Cause-and-effect of linear mechanisms in wall turbulence

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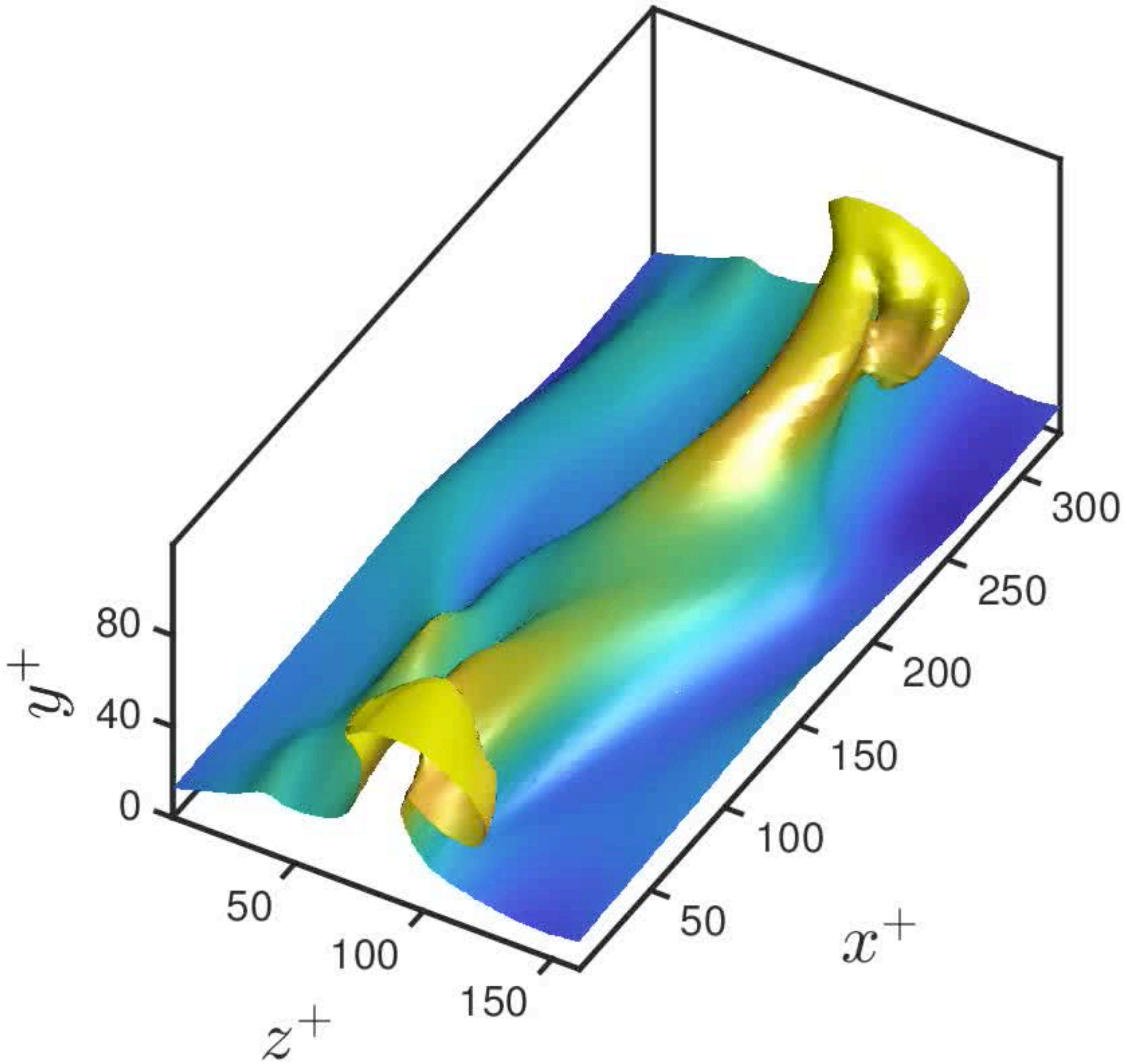
“Chicago, IL”



