Magnetic suppression of zonal flows on a beta plane

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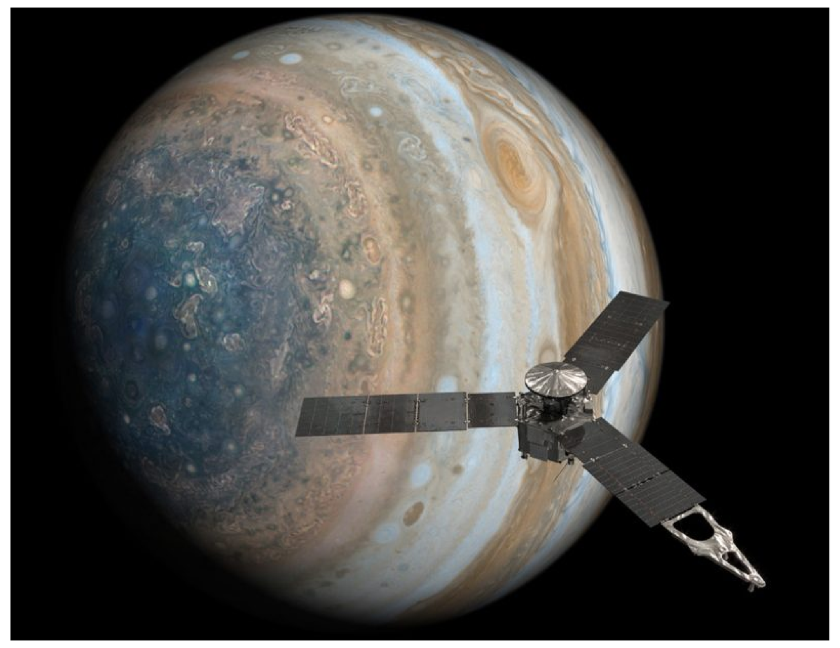
A **new theory** for the interaction of zonal flows and magnetic fields provides possible explanation for why Jupiter's jets terminate at ~3,000 km below the clouds.

Depth of Jupiter's jets



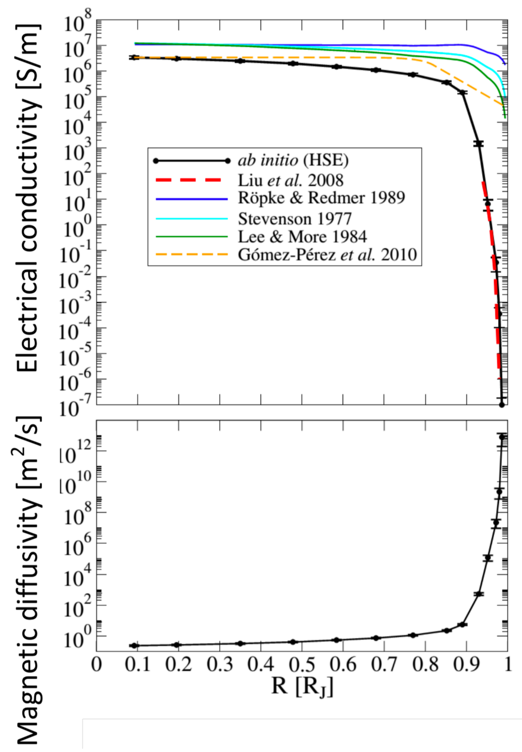
[NASA/JPL]

Juno found that Jupiter's jets go as deep as ~3,000 km below the clouds.

Why the jets stop at that depth?

Inside Jupiter fluid becomes ionized and magnetic fields are strong

- Even before Juno's measurements, it was thought that the depth of zonal flow in Jupiter is set by magnetic effects
- The conductivity of the fluid goes up 11 orders of magnitude within the first 10% of the radius (due to the increasing pressure and ionization)
- Jupiter's planetary magnetic field can start to play a much more important role



[French et al., *Ap. J. Supp. S.* (2012)]

"It's very interesting that (the jets disappear at) about ~3,000 km, because that's about where it might be conducting electricity enough to make a magnetic field.
So, it could be that the magnetic field has something to do with why the belts and zones only go that deep (...) But we don't know this yet; this is just speculation."

