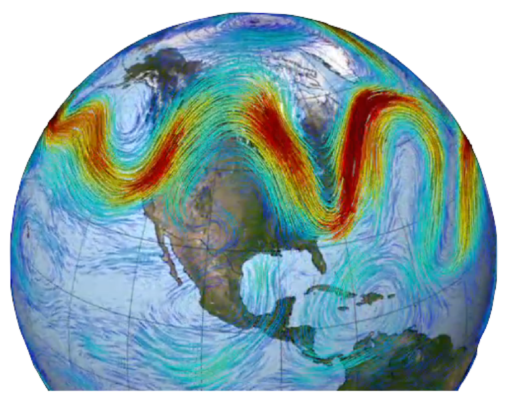
A statistical state dynamics based theory for jet-wave coexistence in beta-plane turbulence

Navid C. Constantinou (UCSD, navid@ucsd.edu) Brian F. Farrell (Harvard) Petros J. Ioannou (National & Kapodistrian University of Athens)

Motivation

Large-scale jets and planetary-scale ***coherent*** waves coexist in planetary turbulence.



NASA/Goddard Space Flight Center

Can this jet–wave coexistence regime exist merely as a consequence of the underlying dynamics and in the absence of topography?

Objectives

Develop a theory for the coexistence of zonal jets and planetary-scale waves in turbulent atmospheres.

- Can we study the stability of turbulent zonal jets?
- Do turbulent zonal jets become unstable to waves?
- How does such an instability equilibrate at finite amplitude?

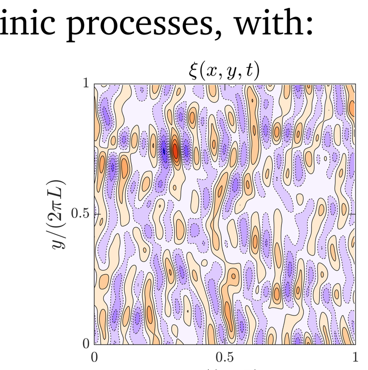
Model: barotropic QG flow on a beta-plane

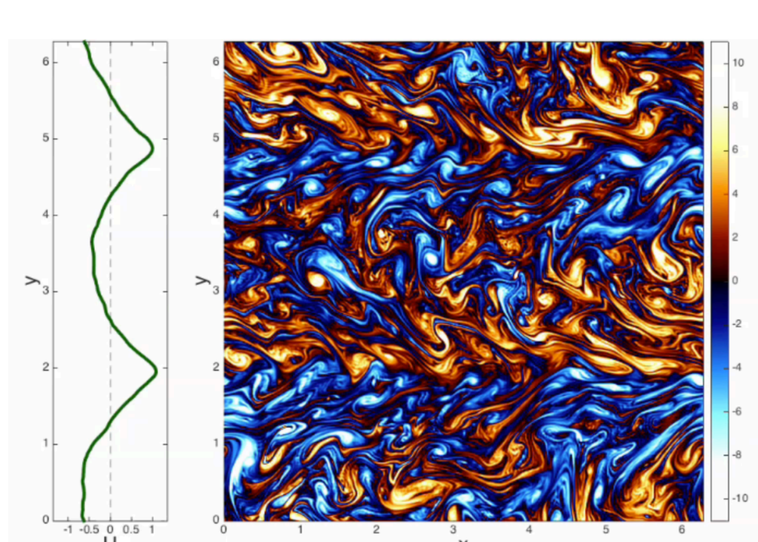
Single-layer (barotropic) quasi-geostrophic setting on a β-plane.
Doubly periodic domain of size $2\pi L \times 2\pi L$.
Flow $\boldsymbol{u} = (u, v)$ is given through streamfunction $\psi(\boldsymbol{x}, t)$ as $u = -\partial_y \psi$, $v = \partial_x \psi$.
Energy injected at small scales through stochastic excitation $\xi(\boldsymbol{x}, t)$.

$$\partial_t \nabla^2 \psi + \mathsf{J}(\psi, \nabla^2\psi) + \beta \partial_x \psi = \underbrace{-(r - \nu\nabla^2)\nabla^2\psi}_{\text{dissipation}} + \underbrace{\sqrt{\varepsilon}\,\xi}_{\text{forcing}} \quad (1)$$

- $\boldsymbol{x} \overset{\text{def}}{=} (x, y)$; x: zonal, y: meridional,
- $\mathsf{J}(a, b) \overset{\text{def}}{=} a_x b_y - a_y b_x$: Jacobian,
- r: Ekman drag coefficient,
- β: planetary vorticity gradient,
- ε: energy injection rate from ξ,
- $\xi(\boldsymbol{x}, t)$: models energy resulting from baroclinic processes, with:

typical length-scale $1/k_f \ll$ domain size,
zero mean $\langle \xi(\boldsymbol{x}, t)\rangle = 0$,
statistically homogeneous,
temporally δ-correlated; spatially correlated,
$\langle \xi(\boldsymbol{x}_a, t_1)\xi(\boldsymbol{x}_b, t_2)\rangle = Q(\boldsymbol{x}_a - \boldsymbol{x}_b)\delta(t_1 - t_2)$.





