# Structure and mechanism of turbulence under dynamical restriction in plane Poiseuille flow

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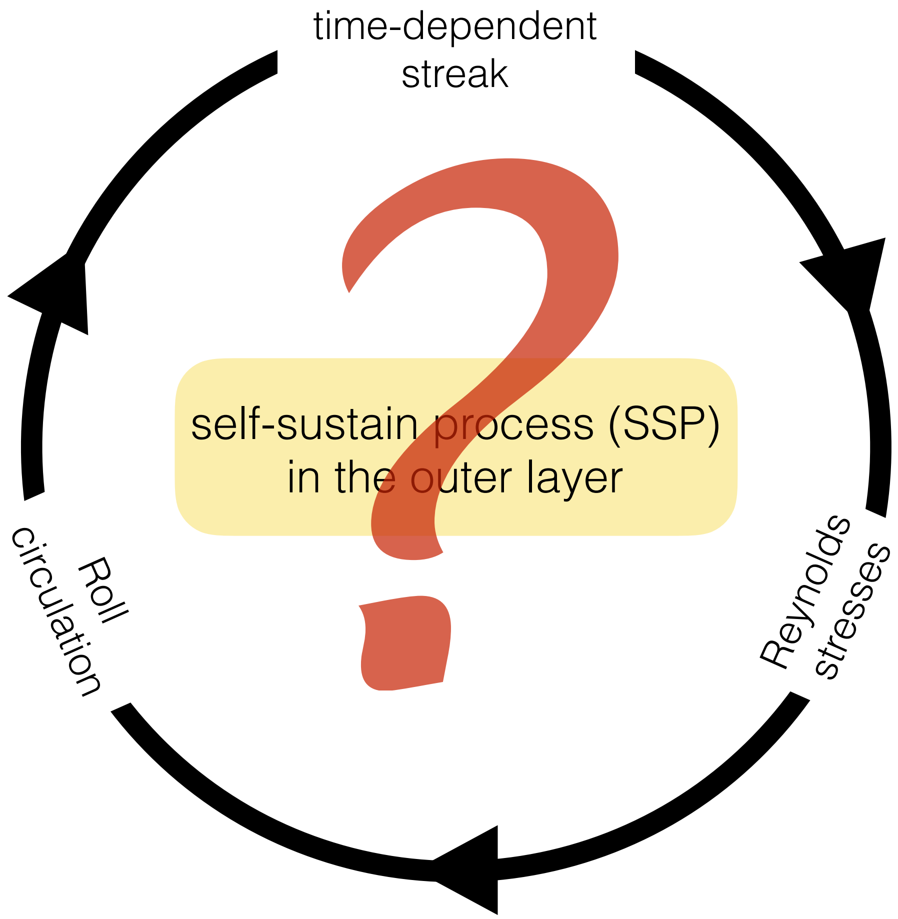
# Question

What maintains the very-large-scale-motions (VLSMs)
in wall-bounded turbulence *in the region away from the walls*?

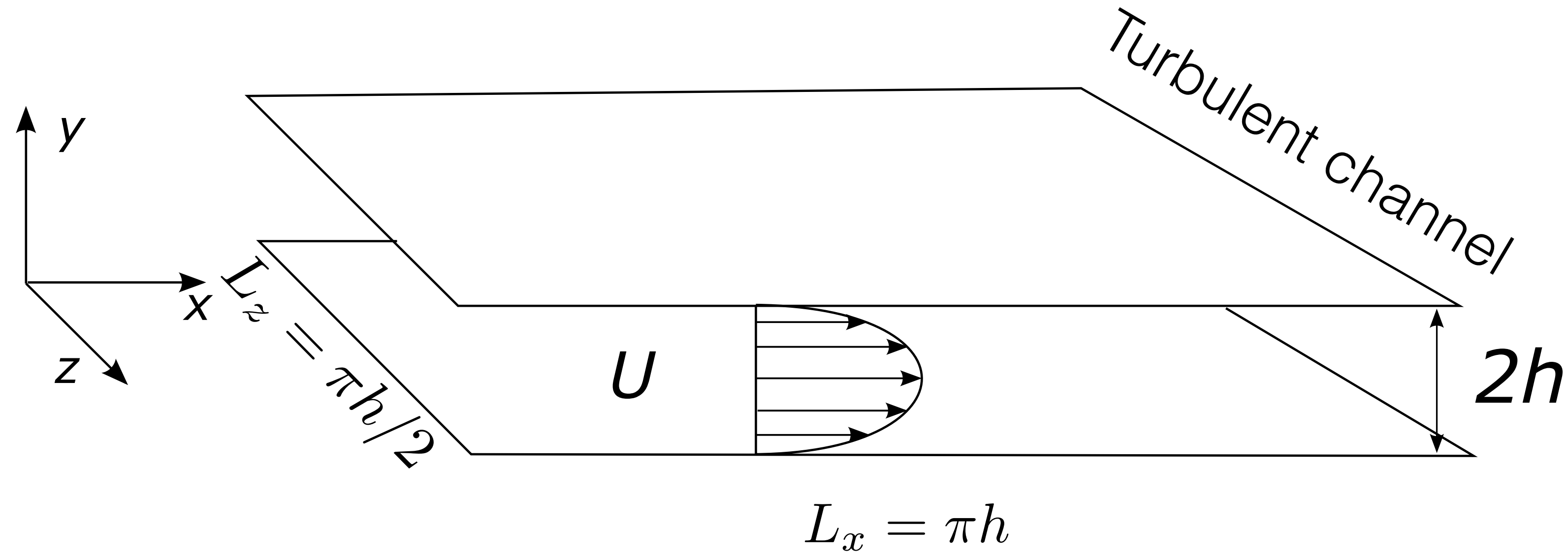
Does a self-sustain process exist *in the outer layer*?




