Emergence and equilibration of zonal winds in turbulent planetary atmospheres

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Turbulent fluids often appear to self-organize forming large-scale zonal structures. Examples from meteorology are the midlatidute polar jet in the Earth's atmosphere and the zonal winds in the atmosphere of Jupiter. These large-scale zonal structures are formed and also maintained by the small-scale baroclinic or barotropic turbulence with which they coexist. We present a new theory, named S3T, that explains the emergence and equilibration at finite amplitude of large-scale zonal flows in planetary turbulence. We apply this theory to make predictions for the emergence of zonal flows from a background of homogeneous turbulence as a function of parameters, in a barotropic fluid on a beta-plane. We show that the transition of a homogeneous turbulent state to an inhomogeneous state, dominated by large-scale zonal flows, occurs as a bifurcation phenomenon. We also show the accuracy of the theory by comparing its predictions to non-linear numerical simulations of the turbulent fluid. This theory provides a vehicle for studying the structural stability of large-scale atmospheric flows and can be used to determine climate sensitivity.

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1 Introduction

Spatially and temporally coherent jets are a common feature of turbulent flows in planetary atmospheres with the banded winds of the giant planets or the Earth's polar front jet constituting familiar examples. Organization of turbulence into large-scale jets is an intriguing scientific problem in its own merit that has important consequences for every day life. In the atmosphere, the balance between the jet and its associated eddies controls the transport of heat, water vapor, trace gases and pollutants, as well as the equator to pole temperature gradient, and the location of storm tracks. Changes in its current structure can therefore have important consequences for both the regional and the global climate.

Fjørtoft (1953) noted that the conservation of both energy and enstrophy in a dissipationless barotropic flow implies that transfer of energy among spatial spectral components results in energy accumulating at the largest scales. This argument provides a conceptual basis for understanding the observed tendency for formation of large-scale structure from small-scale turbulence in planetary atmospheres. However, the observed large-scale structure is dominated by zonal jets with specific form and, moreover, the scale of these jets is distinct from the largest scale in the flow. Rhines (1975) argued that the observed spatial scale of jets in beta-plane turbulence results from arrest of upscale energy transport at the length scale $\sqrt{u/\beta}$, where β is the meridional gradient of planetary vorticity and u is the root mean square velocity in the turbulent fluid. In Rhines's interpretation this is the scale at which the turbulent energy cascade is intercepted by the formation of propagating Rossby waves.

While these results establish a conceptual basis for expecting large-scale zonal structures to form in beta-plane turbulence, the physical mechanism of jet forma-tion, the structure of the jets, and their dependence on parameters remain to be determined.

Observations of the atmospheric circulation (Shepherd 1987) and analysis both of numerical simulations and laboratory experiments (Huang and Robinson 1998, Wordsworth et al. 2008) demonstrated that jets in 2-D planetary turbulent flows are maintained through spectrally non-local interactions rather than by cascade processes. Based on the above observations we will present a non-equilibrium statistical theory for jet formation, named S3T, in which the cascade process does not play a role (Farrell and Ioannou 2003). The S3T is also referred to as CE2 (for second-order cumulant expansion) because it is equivalently obtained by trun-cating the infinite hierarchy of cumulant expansions to second order (Marston et al. 2008). In S3T, jets initially arise as a linear instability of the interaction between an infinitesimal jet perturbation and the associated eddy field (cf. Bakas and Ioannou 2013), and finite amplitude jets result from nonlinear equilibria continuing from these instabilities (Farrell and Ioannou 2007, Srinivasan and Young 2012, Constantinou et al. 2013 (hereafter CFI)). Analysis of this jet formation instability determines the bifurcation structure of the jet formation process as a function of parameters. In addition to jet formation bifurcations, S3T predicts jet breakdown bifurcations as well as the structure of the emergent jets, the structure of the finite amplitude equilibrium jets they continue to, and the structure of the turbulence accompanying the jets.