Model (1) exhibits turbulent zonal jets with embedded large-scale waves that propagate to the west.

Statistical State Dynamics (SSD)

Studying of the dynamics of the statistics (SSD) of the flow reveals instabilities that do not manifest in single flow realization dynamics.

(i) Decompose the flow into "mean" (coherent) + "eddies" (incoherent):

$$\psi(\boldsymbol{x}, t) = \underbrace{\bar\psi(\boldsymbol{x}, t)}_{\text{mean}} + \underbrace{\psi'(\boldsymbol{x}, t)}_{\text{eddy}} \quad (2)$$

Since we are interested in jet and waves, the "mean" should include zonal and non-zonal components.

$$\bar\psi(\boldsymbol{x}, t) \overset{\text{def}}{=} \mathcal{P}_K\psi = \sum_{|k_x \le K\,,\,k_y} \hat\psi(\boldsymbol{k}, t)\,\mathrm{e}^{\mathrm{i}(k_x x + k_y y)} \quad \text{(low zonal wavenumbers)}\,,$$

$$\psi'(\boldsymbol{x}, t) \overset{\text{def}}{=} (1 - \mathcal{P}_K)\psi = \sum_{\substack{|k_x|>K \\ k_y}} \hat\psi(\boldsymbol{k}, t)\,\mathrm{e}^{\mathrm{i}(k_x x + k_y y)} \quad \text{(low zonal wavenumbers)}\,.$$

(ii) Form the same-time, n-point cumulants:

$$C(\boldsymbol{x}_a, \boldsymbol{x}_a, t) \overset{\text{def}}{=} \langle \psi'(\boldsymbol{x}_a, t)\psi'(\boldsymbol{x}_b, t)\rangle\,,$$
$$\Gamma(\boldsymbol{x}_a, \boldsymbol{x}_b, \boldsymbol{x}_c, t) \overset{\text{def}}{=} \langle \psi'(\boldsymbol{x}_a, t)\psi'(\boldsymbol{x}_b, t)\psi'(\boldsymbol{x}_c, t)\rangle\,, \cdots\,.$$

(iii) Neglect 3rd-order cumulants and higher to obtain a ***closed***, ***autonomous***, ***deterministic*** system for the evolution of:

the mean flow (1st cumulant, $\bar\psi$)
&
the 2nd-order eddy statistics (2nd cumulant, C)

$$\partial_t \nabla^2\bar\psi + \mathcal{P}_K[\mathsf{J}(\bar\psi, \nabla^2\bar\psi)] + \beta\partial_x\bar\psi = \underbrace{\mathcal{P}_K\mathcal{R}(C)}_{\substack{\text{Reynolds stress}\\ \text{forcing}}} - (r - \nu\nabla^2)\nabla^2\bar\psi\,, \quad (3)$$

$$\partial_t C = (1 - \mathcal{P}_{Ka})\mathcal{A}_a C + (1 - \mathcal{P}_{Kb})\mathcal{A}_b C + \varepsilon\, Q\,, \quad (4)$$

where

$$\mathcal{R}(C) \overset{\text{def}}{=} -\langle \mathsf{J}(\psi', \nabla^2\psi')\rangle = -\tfrac{1}{2}\boldsymbol{\nabla}\boldsymbol{\cdot}\left[\hat{\mathbf{z}} \times (\boldsymbol{\nabla}_a\nabla_b^2 + \boldsymbol{\nabla}_b\nabla_a^2)C\right]_{a=b}\,,$$
$$\mathcal{A}(\bar\psi) \overset{\text{def}}{=} -\nabla^{-2}(\bar{\boldsymbol{u}}\boldsymbol{\cdot}\boldsymbol{\nabla})\nabla^2 - \nabla^{-2}[\beta\partial_x - (\nabla^2\bar{\boldsymbol{u}})\boldsymbol{\cdot}\boldsymbol{\nabla}] - r + \nu\nabla^2\,.$$

Equations (3)-(4) are referred to as the S3T system or the CE2 system.