thanks to

Coturb Summer Workshop 2019

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Lozano-Duran et al. (2021) Cause-and-effect of linear mechanisms
sustaining wall turbulence, J. Fluid Mech. (accepted; arXiv:2005.05303)
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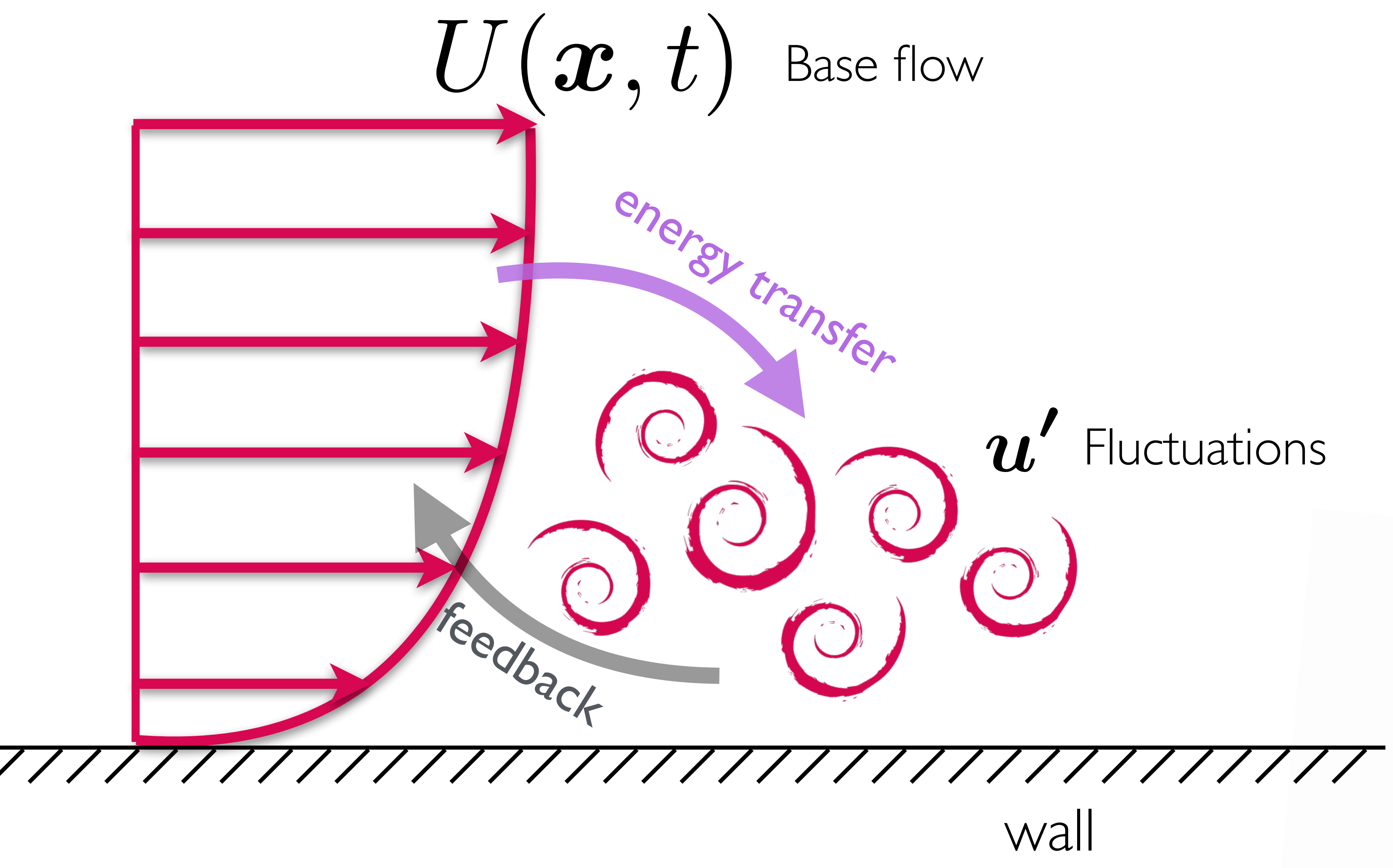
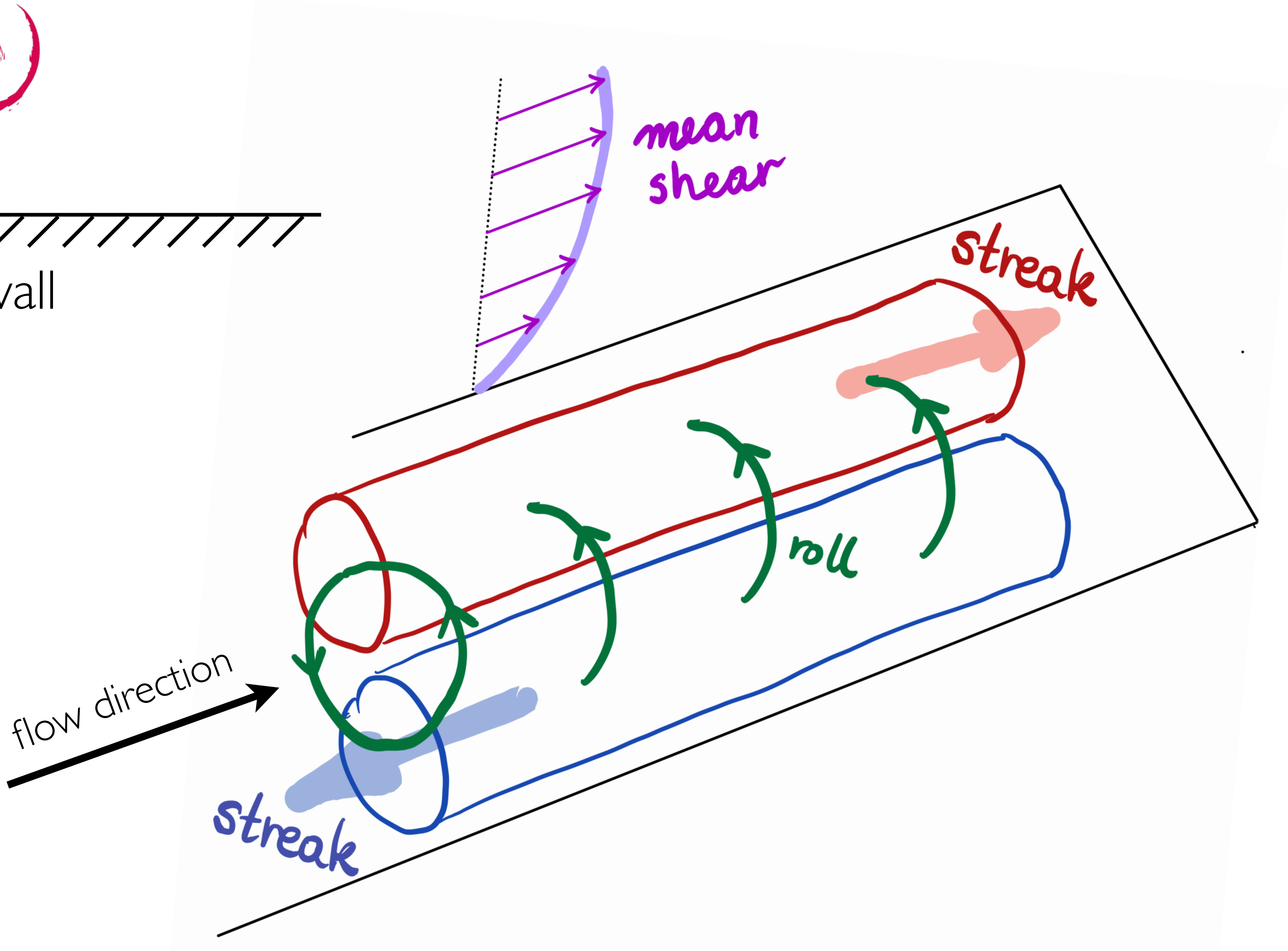
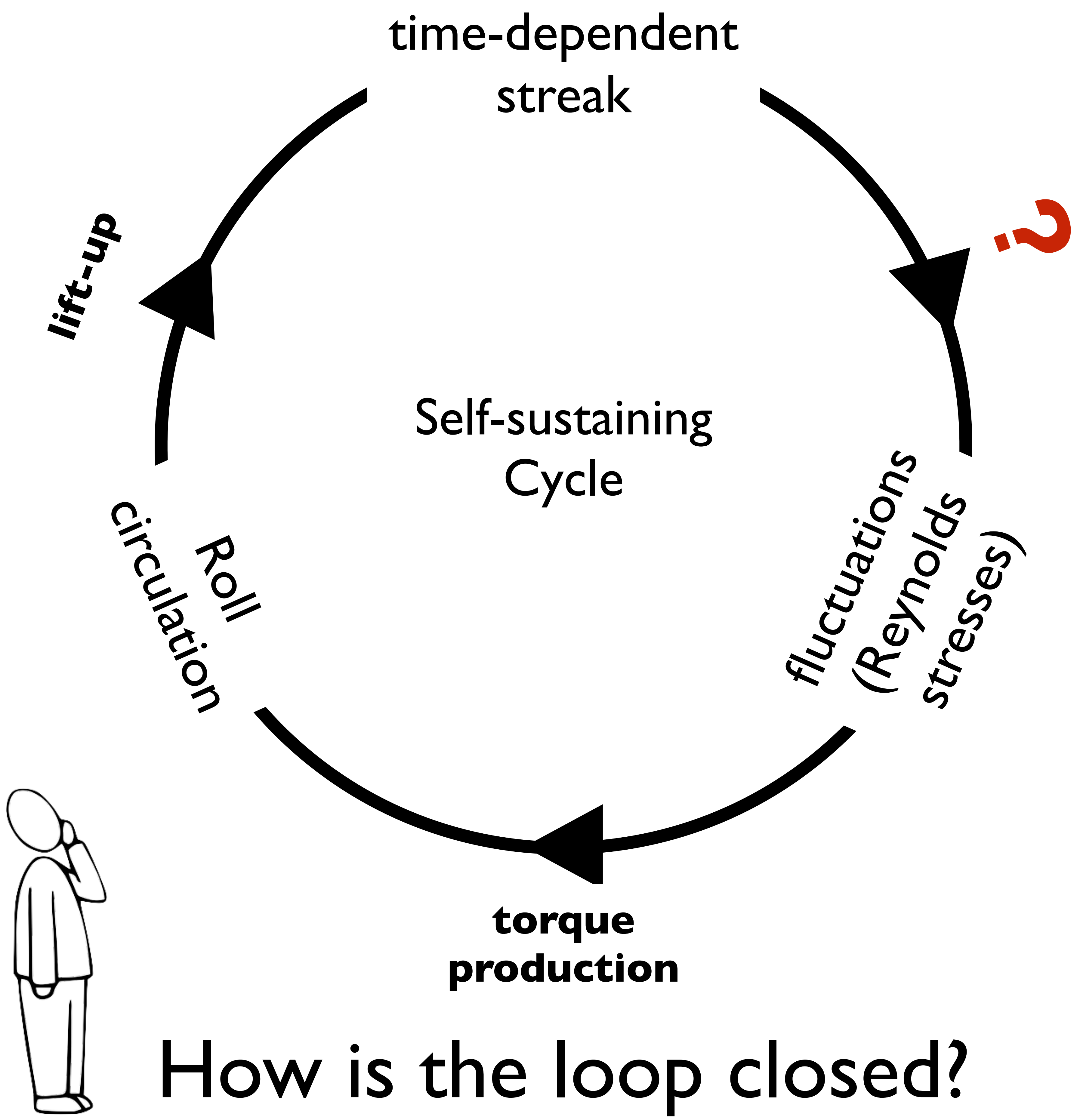
Streaks — Fluctuations — Rolls