Dr Steve Levin, Juno project scientist JPL

Objectives

- Develop a theory to predict the boundary of zonation.
- Dig into the mechanism of how zonal flow suppression occurs.

Simple model: magnetized QG flow on a beta plane

flow (from a streamfunction): $u = -\partial_y \psi$, $v = \partial_x \psi$

relative vorticity $\zeta = \partial_x v - \partial_y u$

magnetic field (from a vector potential):

$$B_x = \underbrace{B_0}_{\substack{\text{constant imposed}\\ \text{azimuthal magnetic field}}} + \partial_y A\ ,\ B_y = -\partial_x A$$

EOM:

$$\partial_t \zeta + \mathsf{J}(\psi, \underbrace{\zeta + \beta y}_{\text{QGPV}}) = \underbrace{\mathsf{J}(A + B_0 y, \nabla^2 A)}_{\text{curl of Lorentz force}} + \nu \nabla^2 \zeta + \underbrace{\xi}_{\substack{\text{hydrodynamic}\\ \text{forcing}}} \qquad (1)$$

$$\partial_t A + \mathsf{J}(\psi, \underbrace{A + B_0 y}_{\substack{\text{total magnetic}\\ \text{vector potential}}}) = \eta \nabla^2 A \qquad (2)$$

- $\mathsf{J}(a,b) = (\partial_x a)(\partial_y b) - (\partial_y a)(\partial_x b)$ is the Jacobian
- doubly-periodic boundary conditions
- forcing ξ is:
  - temporally delta-correlated
  - spatially isotropic with spectrum centered at wavenumber k_f

Previous numerical simulations set the scene

- Tobias, Diamond, & Hughes (2007):
  - DNS on a beta plane, found magnetic suppression of zonal flows for "strong enough" magnetic field. Key parameter is B_0^2/η.
- Tobias, Dagon, & Marston (2011):
  - DNS on the sphere; similar magnetic suppression of zonal flows as on beta plane.
  - Used a quasilinear simplification of the equations. Quasi-linear simulations shows similar magnetic suppression.

But the mechanism of suppression remained elusive.

The zonal flow is driven by Reynolds and Maxwell stresses:

$$\partial_t \bar{u} = \underbrace{-\partial_y \overline{u'v'}}_{\text{Reynolds stress}} - \Big(\underbrace{-\partial_y \overline{B_x' B_y'}}_{\text{Maxwell stress}}\Big) + \nu \partial_y^2 \bar{u}$$

Novel quasilinear analysis shows how magnetic fluctuations suppress zonal flow

Simple picture: Kelvin–Orr shearing waves

- assume long-wavelength mean zonal flow: $\bar{u} = Sy$
- assume non-interacting waves, e.g., $\zeta' = Z_0 e^{ik_x x + (k_y - Sk_x t)y}$
- for a pair of waves compute the resulting Reynolds/Maxwell stress

$$\Delta E_{\text{zonal flow}} \sim S^2 \Big(\overbrace{\frac{k_x^2 - 5k_y^2}{|\boldsymbol{k}|^6} \underbrace{|Z_0|^2}_{\substack{\text{vorticity wave ampl.}\\ \sim\text{strength of}\\ \text{hydrodynamic fluctuations}}}}^{\text{Reynolds stress}} - \overbrace{\frac{k_x^2 - k_y^2}{|\boldsymbol{k}|^2} \underbrace{|A_0|^2}_{\substack{\text{magnetic wave ampl.}\\ \sim\text{strength of}\\ \text{magnetic fluctuations}}}}^{\text{Maxwell stress}} \Big)$$

Maxwell stresses *always act against* the zonal flow.