2 Formulation

Consider quasi-geostrophic dynamics on a barotropic, doubly periodic, beta plane, $(x,y) \in [0,L_x]\mathrm{x}[0,L_y]$. The beta plane is a Cartesian approximation of the surface of a planet at midlatitudes with x being in the zonal (latitudinal) direction and y in the meridional direction. In the absence of forcing and dissipation potential vorticity is conserved, while in the presence of both forcing and dissipation it obeys:

$$\partial_t \zeta + J(\psi, \zeta + \beta y) = -r\zeta + \sqrt{\varepsilon} f \qquad (1)$$

This will be referred to as the nonlinear system (NL). The Jacobian term $J(A,B) \equiv (\partial_x A)(\partial_y B)-(\partial_y A)(\partial_x B)$ represents advection of the absolute vorticity by the velocity field. Linear drag is included with coefficient r, which in geophysical applications represents Ekman dissipation. Term f is an external forcing that parameterizes processes absent in the dynamics (i.e. cascade of energy from baro-clinic to barotropic eddies or convection) and it is modeled here as homogeneous random stirring delta-correlated in time. The amplitude of the excitation is controlled through ε. Inclusion of this term is necessary in this barotropic framework in order to sustain turbulence. The relative vorticity of the fluid is $\zeta=\Delta\psi$, where $\Delta\equiv\partial_{xx}+\partial_{yy}$ is the horizontal Laplacian, and ψ is the streamfunction. Zonal and meridional velocities are respectively: $u=-\partial_y\psi$ and $v=\partial_x\psi$.

For the construction of the theory we proceed as follows:

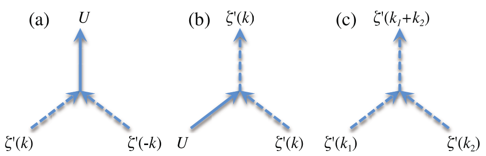

Fig. 1. The nonlinear interactions in the NL system (1) can be classified as follows: (a) two eddies of zonal wavenumbers k and $-k$ combine to form a mean flow (k=0), (b) an eddy of zonal wavenumber k interacts with the mean flow U (a k=0 component) to produce an eddy also at wavenumber k and (c) an eddy with zonal wavenumber k_1 interacts with a k_2 eddy to produce a k_1+k_2 eddy.

1. We write (1) as a system for the evolution of the mean flow, $U \equiv L_x^{-1}\int_0^{L_x} u\,\mathrm{d}x$ and eddy vorticity $\zeta' \equiv \zeta - \overline{\zeta}$. Zonal mean quantities are indicated with a bar or capitals and all deviations from the zonal mean (referred to also as eddies) with primes. The nonlinear interactions in the NL equation (1) are of three types (shown in Fig. 1): two eddies interact to form a mean flow (type (a)), the mean flow interacts with an eddy to produce a distorted eddy (type (b)), and two eddies interact to form another eddy (type (c)). Type (c) interactions redistribute energy among the various eddies and are responsible for the familiar cascade process that fills the eddy energy spectrum. We will neglect type (c) interactions and consider the dynamics that result when only type (a) and type (b) interactions are included. The equation for the evolution of the zonal-mean,

$$\partial_t U = \overline{v'\zeta'} - rU \, , \qquad (2a)$$

contains nonlinear interactions of only type (a) and is retained as is, while retaining only type (b) interactions in the evolution equation for the eddies we obtain

$$\partial_t \zeta' = A(U)\zeta' + \sqrt{\varepsilon} f \, , \qquad (2b)$$

with $A(U) = -U\,\partial_x-[\beta-(\partial_{yy}U)]\,\partial_x\Delta^{-1}-r$. This nonlinear system is the quasi-linear (QL) system associated with the above NL. QL has the advantage that the turbulence associated with it produces without approximation a closure at second order (a CE2).