# Strategy

Use the analytically tractable Reduced NonLinear (**RNL**) system.

# Flow setting




$$\boldsymbol{u}(x,y,z,t) \;=\; \bar{\boldsymbol{u}}(y,z,t) \;+\; \boldsymbol{u}'(x,y,z,t)$$

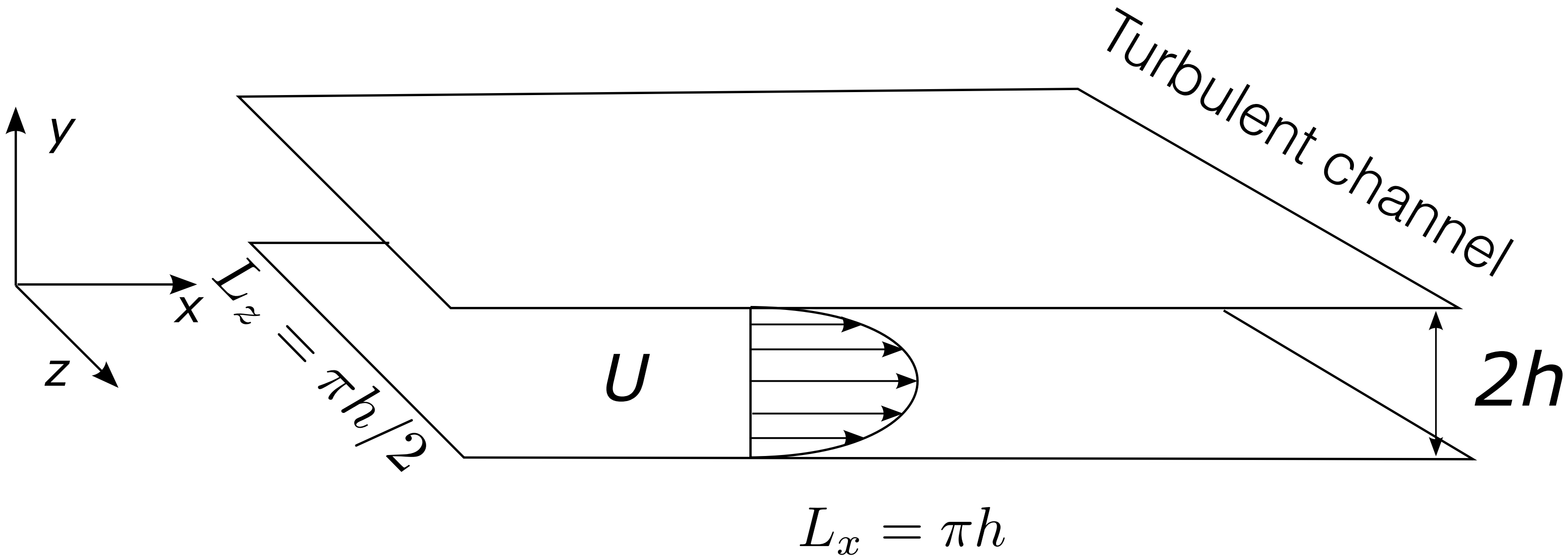
streamwise mean $+$ perturbations

$\bar{\boldsymbol{u}}$ → streak: $U_s = \bar{u} - \frac{1}{L_z}\int_0^{L_z} \bar{u}\,\mathrm{d}z$

$\bar{\boldsymbol{u}}$ → roll: $(\bar{v}, \bar{w})$

Navier-Stokes (**NS**) → two set of equations for $\bar{\boldsymbol{u}}$ and $\boldsymbol{u}'$

# Flow setting




$$\boldsymbol{u}(x,y,z,t) \;=\; \bar{\boldsymbol{u}}(y,z,t) \;+\; \boldsymbol{u}'(x,y,z,t)$$

streamwise mean $+$ perturbations

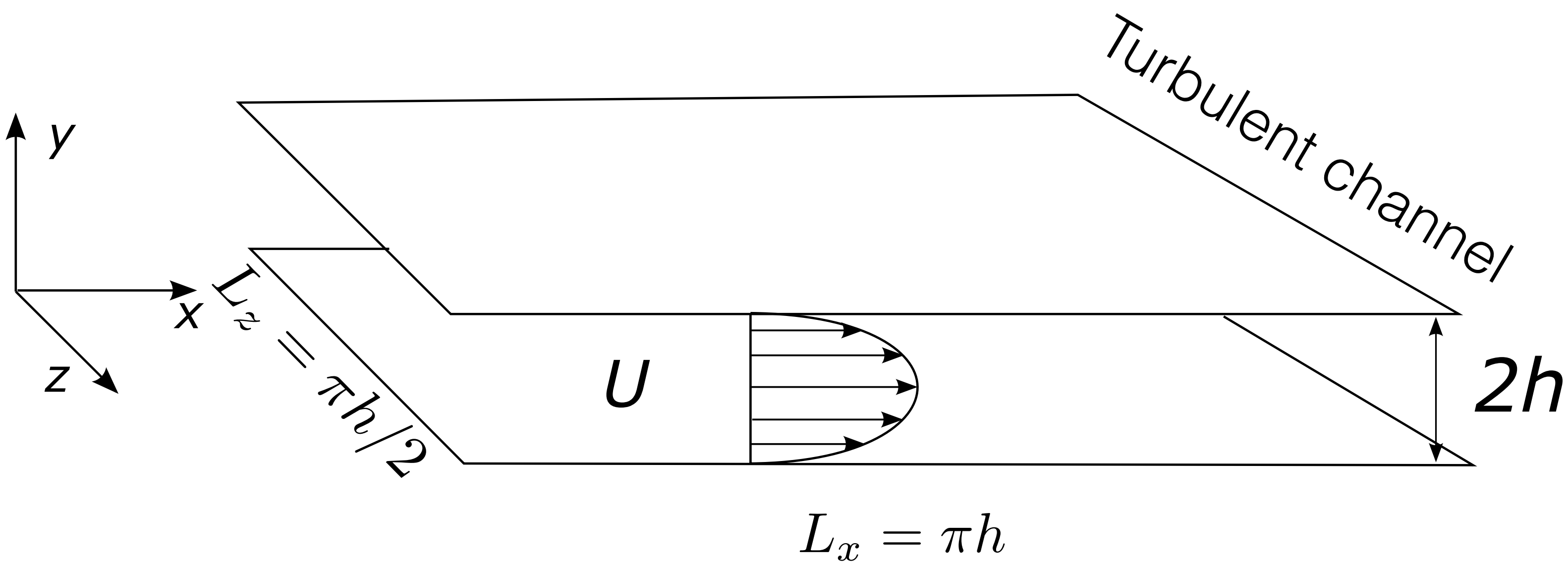
$$\bar{\boldsymbol{u}}_t + \bar{\boldsymbol{u}}\cdot\boldsymbol{\nabla}\bar{\boldsymbol{u}} + \boldsymbol{\nabla}\bar{p} - \nu\nabla^2\bar{\boldsymbol{u}} = -\overline{\boldsymbol{u}'\cdot\boldsymbol{\nabla}\boldsymbol{u}'}$$