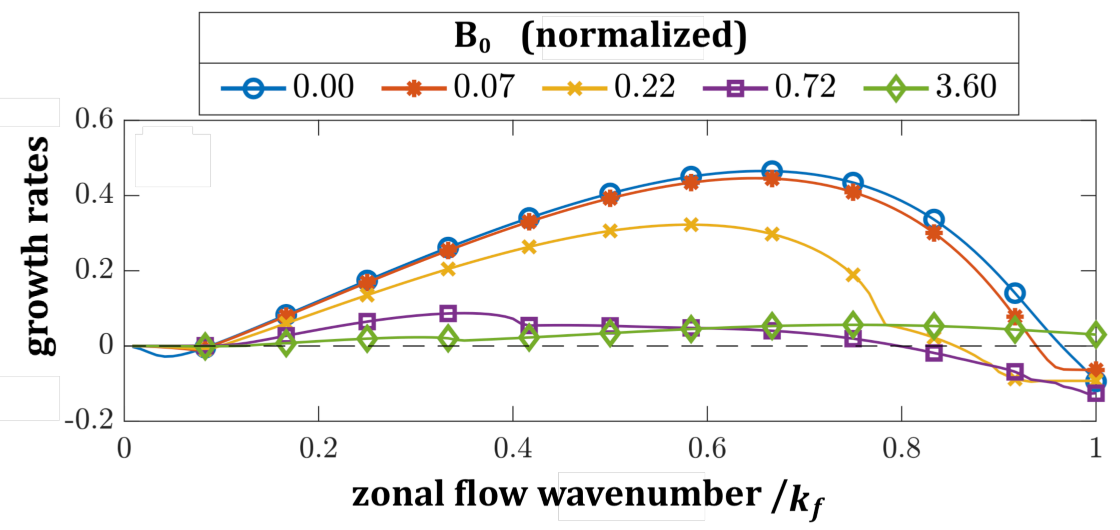
More elaborate: Jet-forming instability using the inhomogeneous statistical turbulent closure at 2nd order (CE2)

- from (1)-(2) $\longrightarrow$ derive **dynamics** for the same-time flow statistics:

$$\underbrace{\bar{\psi}(\boldsymbol{x})}_{\text{mean flow}}\ ,\ \underbrace{\overline{\psi'(\boldsymbol{x}_1)\psi'(\boldsymbol{x}_2)}}_{\substack{\text{2nd-order}\\ \text{eddy statistics}}}\ ,\ \underbrace{\overline{\psi'(\boldsymbol{x}_1)\psi'(\boldsymbol{x}_2)\psi'(\boldsymbol{x}_3)}}_{\substack{\text{3rd-order}\\ \text{eddy statistics}}}\ , \dots$$

- turbulent closure problem avoided $\longrightarrow$ neglect $\geq$ 3rd-order statistics
- the homogeneous turbulent state with no mean flow is a fixed point
- study the linear stability of the homogeneous ***turbulent*** state
  (we've linearized about a state of zero mean flow, zero mean magnetic field and homogeneous eddy flow and eddy magnetic fields)

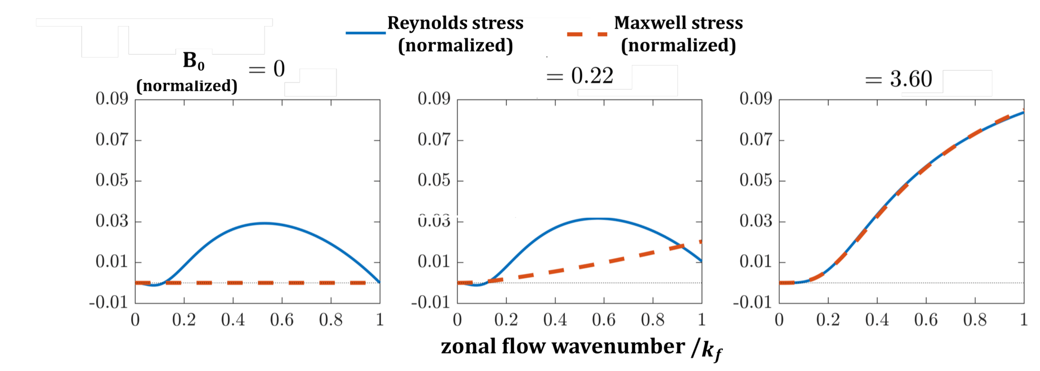
[Inhomogeneous statistical turbulent closure at 2nd order have been used to explain zonal flow formation in non-magnetized fluids; see, e.g., Farrell & Ioannou JAS (2003, 2007), Constantinou et al., JAS (2014, 2016), Marston et al., JAS (2008) Tobias et al, ApJ (2011); PRL (2013), Srinivasan & Young, JAS (2012, 2014), Parker & Krommes, PoP (2013), NJP (2014), Squire & Bhattacharjee, PRL (2015)]



