Magnetic eddy viscosity of mean shear flows in 2D magnetohydrodynamics: possible application to gas giants' interiors

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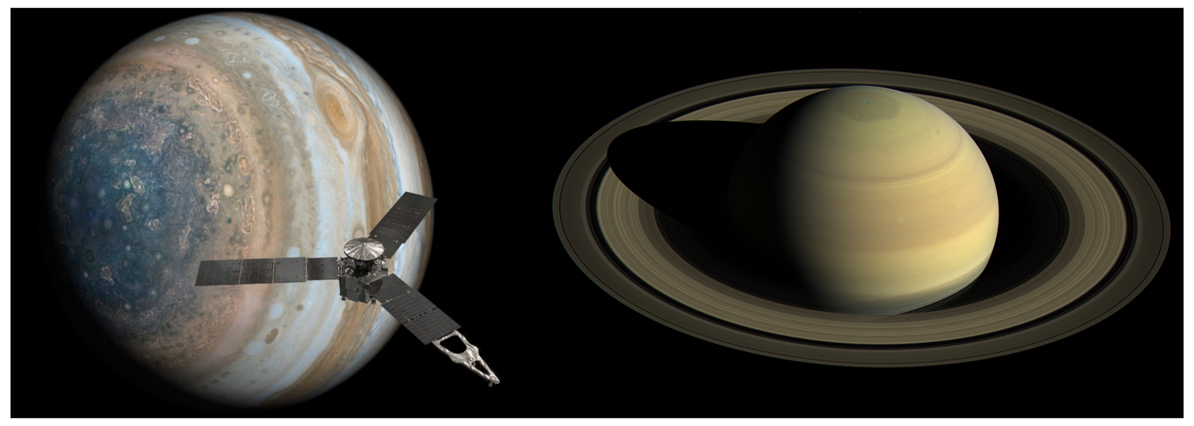
Magnetic induction in MHD causes **drag** for magnetic Reynolds number $\mathrm{Rm} \ll 1$.

We show that induction by a mean shear flow for $\mathrm{Rm} \gg 1$ leads to an effective **magnetic eddy viscosity** acting on zonal flow.

Magnetic eddy viscosity leads to transport of angular momentum and may be of importance to zonal flows in the interior of gas giants.

Depth of Jupiter's and Saturn's zonal flows



[NASA/JPL]

Juno found that Jupiter's jets go as deep as ~3,000 km below the clouds. Similarly, *Cassini* found that Saturn's jets reach depth of ~8,500 km.

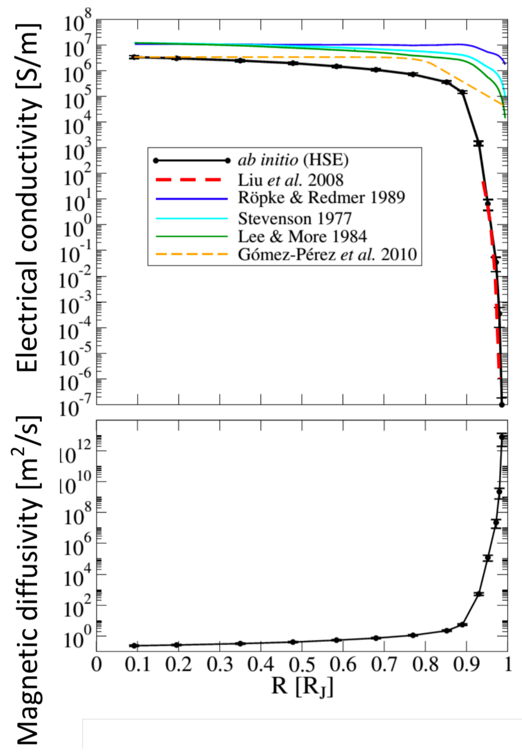
[Kaspi et al. 2018, Guillot et al. 2018, Iess et al. 2019, Galanti et al. 2019]

Why the jets stop at that depth?

Inside Gas Giants fluid becomes ionized and magnetic fields are strong

Jupiter:

- Even before *Juno*, it was thought that the depth of zonal flow in Jupiter is set by magnetic effects.
- The conductivity of the fluid goes up 11 orders of magnitude within the first 10% of the radius (due to the increasing pressure and ionization).
- Jupiter's planetary magnetic field can start being important.