$$\boldsymbol{u}'_t + \bar{\boldsymbol{u}}\cdot\boldsymbol{\nabla}\boldsymbol{u}' + \boldsymbol{u}'\cdot\boldsymbol{\nabla}\bar{\boldsymbol{u}} + \boldsymbol{\nabla}p' - \nu\nabla^2\boldsymbol{u}' = -\left(\boldsymbol{u}'\cdot\boldsymbol{\nabla}\boldsymbol{u}' - \overline{\boldsymbol{u}'\cdot\boldsymbol{\nabla}\boldsymbol{u}'}\right)$$

$$\boldsymbol{\nabla}\cdot\bar{\boldsymbol{u}} = \boldsymbol{\nabla}\cdot\boldsymbol{u}' = 0$$

Navier-Stokes
(**NS**)

# Flow setting




$$\boldsymbol{u}(x,y,z,t) \;=\; \bar{\boldsymbol{u}}(y,z,t) \;+\; \boldsymbol{u}'(x,y,z,t)$$

streamwise mean $+$ perturbations

$$\bar{\boldsymbol{u}}_t + \bar{\boldsymbol{u}}\cdot\boldsymbol{\nabla}\bar{\boldsymbol{u}} + \boldsymbol{\nabla}\bar{p} - \nu\nabla^2\bar{\boldsymbol{u}} = -\overline{\boldsymbol{u}'\cdot\boldsymbol{\nabla}\boldsymbol{u}'}$$

No perturbation cascades in ***x***!

$$\boldsymbol{u}'_t + \bar{\boldsymbol{u}}\cdot\boldsymbol{\nabla}\boldsymbol{u}' + \boldsymbol{u}'\cdot\boldsymbol{\nabla}\bar{\boldsymbol{u}} + \boldsymbol{\nabla}p' - \nu\nabla^2\boldsymbol{u}' = \cancel{-\left(\boldsymbol{u}'\cdot\boldsymbol{\nabla}\boldsymbol{u}' - \overline{\boldsymbol{u}'\cdot\boldsymbol{\nabla}\boldsymbol{u}'}\right)}$$

$$\boldsymbol{\nabla}\cdot\bar{\boldsymbol{u}} = \boldsymbol{\nabla}\cdot\boldsymbol{u}' = 0$$

Restricted NonLinear
(**RNL**)

# **RNL** sustains turbulence

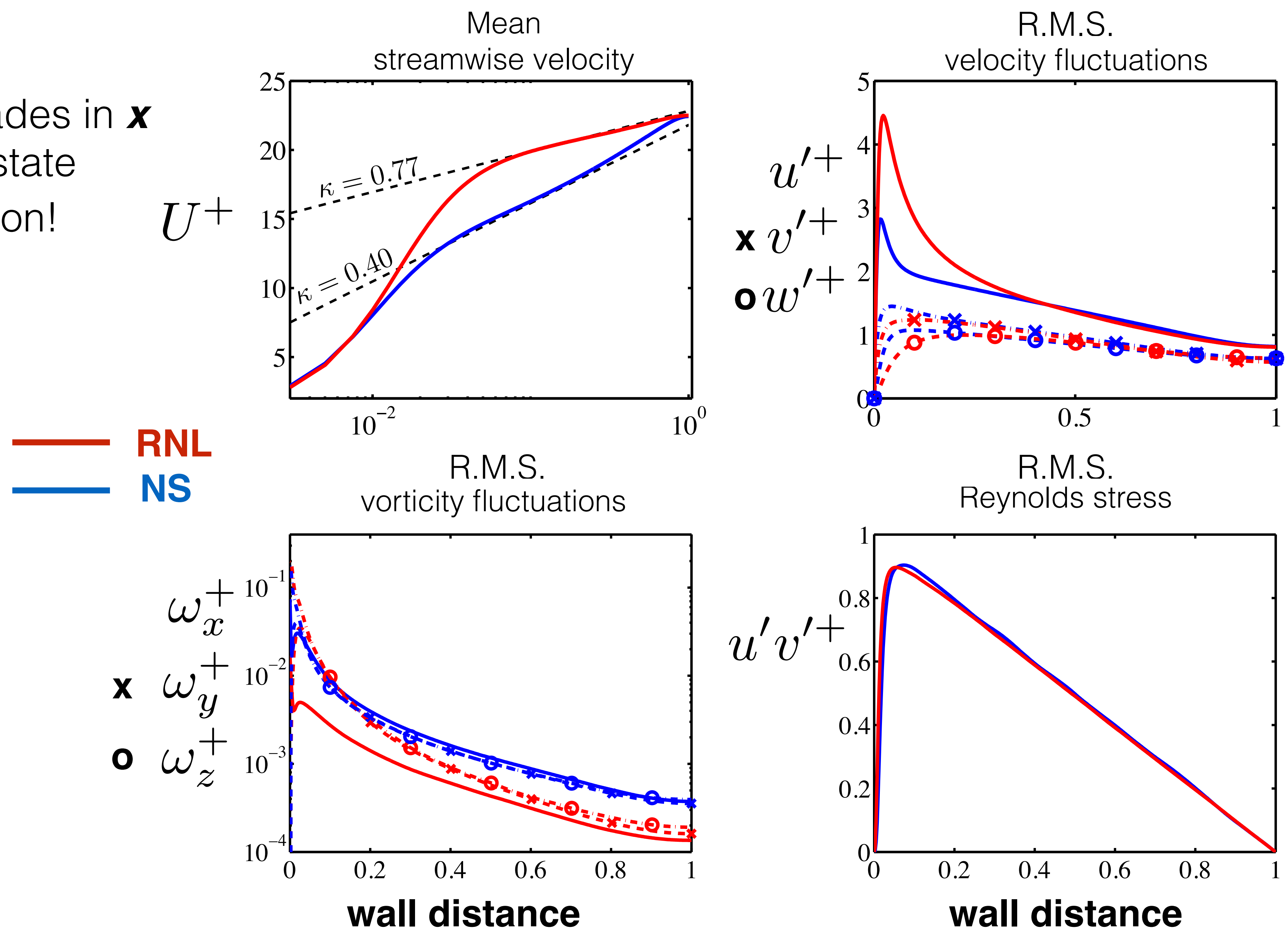
*Despite* the absence of cascades in ***x***
**RNL** sustains a turbulent state
*and* with ≈same dissipation!