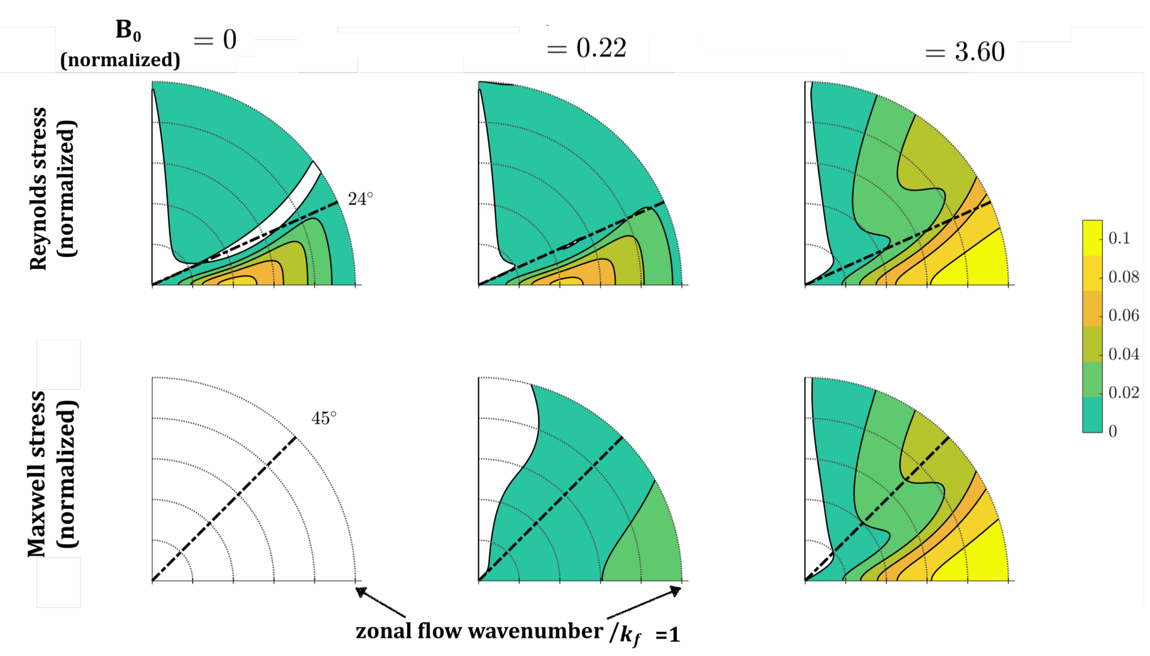
As B_0 increases the jet-forming instability is suppressed.