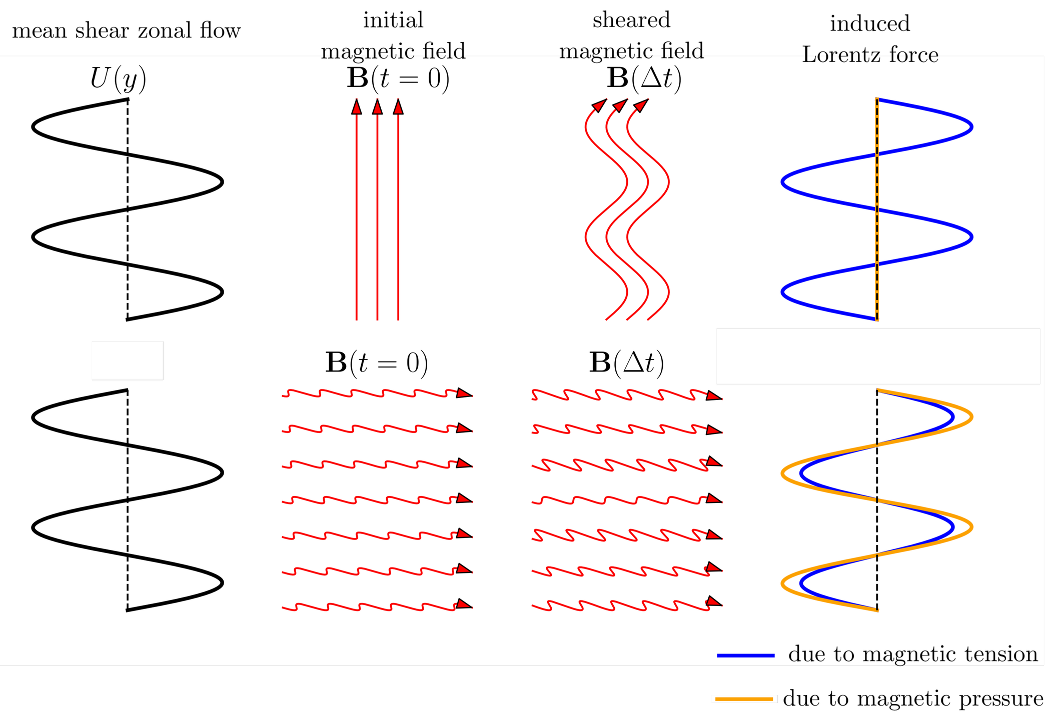


[French et al., *ApJ Supp. S.* (2012)]

Objectives

- Study the effect of mean shear flows on the induced Lorentz force in a turbulent MHD flow.
- Discuss potential application in gas giants' interior.

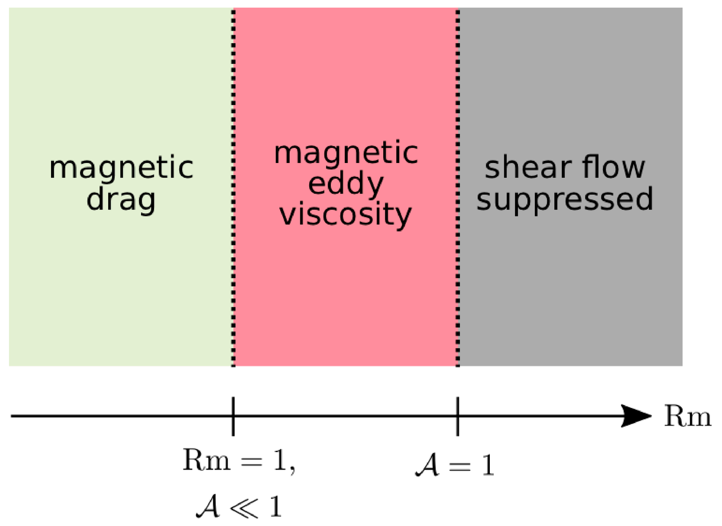
Lorentz force induced by shearing of magnetic field



[Magnetic response to a mean shear flow expected in turbulent regime]

Parameter diagram

$$\mathcal{A} = \frac{|\boldsymbol{J}\times\boldsymbol{B}|}{|\rho\boldsymbol{v}\cdot\boldsymbol{\nabla}\boldsymbol{v}|} = \frac{\text{Lorentz force}}{\text{inertial force}} \left(\approx \frac{\text{magnetic energy}}{\text{kinetic energy}}\right)$$



For Rm > 1 but while still $\mathcal{A} \ll 1$ (dominated by hydrodynamic fluctuations), the magnetic fields induce an *effective magnetic viscosity* rather than a *magnetic drag*.