Navier-Stokes
(**NS**)

$$Re_\tau = 940$$

Restricted NonLinear
(**RNL**)

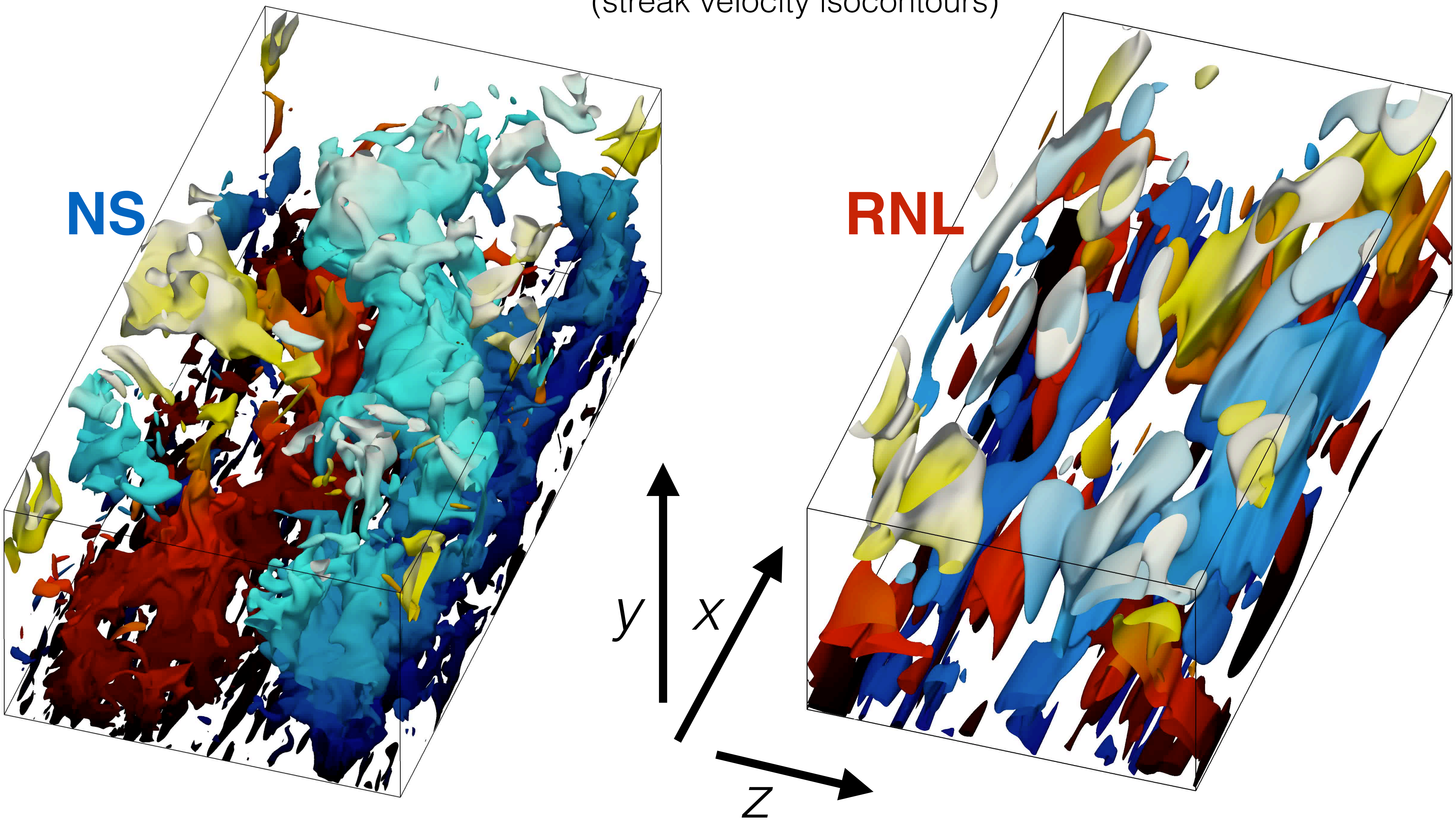
$$Re_\tau = 883$$



# RNL exhibits VLSMs

Streak $$U_s = \bar{u} - \frac{1}{L_z}\int_0^{L_z} \bar{u}\,\mathrm{d}z$$

(streak velocity isocontours)