Coherent roll–streak structure and turbulent fluctuations actively participate in a self-sustaining cycle

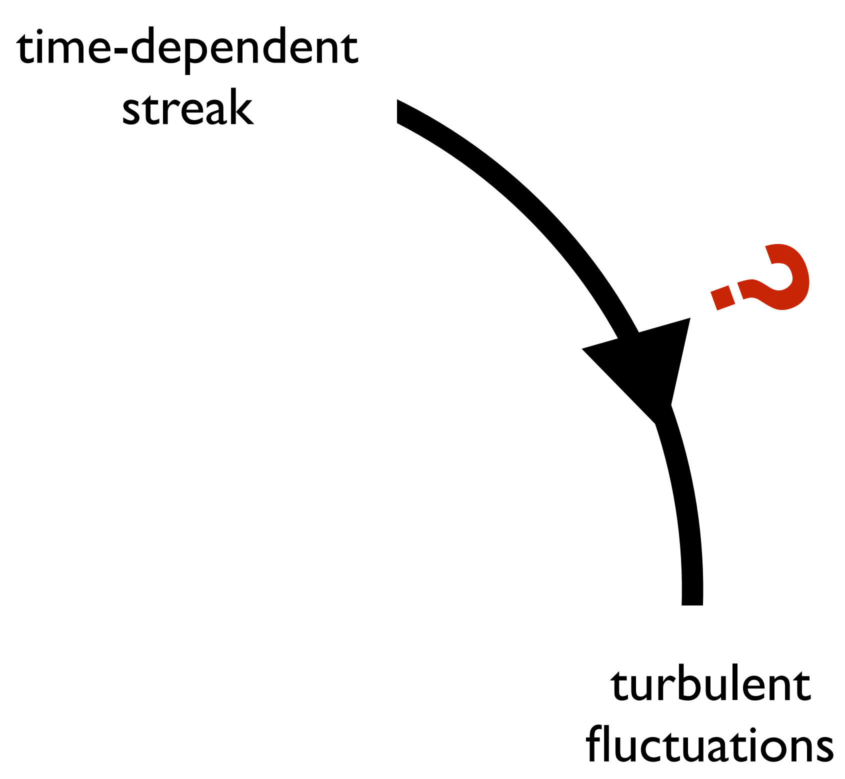






How is the loop closed?