Magnetic eddy viscosity (and Negative eddy viscosity)

Zonal mean–eddy decomposition: $f = \underbrace{\overline{f}}_{\text{zonal mean}} + f'$

$$\underbrace{\boldsymbol{F}_{\mathrm{L}} = \boldsymbol{J}\times\boldsymbol{B}}_{\text{Lorentz force}} = \boldsymbol{\nabla}\cdot\underbrace{\left[\tfrac{1}{\mu_0}(B_iB_j - \tfrac{1}{2}B^2\delta_{ij})\right]}_{\text{Maxwell stress-tensor}} \Rightarrow \overline{F_{\mathrm{L}x}} = \tfrac{1}{\mu_0}\partial_y\big(\overline{B_xB_y}\big)$$

induction eq. for $\boldsymbol{\nabla}\cdot\boldsymbol{v}=0$: $\quad \partial_t\boldsymbol{B} = \boldsymbol{B}\cdot\boldsymbol{\nabla}\boldsymbol{v} - \boldsymbol{v}\cdot\boldsymbol{\nabla}\boldsymbol{B}$

$$\boldsymbol{B}(t=0)\ \&\ \boldsymbol{v} = U(y)\,\hat{\boldsymbol{x}} \quad \xrightarrow{\text{after }\Delta t} \quad \underbrace{\Delta\overline{B_xB_y}}_{=\overline{B_xB_y}(\Delta t)-\overline{B_xB_y}(t=0)} = \Delta t\,\overline{B_y^2}\,\partial_yU + \mathcal{O}(\Delta t^2)$$

This should be valid up to $\Delta t \approx \tau_{\mathrm{cor}}$ (turbulent decorrelation time):

$$\overline{F_{\mathrm{L},x}} \approx \partial_y\Big(\underbrace{\alpha\frac{\overline{B_y^2}}{\mu_0}\tau_{\mathrm{cor}}}_{\text{magnetic viscosity}}\,\partial_yU\Big) \qquad (1)$$

predicts Maxwell stress as a function of mean flow shear

Combining with turbulent eddy viscosity: $-\rho\partial_y\overline{uv} \approx \partial_y\big(\underbrace{-\gamma\rho\tau_{\mathrm{cor}}\overline{v^2}}_{\text{negative eddy viscosity}}\,\partial_yU\big)$:

$$\partial_t(\overline{\rho u})\big|_{\mathrm{turb}} = \partial_y\frac{\overline{B_xB_y}}{\mu_0} - \rho\partial_y\overline{uv} = \partial_y\Big[\Big(\alpha\frac{\overline{B_y^2}}{\mu_0} - \gamma\rho\overline{v^2}\Big)\tau_{\mathrm{cor}}\,\partial_yU\Big] \qquad (2)$$

2D MHD quasi-geostrophic flow on a beta plane

2D flow (from a streamfunction): $\boldsymbol{v} = \hat{\boldsymbol{z}}\times\boldsymbol{\nabla}\psi$

Coriolis $f_0 + \beta y$, relative vorticity $\zeta = \hat{\boldsymbol{z}}\cdot(\boldsymbol{\nabla}\times\boldsymbol{v})$

magnetic field: $\boldsymbol{B} = \underbrace{\boldsymbol{B}_0}_{\text{constant, imposed}} + \tilde{\boldsymbol{B}} = \boldsymbol{\nabla}\times(\underbrace{A\,\hat{\boldsymbol{z}}}_{\text{vector potential}})$