Mechanism of suppression

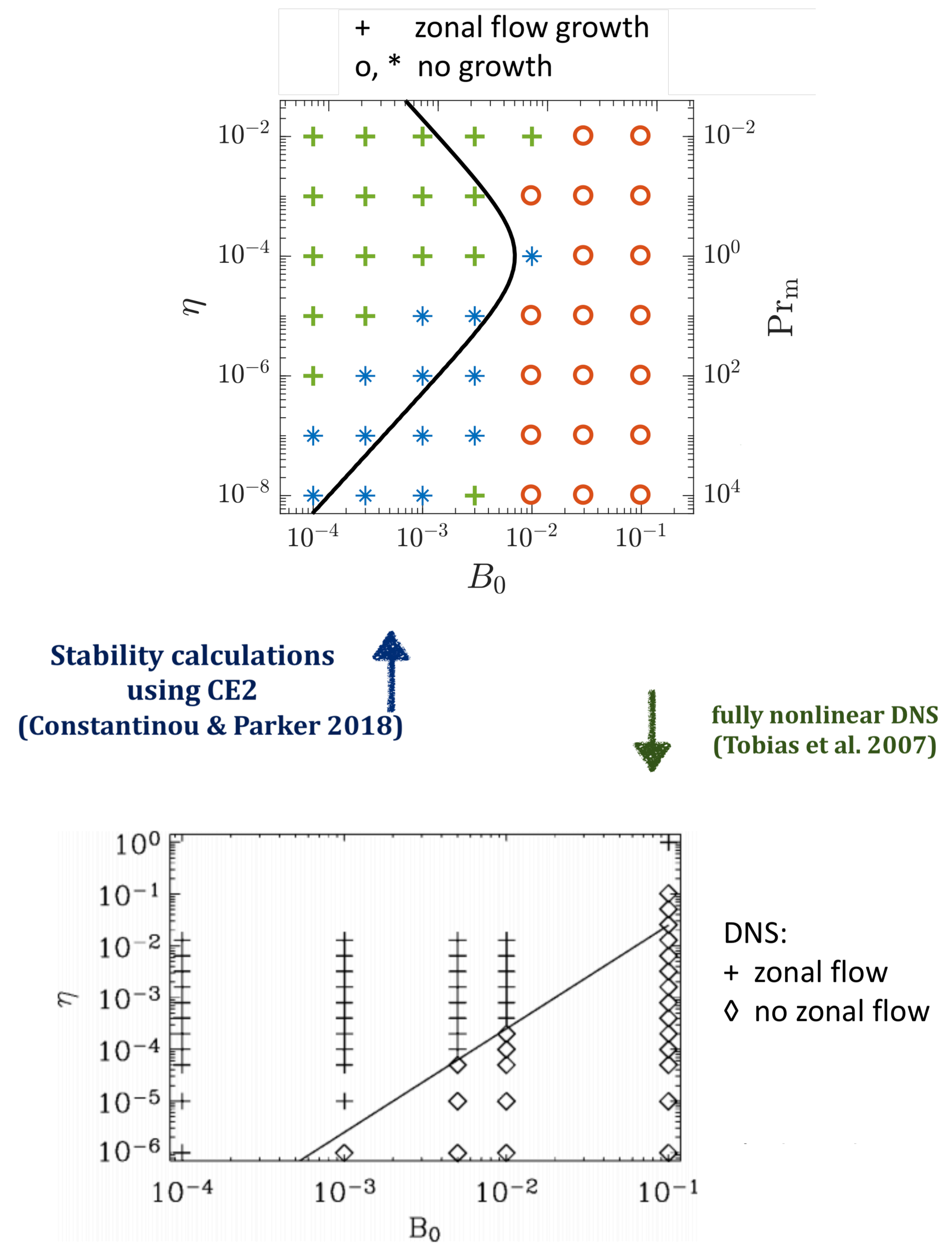
As B_0 increases, the Maxwell stress grows and cancels the reinforcing Reynolds stress.



The cancellation occurs for each spectral component of the eddy field:



CE2 instability results agree well with DNS



Conclusions

- Magnetic fluctuations can suppress zonal flow through the Maxwell stress when the resistivity is sufficiently small
  - Consistent with earlier results of Tobias et al. (2007, 2011)
  - Here, we found that the suppression can be explained quasilinearly, and even occurs with weak zonal flows, without requiring nonlinear effects.
  - The growth rate of zonal flow is suppressed.
- Results may explain the extent of the zonal jets in Jupiter or Saturn.

Constantinou & Parker (2018) Magnetic suppression of zonal flows on a beta plane. *The Astrophysical Journal*, **863**, 46

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