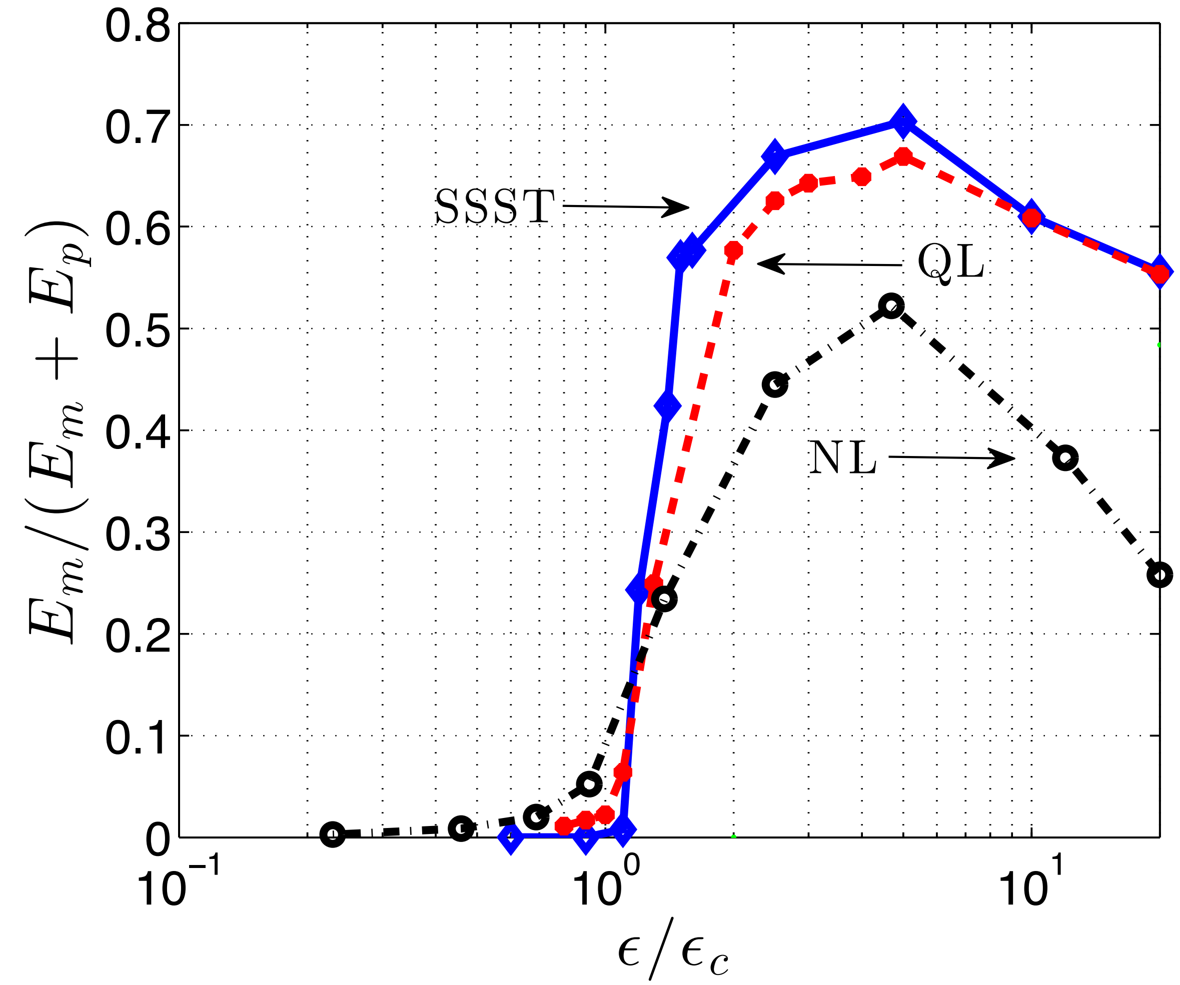






Bifurcation

r = 0.1



Conclusions

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- jet emergence in barotropic beta-plane turbulence or in drift-wave turbulence is a result of the cooperative mean flow/perturbation SSST instability

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- jet emergence in barotropic beta-plane turbulence or in drift-wave turbulence is a result of the cooperative mean flow/perturbation SSST instability
- SSST provides prognostic theory for the emergence & equilibration of zonal flows and also for the prediction of the characteristics of the emergent flows

Thank you

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Constantinou, N.C, Ioannou, P.J. and Farrell, B.F., 2012:
Emergence and equilibration of jets in beta-plane turbulence. (arXiv:1208.5665)