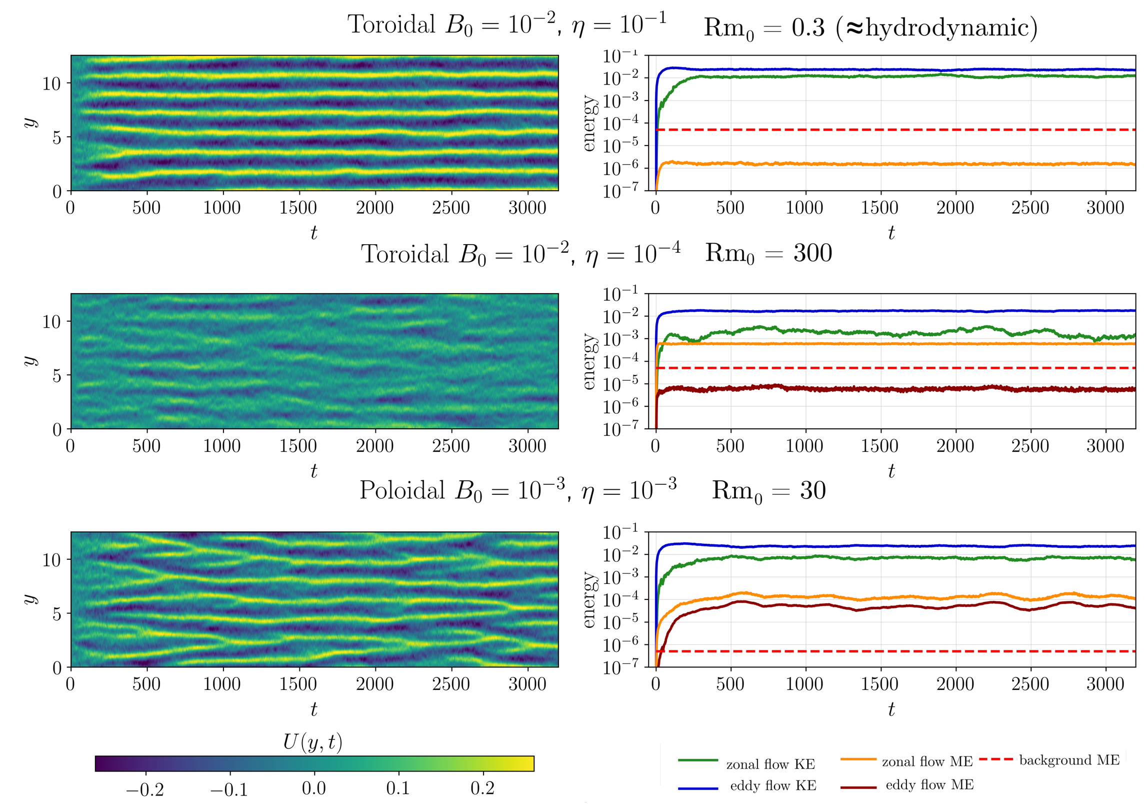
$A = (-B_{0y}\,x + B_{0x}\,y + \tilde{A})\,\hat{\boldsymbol{z}}$, current: $\boldsymbol{J} = \boldsymbol{\nabla}\times\boldsymbol{B} = -\nabla^2A\,\hat{\boldsymbol{z}}$

$$(\partial_t + \boldsymbol{v}\cdot\boldsymbol{\nabla})(\underbrace{\zeta+\beta y}_{\text{total vorticity}}) = \underbrace{-\boldsymbol{B}\cdot\boldsymbol{\nabla}\nabla^2A}_{\text{curl of Lorentz force}} \ \underbrace{-\kappa\zeta + \nu\nabla^2\zeta}_{\text{drag \& viscosity}} + \underbrace{\xi}_{\text{stochastic small-scale isotropic forcing}}$$

$$(\partial_t + \boldsymbol{v}\cdot\boldsymbol{\nabla})A = \underbrace{\eta\nabla^2A}_{\text{resistivity}}$$

- imposed $\boldsymbol{B}_0$: toroidal $\to \boldsymbol{B}_0 = B_0\,\hat{\boldsymbol{x}}$ / poloidal $\to \boldsymbol{B}_0 = B_0\,\hat{\boldsymbol{y}}$
- $4\pi\times4\pi$ domain – doubly-periodic boundary conditions
- $\beta = 10$, $\kappa = 10^{-2}$, $\nu = 10^{-4}$, $k_f = 12$ (forcing wavenumber)
- Reference magnetic Reynolds number $\mathrm{Rm}_0 = L_0V_0/\eta$
  L_0, V_0 length and velocity scales in hydrodynamic regime (large η)