Proposed mechanism for energy transfer to turbulent fluctuations



Modal instabilities of the streak

[Waleffe 1997, Kawahara 2003, Hack & Moin 2018, …]

Transient growth due to non-normality of linear operator $\mathscr{L}$

[Schoppa & Hussain (2002), Farrell & Ioannou (2012), Giovanetti et al. (2017),…]

Neutral modes — vortex-wave interactions

[Hall & Smith (1988), Hall & Sherwin (2010),…]

Parametric instability (enhanced energy transfer due to time-varying $U(y, z, t)$)

[Farrell & Ioannou (2012), Farrell et al. (2016),…]

Linear and nonlinear processes

incompressible
Navier—Stokes

$$\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u} \cdot \nabla \boldsymbol{u} = -\frac{1}{\rho} \nabla p + \nu \nabla^2 \boldsymbol{u} \qquad \nabla \cdot \boldsymbol{u} = 0$$

Linear and nonlinear processes

incompressible Navier—Stokes

$$\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u} \cdot \nabla \boldsymbol{u} = -\frac{1}{\rho}\nabla p + \nu \nabla^2 \boldsymbol{u} \qquad \nabla \cdot \boldsymbol{u} = 0$$

decompose the flow as $\boldsymbol{u} = \boldsymbol{U} + \boldsymbol{u}'$, $\boldsymbol{U} \equiv \langle \boldsymbol{u} \rangle$

Streaky base flow
$\boldsymbol{U} = U(y, z, t)\,\hat{\boldsymbol{x}}$
(only x-component)

$$U(y, z, t) \equiv \int u(x, y, z, t)\,\mathrm{d}x \big/ L_x$$

$$\frac{\partial \boldsymbol{U}}{\partial t} + \boldsymbol{U} \cdot \nabla \boldsymbol{U} = -\frac{1}{\rho}\nabla P + \nu \nabla^2 \boldsymbol{U} - \underbrace{\langle \boldsymbol{u}' \cdot \nabla \boldsymbol{u}' \rangle}_{\text{Reynolds stresses}} \qquad \nabla \cdot \boldsymbol{U} = 0$$

$$\frac{\partial \boldsymbol{u}'}{\partial t} = \underbrace{\mathscr{L}(\boldsymbol{U})\,\boldsymbol{u}'}_{\substack{\text{linear}\\ \text{processes}}} + \underbrace{\mathscr{N}(\boldsymbol{u}')}_{\substack{\text{nonlinear}\\ \text{processes}}}$$

Linear and nonlinear processes

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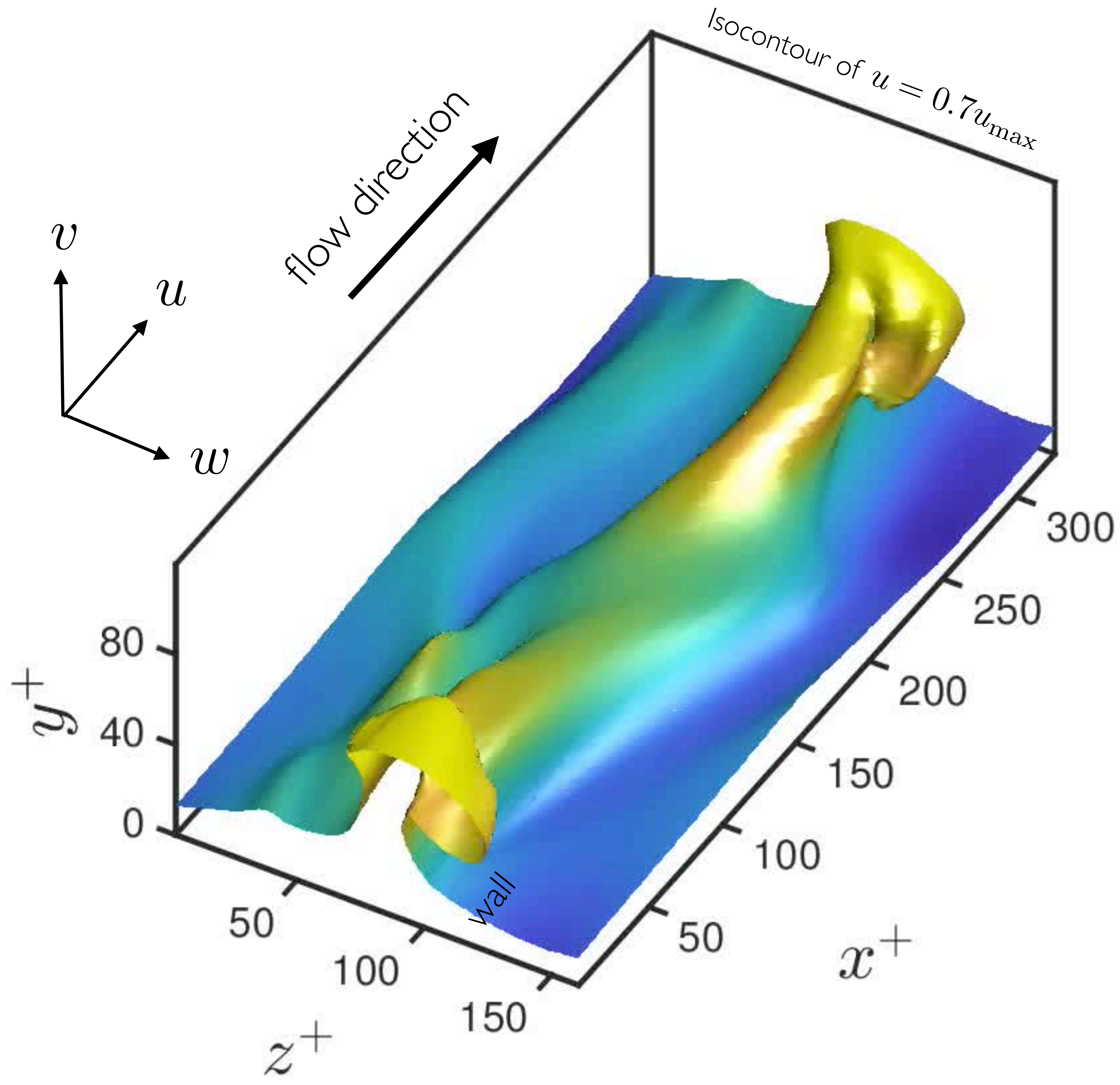
We *don't linearise* about a solution $\boldsymbol{U}$!