**https://vimeo.com/175726749**

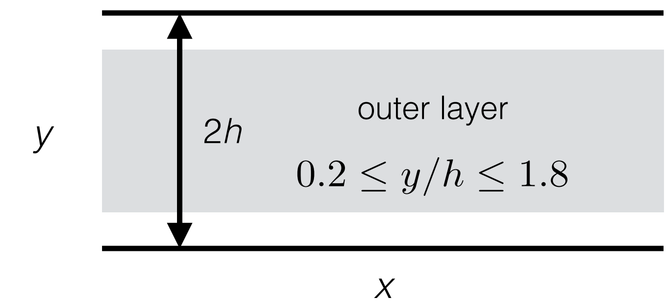
**https://vimeo.com/188194159**

Who supports the streak in the outer layer?

# Maintenance of the streak in the outer layer

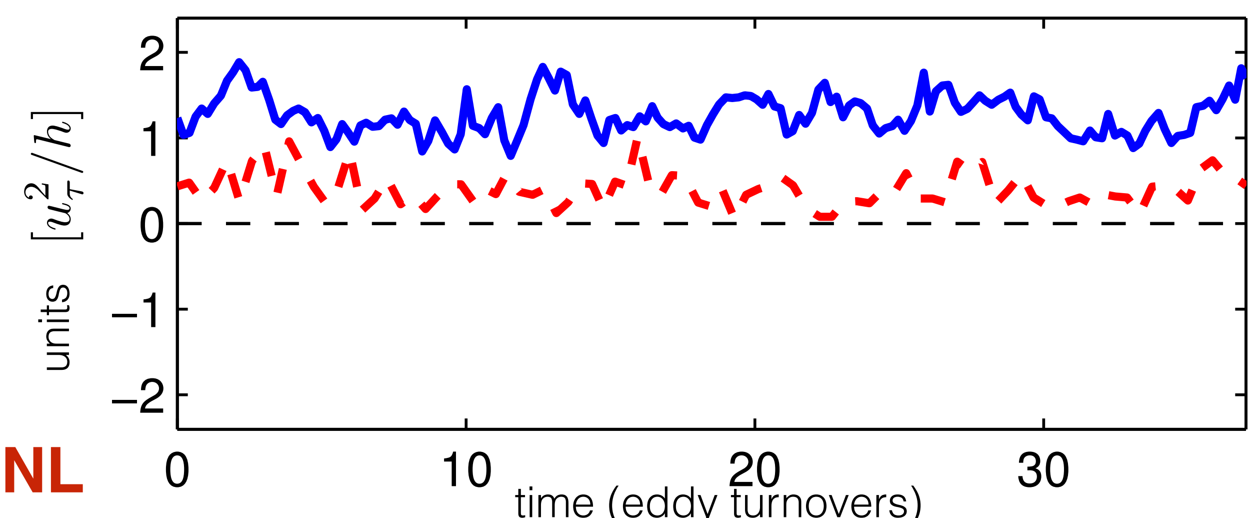
$$\frac{\partial |U_s|}{\partial t} = \left( \begin{array}{c} \text{lift-up} \\ \text{due to roll } (\bar{v},\bar{w}) \end{array} \right) + \boldsymbol{\nabla}\cdot \left( \begin{array}{c} \text{Reynolds stresses} \\ \text{due to } \boldsymbol{u}' \end{array} \right) + \left( \text{dissipation} \right)$$