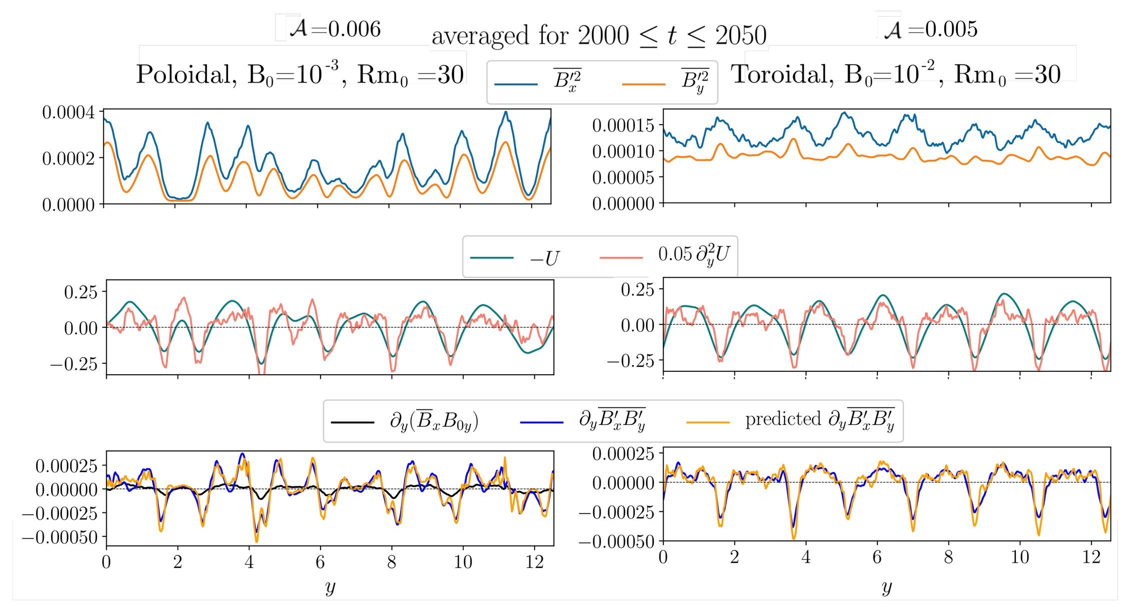
Magnetic fields disrupt zonation



As B_0 increases zonal flow is suppressed.

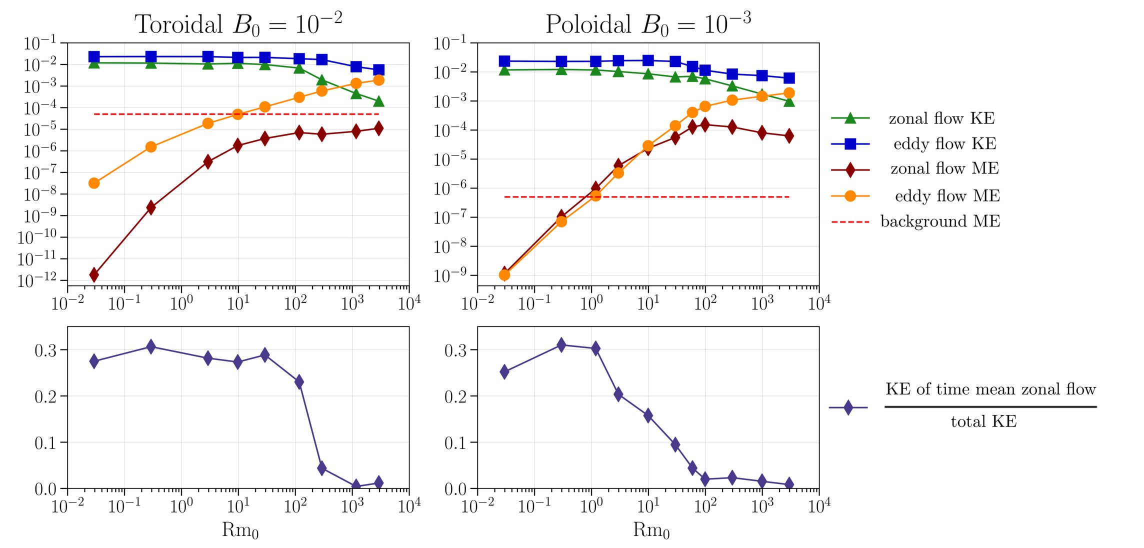
(Constantinou & Parker, *ApJ*, 2018)

How good is the Maxwell stress prediction eq. (1)?