We *decompose* the flow and call "linear" anything included in $\mathscr{L}(\boldsymbol{U})\boldsymbol{u}'$.

Different choice for U can make a process included in $\mathscr{L}(\boldsymbol{U})\boldsymbol{u}'$ to become part of $\mathscr{N}(\boldsymbol{u}')$.

Problem set-up: minimal turbulent channel

Half channel flow
constant pressure gradient



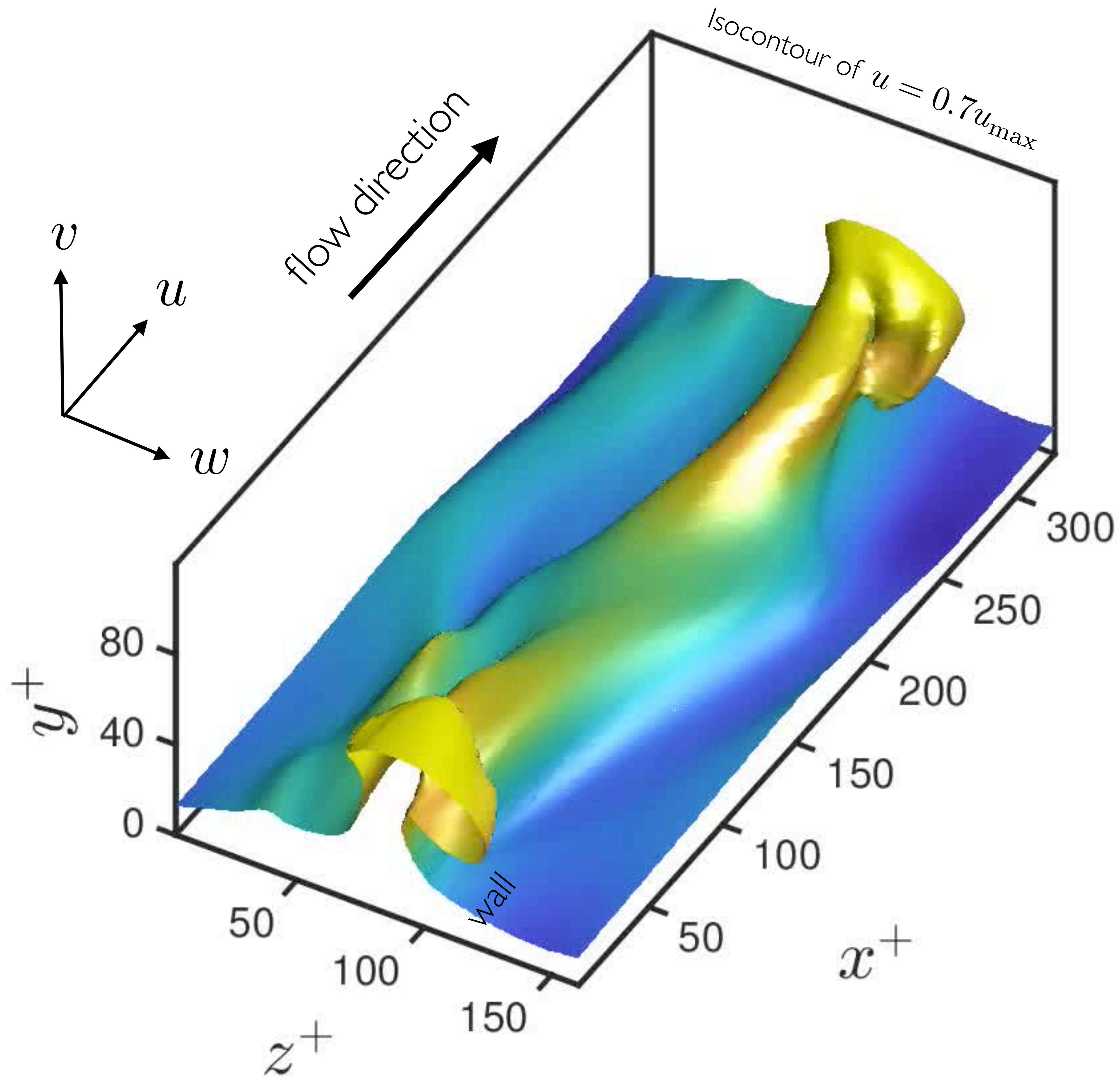
Solution by
Direct Numerical Simulation

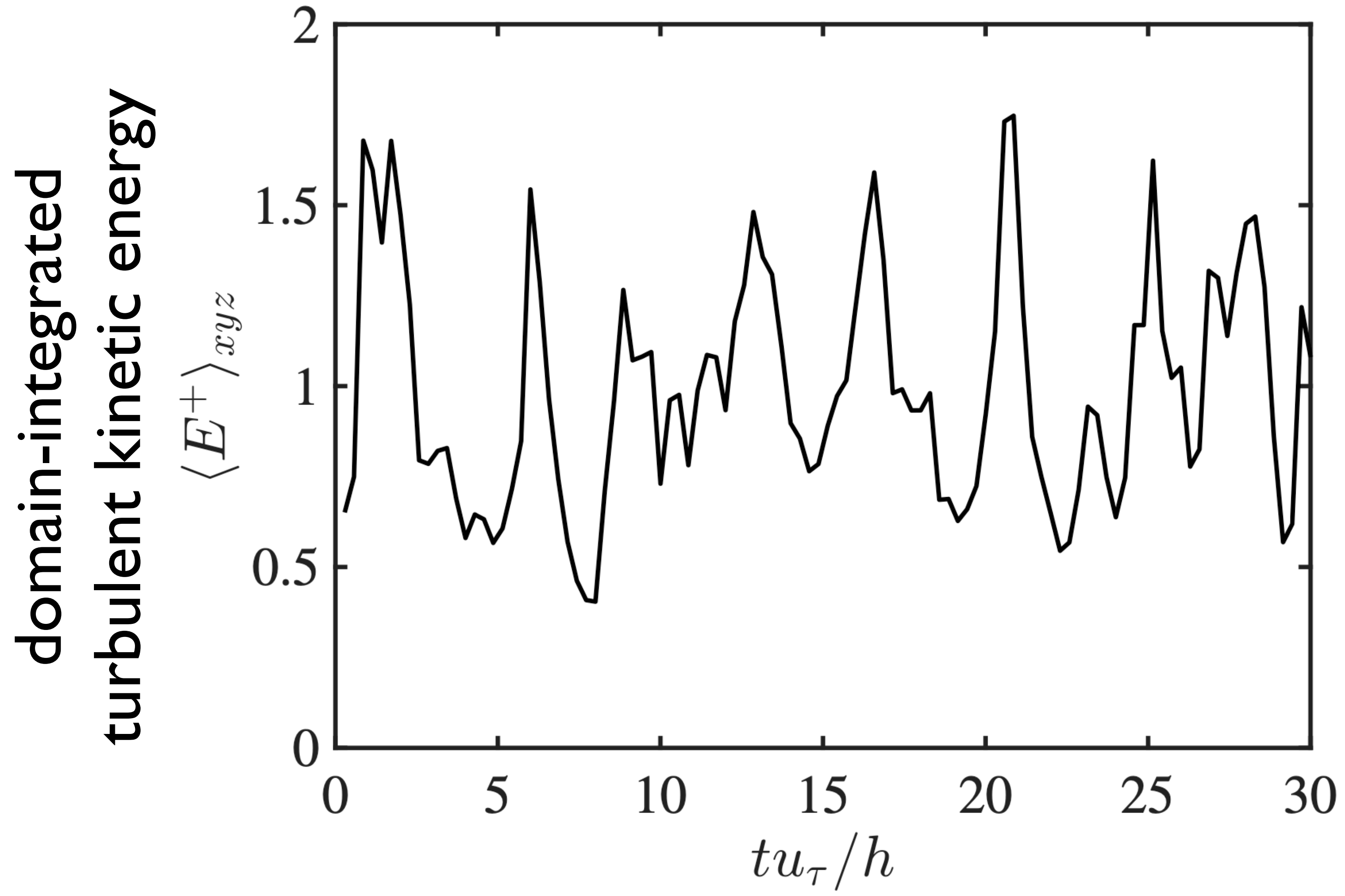
$Re_\tau = 184$

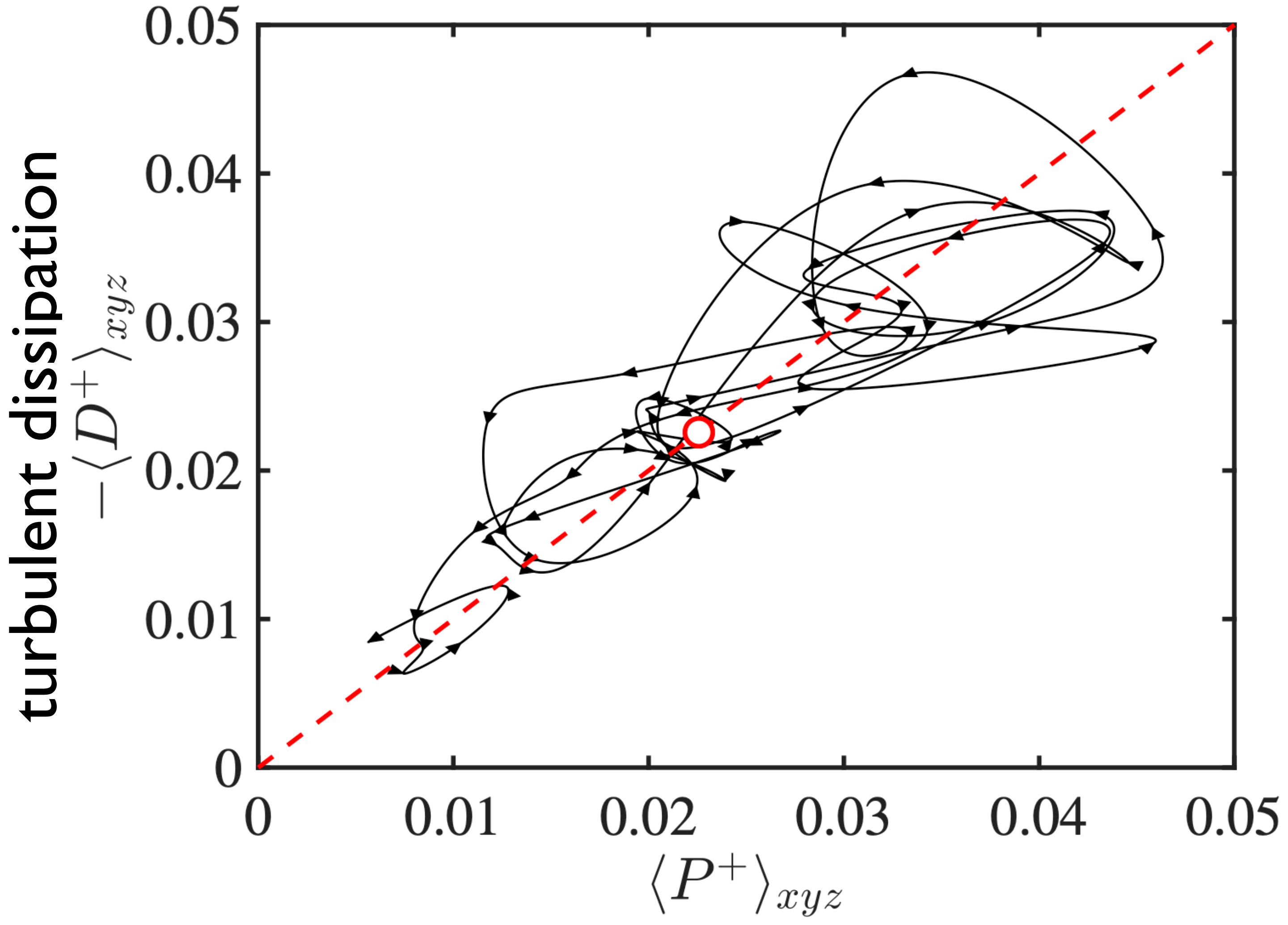
h wall-normal height u_τ friction velocity

Problem set-up: minimal turbulent channel

Half channel flow
constant pressure gradient







Solution by
Direct Numerical Simulation

$Re_\tau = 184$

We run DNS for >600h/u_τ and keep *all* snapshots of base flow $U(y, z, t)$

h wall-normal height u_τ friction velocity

Two ways to assess various mechanisms

Interrogate DNS output

non-intrusive

Sensibly modify equations of motion to *preclude* some mechanisms

allows infer casual relationships

Modal instabilities of the streaky base flow

$$\underbrace{\mathscr{L}(U(y,z,t)) = \mathscr{U} \begin{pmatrix} \lambda_1 + \mathrm{i}\omega_1 & & & \\ & \lambda_2 + \mathrm{i}\omega_2 & & \\ & & \lambda_3 + \mathrm{i}\omega_3 & \\ & & & \ddots \end{pmatrix} \mathscr{U}^{-1}}_{\text{Eigen-decomposition of } \mathscr{L}}$$