averaged over the outer layer




lift-up

streak *gains* energy from the lift-up due to the roll

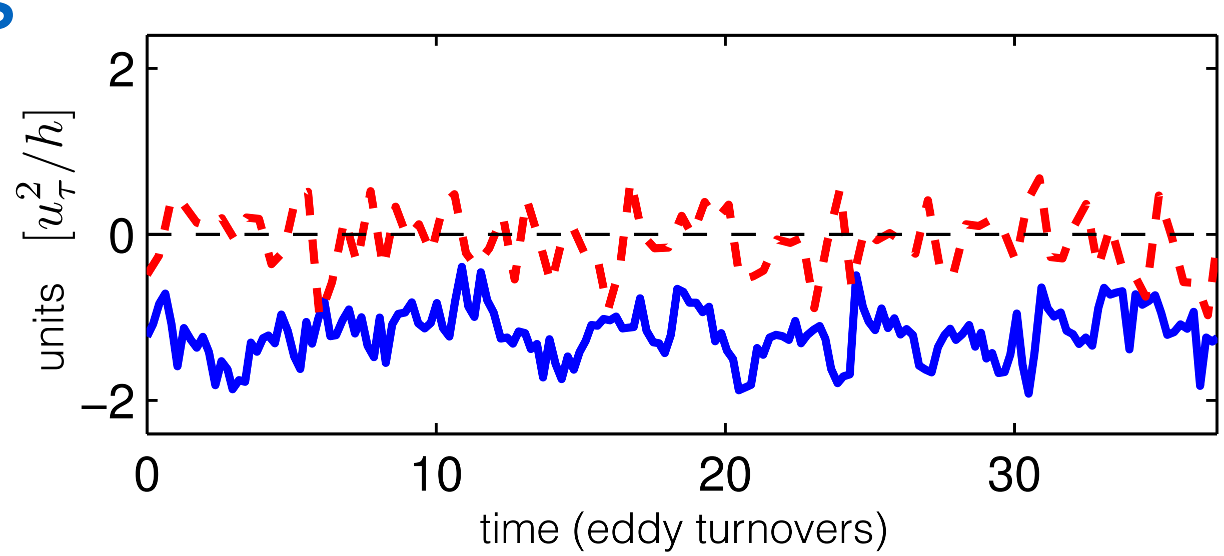



| | mean |
|---|---|
| NS | 1.9 |
| RNL | 0.4 |

RNL
NS

Reynolds stresses

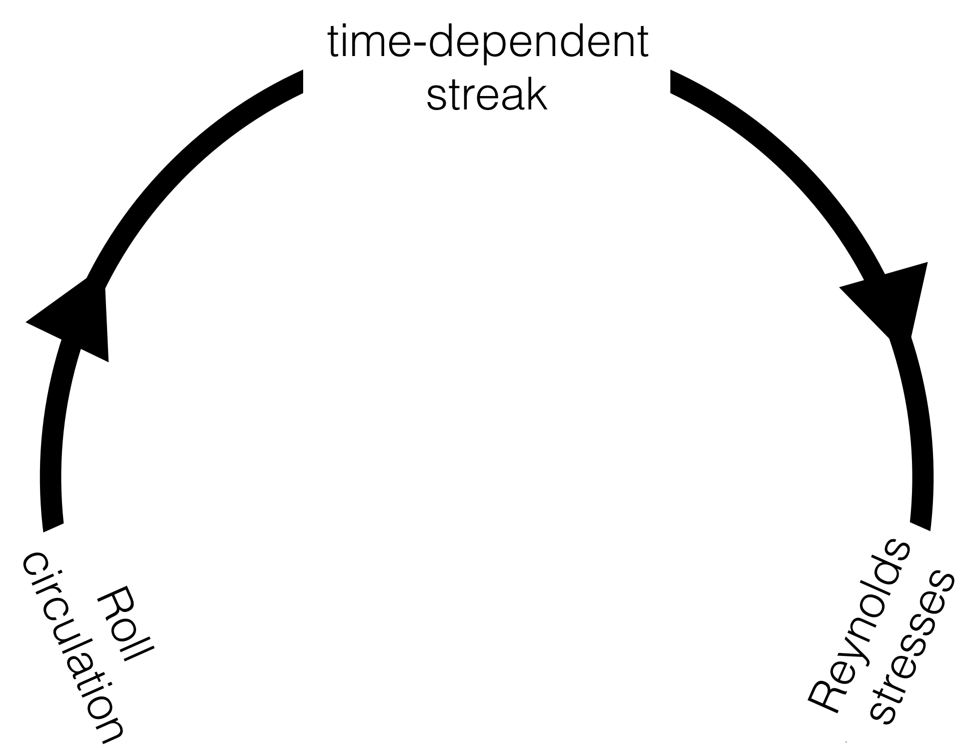
streak *looses* energy to perturbations




| | mean |
|---|---|
| NS | -1.2 |
| RNL | -0.1 |

# in the outer layer of both

Navier-Stokes (**NS**) & Restricted NonLinear (**RNL**)

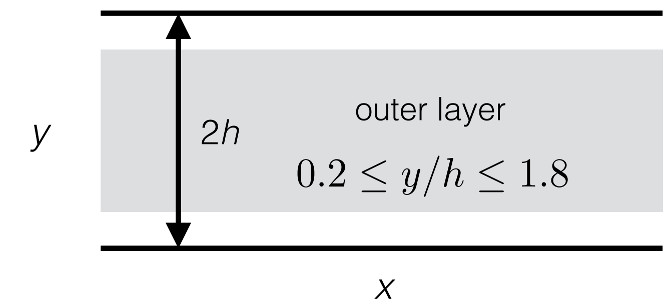



Who feeds the roll circulation in the outer layer?

# Maintenance of the roll in the outer layer

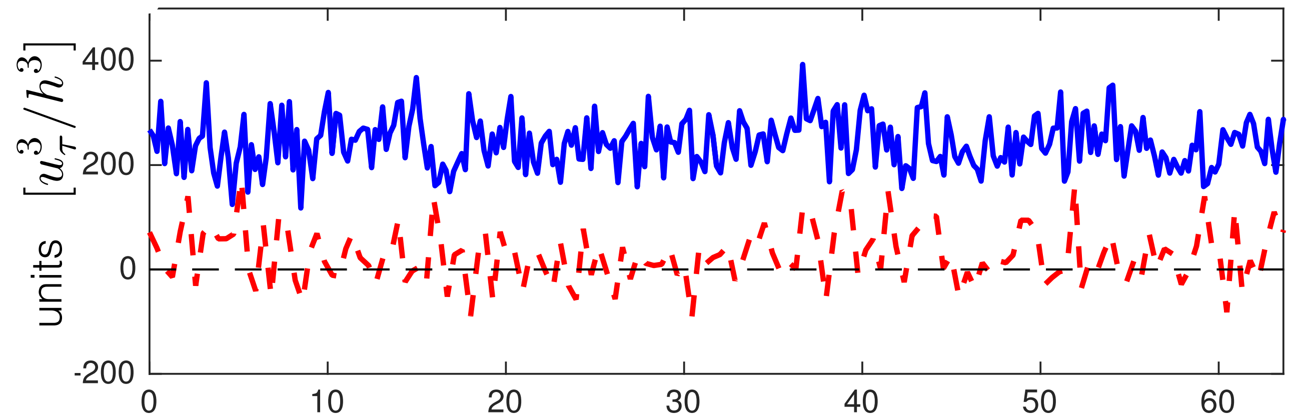
$$\boldsymbol{\omega} = \boldsymbol{\nabla} \times \boldsymbol{u}$$

within the outer layer $\quad Q = \displaystyle\int_{\substack{\text{outer layer}\\ \text{(gray area)}}} \tfrac{1}{2}\bar{\omega}_x^2 \,\mathrm{d}z\,\mathrm{d}y$




$$\frac{\mathrm{d}Q}{\mathrm{d}t} = \left(\begin{array}{c}\text{cross-stream advection}\\ \text{of } \overline{\omega_x}^2 \text{ into outer layer}\\ \text{due to roll } (\bar{v},\bar{w})\end{array}\right) + \left(\begin{array}{c}\text{Reynolds stress torque}\\ \text{production in outer layer}\\ \text{due to } \boldsymbol{u}'\end{array}\right) + \left(\text{dissipation}\right)$$