Maxwell stress are well approximated as "magnetic viscosity" through eq. (1) [with $\alpha \approx 1$].

Results from 2D MHD simulations



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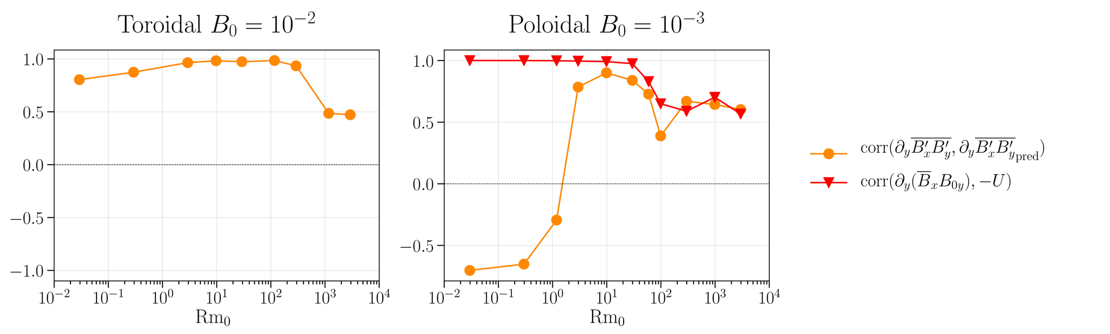
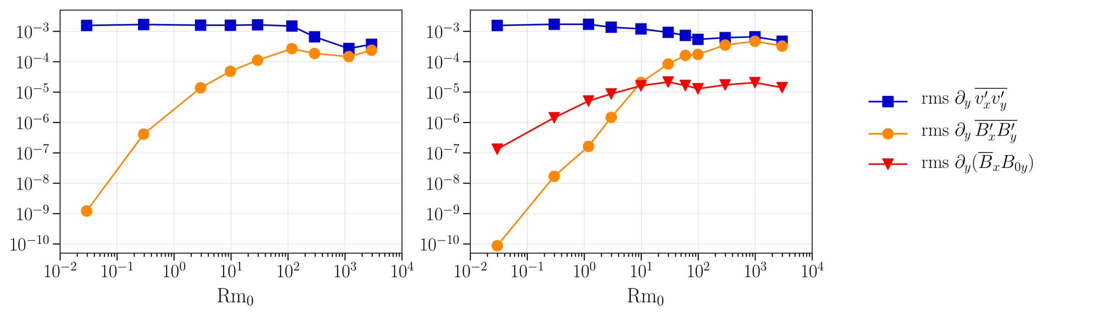


As we cross $\mathrm{Rm}_0 = 1$ the Lorentz force transitions from magnetic drag ($\propto -U$) to magnetic viscosity ($\propto \partial_y^2U$).



Magnetic viscosity cancels Reynold stresses, *not* magnetic drag.

Application to Jupiter's and Saturn's interiors

Given the flow at the outer atmosphere, how strong magnetic fluctuations we need so that magnetic viscosity cancels negative eddy viscosity (RHS of eq. (2) vanishes)?

Using *(i)* outer atmosphere typical scales, *(ii)* $\overline{B_y^2} \approx \frac{1}{2}\overline{B^2}$, *(iii)* estimates for mean magnetic field B_0, and *(iv)* $B^2 = B_0^2\,\mathrm{Rm}$ scaling we find:

critical: $\mathrm{Rm}_{\mathrm{Jupiter}} = 60$, $\mathrm{Rm}_{\mathrm{Saturn}} = 700$

Resistivity η is low enough for Rm to attain the above critical value at:

$0.96R_{\mathrm{Jupiter}} = 3500\,\mathrm{km}$ and $0.80R_{\mathrm{Saturn}} = 11\,000\,\mathrm{km}$

(see paper and refs therein)

Conclusions

- We show that for high Rm but in a regime dominated by hydrodynamic fluctuations ($\mathcal{A} \ll 1$), the magnetic fluctuations induce an effective magnetic viscosity on the mean flow.
- Magnetic viscosity mechanism is 3D (see paper); simulations presented here are 2D.
- Potential application in determining the depth of zonal flows in the interior of gas giants.

Parker & Constantinou (2019) Magnetic eddy viscosity of mean shear flows in 2D magnetohydrodynamics. (in review, arXiv:1902.01105)

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