[Farrell & Ioannou 2003; Marston, Conover, Schneider 2008]

SSD describes turbulent statistical flow equilibria

Equilibria comprise: a mean flow $\bar{\boldsymbol{u}}^e$ and an eddy covariance C^e.

For example, the homogeneous turbulent state:

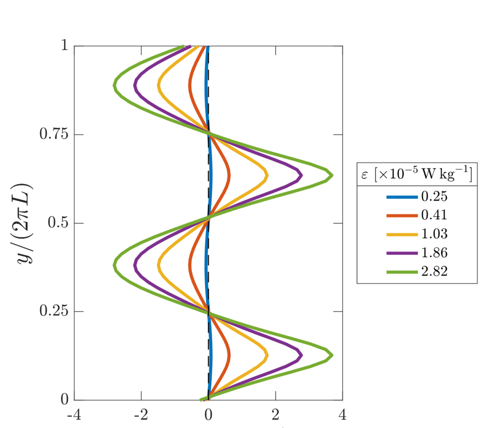
$$\underset{\text{no mean flow}}{\bar{\boldsymbol{u}}^e = (0, 0)} \quad \& \quad \underset{\text{homogeneous eddy statistics}}{C^e(\boldsymbol{x}_a - \boldsymbol{x}_b)} \quad (5)$$

[Farrell & Ioannou 2003, 2007; Srinivasan & Young 2012; Constantinou, Farrell & Ioannou 2014; Bakas & Ioannou 2011, 2013, 2014; Parker & Krommes 2013, 2014; Bakas, Constantinou & Ioannou 2015]

Here we are interested in equilibria:

$$\underset{\text{zonal jets}}{\bar{\boldsymbol{u}}^e = (\bar u^e(y), 0)} \quad \& \quad \underset{\text{inhomogeneous eddy statistics}}{C^e(x_a - x_b, y_a, y_b, t)} \quad (6)$$

$2\pi L = 1200\,\mathrm{km}$
typical time-scale $T = 6$ days
$\beta = 1.6 \times 10^{-11}\,\mathrm{m^{-1}s^{-1}}$
$r = 1/(40\,\text{days})$
$\nu = 2.7 \times 10^4\,\mathrm{m^2s^{-1}}$
we find, e.g., a series of 2-jet equilibria as we vary ε



+ associated turbulence
$C^e(x_a - x_b, y_a, y_b)$

Equilibria (5) or (6) are ***always hydrodynamically stable***:

$$\max(\mathrm{Re}\{\mathrm{eig}[\mathcal{A}(\bar\psi^e)]\}) < 0\,.$$

The interaction of the turbulence with mean flow through the Reynolds stress forcing term $\mathcal{R}(C)$ may render (5) or (6) ***unstable***!

SSD addresses the stability of turbulent flow equilibria

Perturbations $\delta\bar\psi$ and δC about equilibria (5) or (6) and satisfy the linearized version of (3)-(4):

$$\partial_t\delta\bar\psi = \mathcal{P}_K[\mathcal{A}^e\,\delta\bar\psi] + \underbrace{\nabla^{-2}\mathcal{P}_K\mathcal{R}(\delta C)}_{\substack{\text{Reynolds stress}\\ \text{coupling}}}\,, \quad (7)$$

$$\partial_t\delta C = (1 - \mathcal{P}_{Ka})[\delta\mathcal{A}_a C^e + \mathcal{A}_a^e\delta C] + (1 - \mathcal{P}_{Kb})[\delta\mathcal{A}_b C^e + \mathcal{A}_b^e\delta C]\,, \quad (8)$$

where $\mathcal{A}^e \overset{\text{def}}{=} \mathcal{A}(\bar\psi^e)$ and $\delta\mathcal{A} \overset{\text{def}}{=} \mathcal{A}(\bar\psi^e + \delta\bar\psi) - \mathcal{A}(\bar\psi^e)$.
Eigenanalysis of the resulting system determines the stability of ***turbulent statistical equilibria***.
With (7)-(8) we can study the ***stability*** of equilibria (5) or (6) that consists both of a mean flow and eddy statistics.

Mean flow perturbations eigenfunctions are either:

- zonal jet: $\delta\bar\psi(y)\,\mathrm{e}^{\sigma t}$, or
- large-scale wave: $\delta\bar\psi(y)\mathrm{e}^{\mathrm{i}k_x x}\mathrm{e}^{\sigma t}$.