2. We consider an ensemble of eddy realizations over the latitude circle, chara-cterized by the eddy spatial vorticity covariance between points $x_\alpha=(x_\alpha,y_\alpha)$ and $x_\beta=(x_\beta,y_\beta)$ of the flow, $C_{\alpha\beta}(t) \equiv C(x_\alpha,x_\beta,t)= \langle\zeta'(\mathbf{x}_\alpha,t)\zeta'(\mathbf{x}_\beta,t)\rangle$ (brackets denote ensemble average over realizations of the stochastic excitation f). Taking the ensemble average of (2b) we obtain the covariance evolution equation:

$$\partial_t C_{\alpha\beta} = [A_\alpha(U) + A_\beta(U)]\, C_{\alpha\beta} + \varepsilon\, \Xi_{\alpha\beta} \, , \qquad (3a)$$

with $A_j(U)$ evaluated at points $\mathbf{x}_j$.
$\Xi_{\alpha\beta} \equiv \Xi(\mathbf{x}_\alpha,\mathbf{x}_\beta)$ is the spatial covariance of the forcing under the assumption that the forcing field has zero mean and satisfies $\langle f(\mathbf{x}_\alpha,t_1)\, f(\mathbf{x}_\beta,t_2)\rangle = \Xi(\mathbf{x}_\alpha-\mathbf{x}_\beta)\delta(t_1-t_2)$.
Note that while f is temporally delta-correlated, it will be assumed that it has a finite spatial correlation. Here we consider cases with $\Xi_{\alpha\beta}\sim\Sigma_k \cos[k(x_\alpha-x_\beta)]\exp[-(y_\alpha-y_\beta)^2/(2s^2)]$ and $s\approx0.02L_y$.

3. Under the ergodic assumption that the ensemble average is equal to the zonal average, $\overline{v'\zeta'}$ in (2a) becomes a linear function of C and (2a) takes the form:

$$\partial_t U = \tfrac{1}{2}[(\partial_{x_\alpha}\Delta_\alpha^{-1} + \partial_{x_\beta}\Delta_\beta^{-1})C_{\alpha\beta}]_{\mathbf{x}_\alpha=\mathbf{x}_\beta} - r_{\mathrm{m}}U \ , \qquad (3b)$$

forming in this way with (3a) a closed, autonomous, fluctuation-free deterministic dynamical system for the evolution of the mean flow U and its associated second order eddy statistics C. It constitutes a second order closure (a CE2) for the turbulent state dynamics and is referred to as the S3T system. Note that mean flow U may be dissipated with coefficient r_{m} which may be different from the damping coefficient of the eddies. This asymmetric damping can be regarded as a model for approximating jet dynamics in actual baroclinic flows in which the upper level jet is lightly damped, while the active baroclinic turbulence generating scales are strongly Ekman damped (for discussion cf. CFI).

The equilibrium solutions of the S3T system, denoted as (U^e,C^e), define stationary statistical states of the turbulent flow. These turbulent equilibria comprise of a zonal mean flow, U^e, and of its second-order eddy statistics, C^e. When these equilibria are stable they correspond to statistically steady states of the flow. When they become unstable as a parameter changes, structural instability of the turbulent state occurs and the system bifurcates to a new regime. Such abrupt reorganizations can be seen in both observations and in general circulation models of the Earth's atmosphere and have been proposed to provide a mechanism for abrupt climate change (Farrell and Ioannou 2003, Wunsch 2003).

3 Emergence of jets out of homogeneous turbulence

The NL system (1) for low values of the stochastic excitation amplitude, ε, produces a turbulent state that is homogeneous with no jets. As we increase the forcing amplitude jets emerge at some critical excitation amplitude. The jets are at first weak but as ε increases the jets equilibrate to higher amplitude. The ratio of the zonal-mean flow energy

$$E_m \equiv L_y^{-1}\int_0^{L_y} \tfrac{1}{2}U^2\, \mathrm{d}y$$

over the total energy of the flow

$$E \equiv (L_x L_y)^{-1}\int_0^{L_x}\int_0^{L_y} \tfrac{1}{2}(u^2+v^2)\,\mathrm{d}y\mathrm{d}x$$

measuring the relative strength of the jets, is plotted for this NL experiment as a function of ε in Fig. 2. This figure suggests that a bifurcation phenomenon may underlie the symmetry breaking of the homo-geneous state and the emergence of jets. The QL system (2) reproduces the behavior of the NL, as can be seen from Fig. 3. This shows that the cascade process, which is absent in the QL system, is not responsible for jet formation in this problem. The S3T system will now be used to make predictions for the emergence of jets and also make predictions for their equilibrated amplitudes. The homogeneous equilibrium (U^e=0,C^e=$\varepsilon\Xi/(2r)$) is always an equilibrium solution of the S3T system (3). However, this equilibrium becomes unstable at a critical value ε_c. At this forcing amplitude the S3T dynamics predict that the homogeneous equilibrium is no longer tenable and the flow bifurcates to a flow with jets.