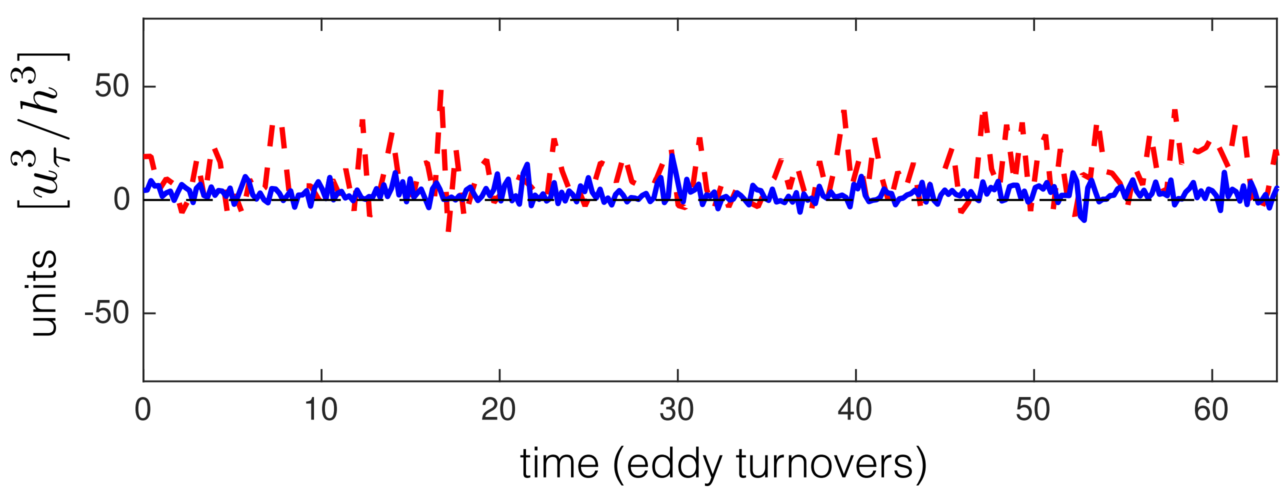
Reynolds stress torque production in outer layer