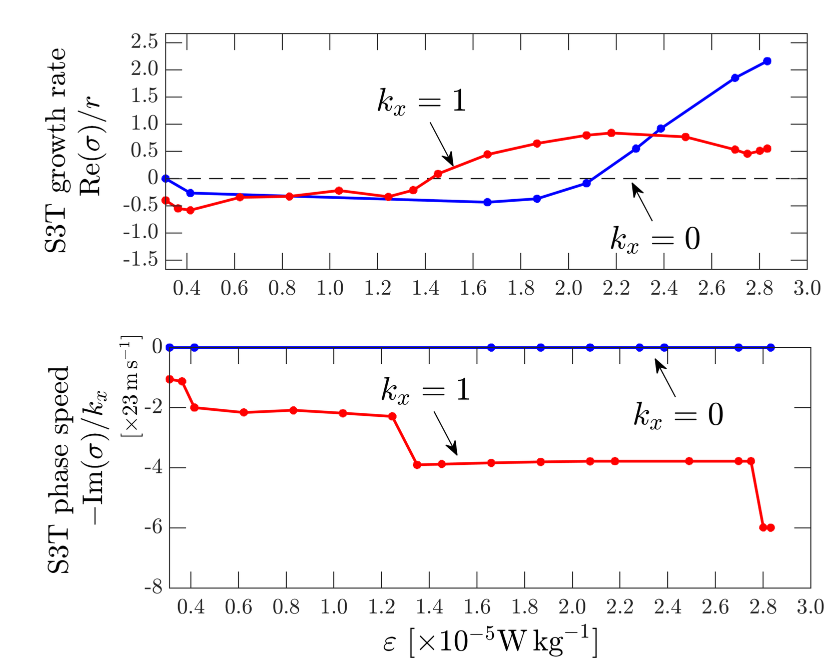
Classical hydrodynamic stability of the jets $\bar\psi^e(y)$ is a subset of (7)-(8) (when we do not allow perturbations in the eddy statistics $(\delta\bar\psi, \delta C = 0)$).

The Reynolds stress coupling term may induce instability when the general mean flow–turbulence perturbations $(\delta\bar\psi, \delta C)$ are considered.

$$\partial_t\delta\bar\psi = \mathcal{P}_K[\mathcal{A}^e\,\delta\bar\psi]$$

The stability of the 2-jet turbulent equilibria

What is the stability of the 2-jet turbulent equilibria (6) as ε increases?



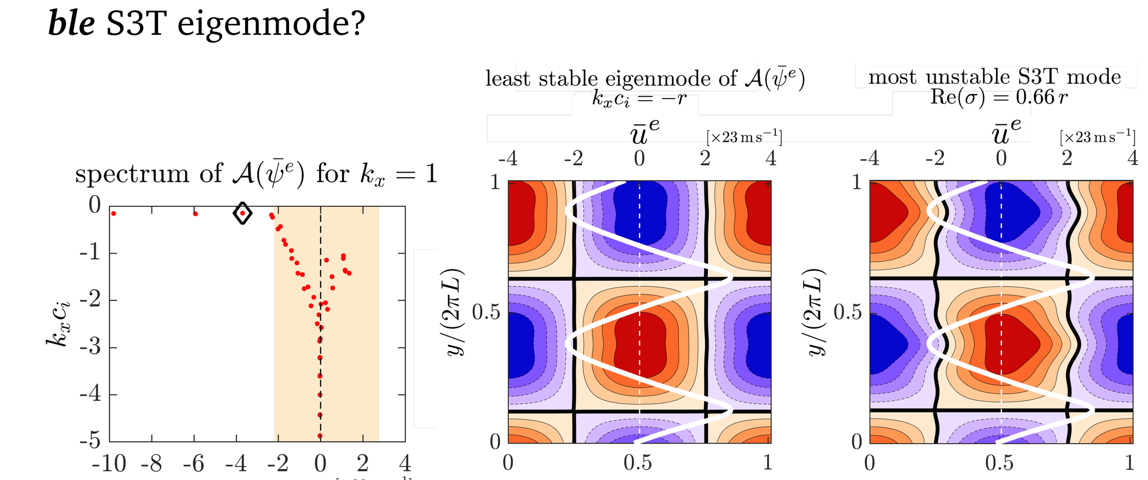
The jets first become unstable to large-scale wave mean flow perturturbations $\delta\bar\psi(y)\,\mathrm{e}^{\mathrm{i}k_x x}$.