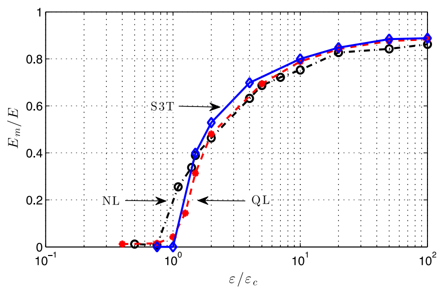

Fig. 2. Bifurcation structure comparison for jet formation in S3T, QL, and NL. Shown is E/E_m vs. forcing amplitude supercriticality, $\varepsilon/\varepsilon_c$. S3T predicts that the homogeneous flow becomes unstable at $\varepsilon/\varepsilon_c$=1 and a symmetry breaking bifur-cation occurs at this ε, whereupon jets emerge. Agreement between NL and S3T argues that zonal jet formation is a bifurcation phenomenon and that S3T predicts both the inception of the instability and the finite amplitude equilibration of the emergent flows. Zonal wavenumbers k=(2π/L_x)x (1,…,14) are forced, β=10, r=0.1, r_{m}=0.01. (Adapted from CFI, © 2014 AMS.)

When such stability analysis is performed for the specific forcing we obtain that the ε_c predicted by S3T stability is precisely the value for which jets start emerging in both the NL and QL simulations (see Fig. 2). It can be seen that both predictions of S3T for the critical ε_c and also for the final equilibrium amplitude of the jets are very accu-rately reflected in sample NL and QL integrations.

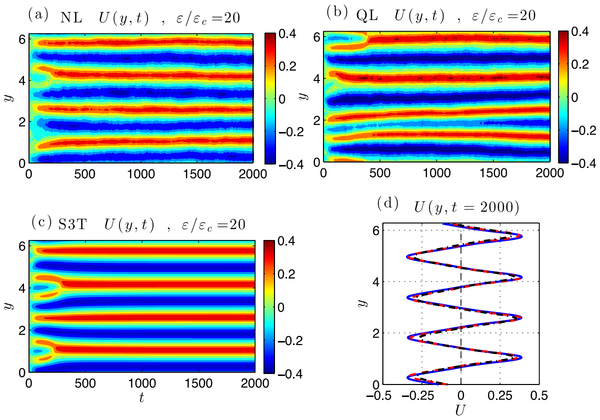

Fig. 3. Hovmöller diagrams of jet emergence in NL, QL and S3T simulations with typical non-isotropic forcing at $\varepsilon/\varepsilon_c$=20. Shown are *U(y,t)* for the (a) NL, (b) QL and (c) S3T simulations and (d) the equilibrium jets in the NL (dash-dot), QL (dashed), and S3T (solid) simulations. It can be seen that S3T pre-dicts the structure, growth and equilibration of immo-derately forced jets in both the QL and NL simulations. Other parameters as in Fig. 2. (Adapted from CFI, © 2014 AMS.)

4 Conclusions

We have shown that jet emergence in planetary atmospheres is a bifurcation phenomenon that can be fully analyzed and accounted by S3T. S3T shows that jets emerge from a cooperative instability of the interaction between an ensemble of turbulence realizations with the mean flow in the absence of turbulent cascades. This instability is a cooperative instability of the statistical dynamics of the turbu-lence and is accurately represented in the S3T system, which comprises a second-order cumulant closure of the full statistical dynamics of the turbulent flow. S3T also predicts the formation of jets at finite amplitude. We have shown that sample integrations of the nonlinear equations shadow the predictions about the statistics of the turbulence that are obtained from S3T.

In general, S3T provides the dynamics of the first two moments of the climate of a planet and can be used to obtain understanding of the structural stability of the present climate state to parameter changes. In this study we have verified that S3T can accurately predict abrupt transition to a different climate state, which is chara-cterized by another zonal state.

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