RNL

NS

cross-stream advection of $\overline{\omega_x}^2$ into outer layer

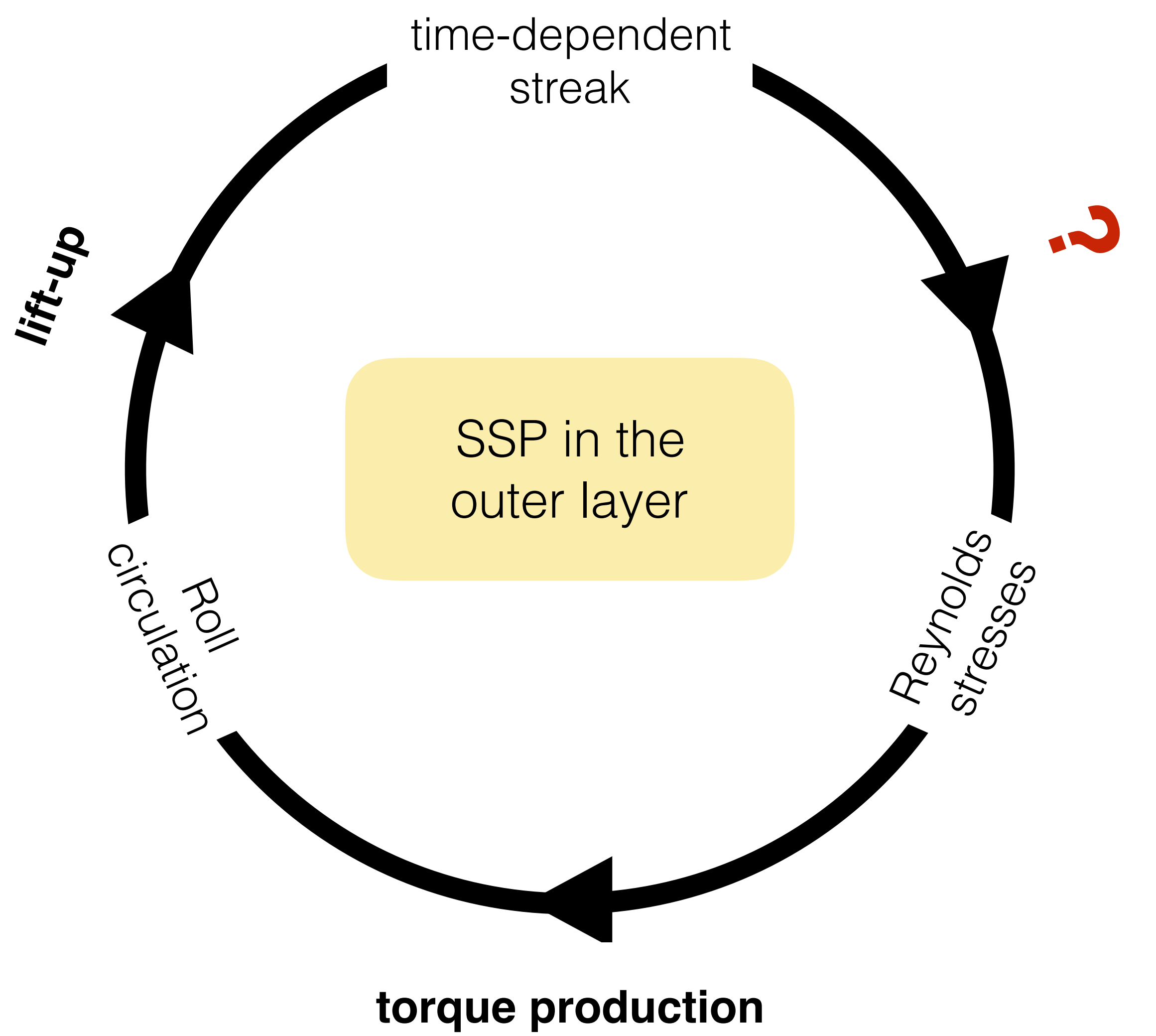



| | mean |
|---|---|
| NS | 242 |
| RNL | 29 |
| NS | 2.9 |
| RNL | 11 |

the roll in the outer layer is maintained by *in situ* torque production by Reynolds stresses

# in the outer layer of both

Navier-Stokes (**NS**) **&** Restricted NonLinear (**RNL**)



Within **RNL** we understand the regeneration cycle in the outer layer:

Farrell & Ioannou (2012), *J. Fluid Mech.*, **708**, 149-196

perturbations grow due to the parametric instability of the time-dependent streak

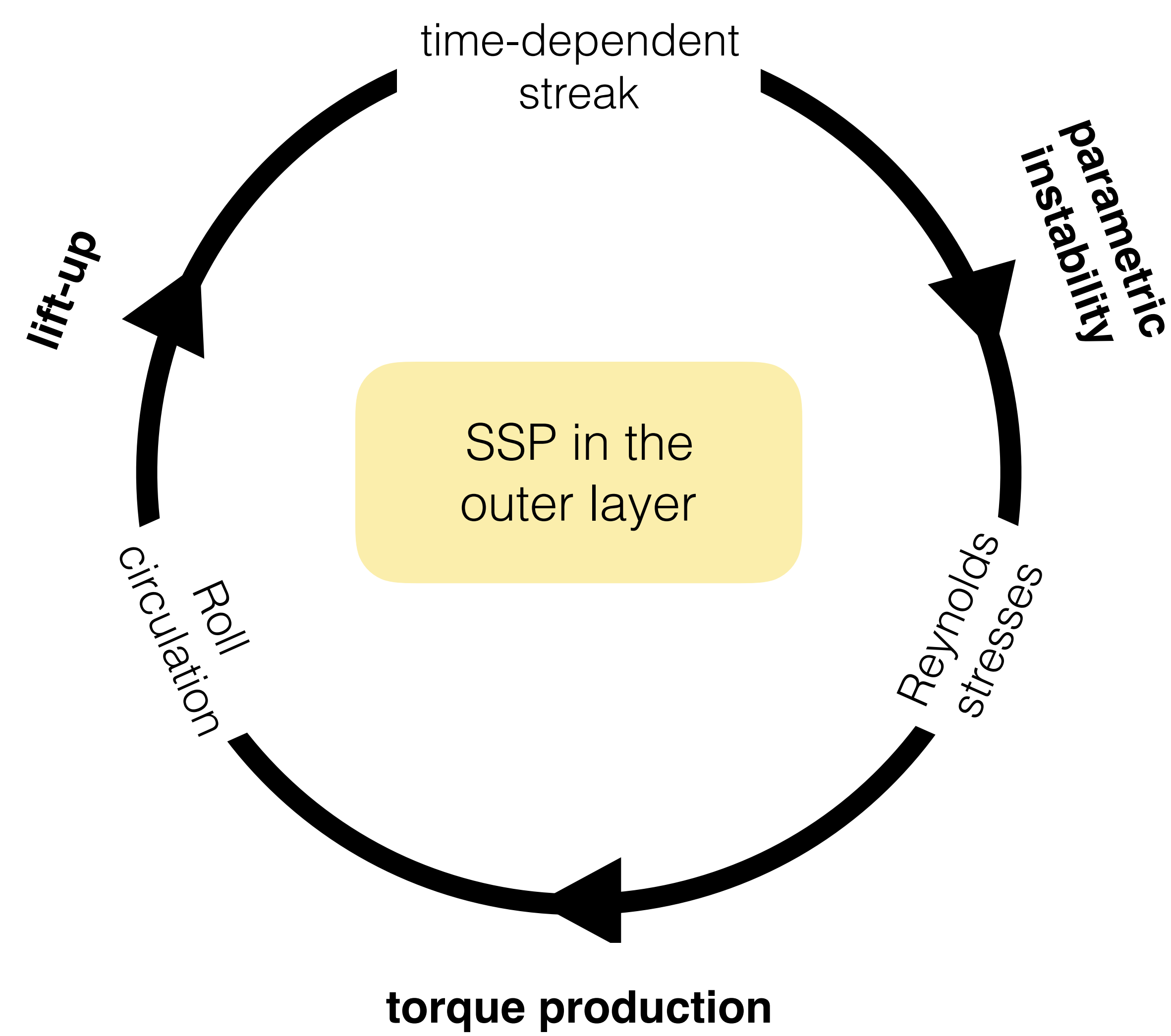
We demonstrated that **NS** and **RNL** show similar behavior,
(mean flow profiles, one-point perturbation statistics,
Reynolds stresses, perturbation spectra, etc…)
and similar streak & roll energetics.

**Conclusion**:

In the outer layer, the *same* regeneration mechanism exists
in both **NS** as in **RNL**.

# in the outer layer of both

Navier-Stokes (**NS**) & Restricted NonLinear (**RNL**)




for more details see:

Farrell et. al. (2016). A statistical state dynamics-based study of the structure and mechanism of large-scale motions in plane Poiseuille flow. *J. Fluid Mech.*, **809**, pp. 290-315, doi:10.1017/jfm.2016.661

*thanks!*