A closer look at the 2-jet equilibrium at $\varepsilon = 1.86 \times 10^{-5}\,\mathbf{W\,kg^{-1}}$

The jets at $\varepsilon = 1.86 \times 10^{-5}\,\mathrm{W\,kg^{-1}}$ are:

- ***stable*** in the classical hydrodynamic sense,
- ***stable*** to $(\delta\bar\psi(y), \delta C)$ with ***zonal*** mean flow perturbations, $k_x = 0$,
- ***unstable*** to $(\delta\bar\psi(y)\mathrm{e}^{\mathrm{i}k_x x}, \delta C)$ with ***non-zonal*** mean flow perturbations, $k_x = 1$.

How does the least stable eigenmode of $\mathcal{A}(\bar\psi^e)$ compare with the ***unstable*** S3T eigenmode?



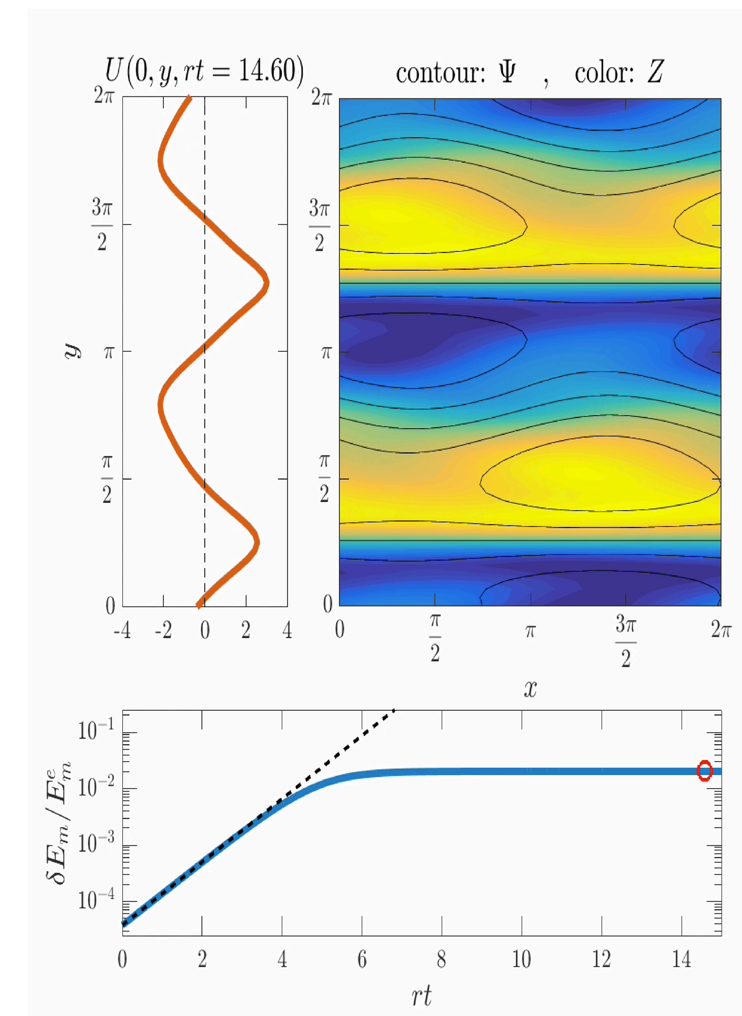
Large-scale $k_x = 1$ waves that are damped modes of $\mathcal{A}(\bar\psi^e)$ in the laminar jet, are transformed into exponentially growing waves by interaction with the incoherent small-scale turbulence.

This interaction results in a change in the mode structure allowing the mode to tap the energy of the mean jet.

Equilibration of the wave instability

But the applicability of SSD does not stop at predicting the instability...

SSD dynamics further predict how the turbulent zonal jet equilibrium (6) evolves to a time-depentent turbulent state with a mean flow consisting of a large-scale wave embedded in the 2-jet mean flow.



Conclusions

- Planetary turbulence may ***bifurcate*** from a state with

zonal jets & turbulence → to a state with → zonal jets, large-scale waves & turbulence

- These large-scale waves are equilibrated external Rossby waves destabilized by the turbulence.
- We provide a new mechanism for understanding planetary-scale waves in the atmosphere. This theory may provide explanation for the existence of the ovals that are embedded in the turbulent jets of the outer planets (e.g. Jupiter).

Journal reference

Constantinou, Farrell and Ioannou (2016). Statistical state dynamics of jet—wave coexistence in barotropic beta-plane turbulence. ***J. Atmos. Sci.*, 73 (5)**, 2229-2253, doi:10.1175/JAS-D-15-0288.1

Acknowledgements

NOAA Climate & Global Change Postdoctoral Fellowship Program
National Science Foundation, grant AGS-1246929