Eigenvalues

growth rates
$\lambda_1 \geq \lambda_2 \geq \cdots$

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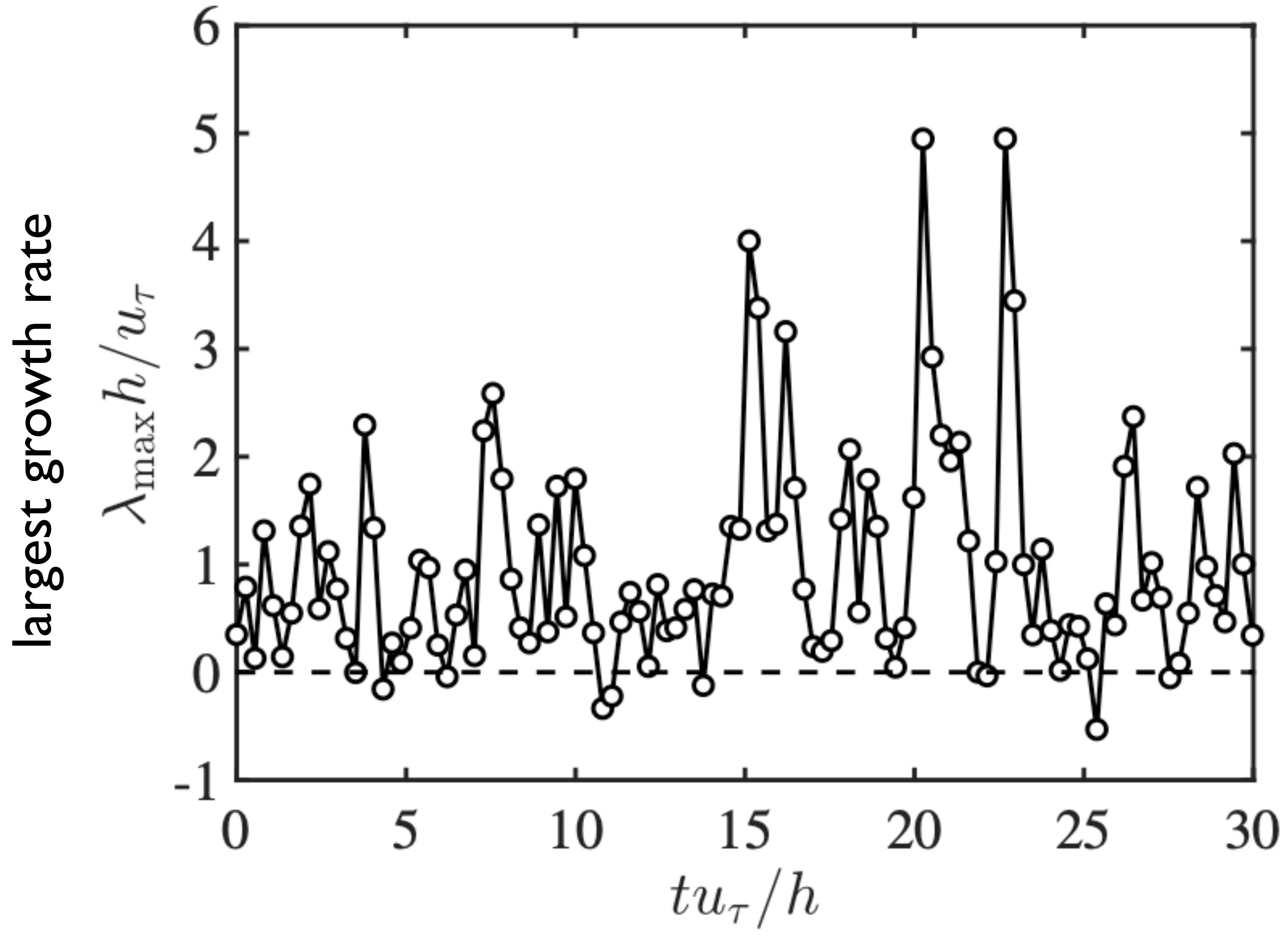
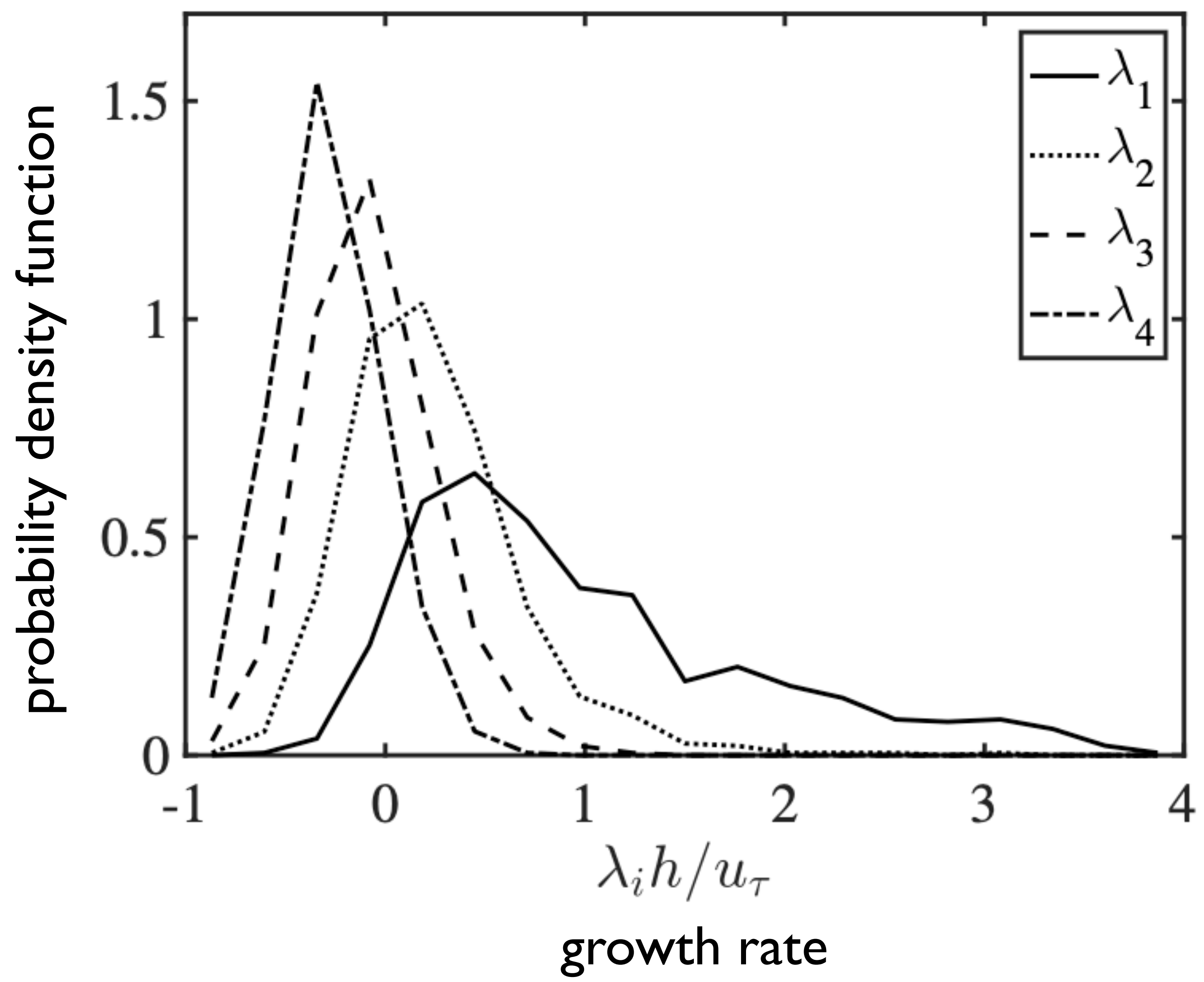
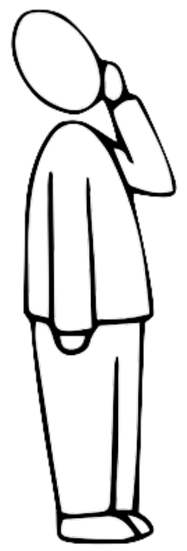




Autocorrelation of $U \Rightarrow$ base flow changes (at least) ~3 x slower than e-folding $1/\lambda$
$\Rightarrow$ modal instabilities *do* have time to grow

If modal instabilities are *crucial* for the self-sustaining cycle

the flow should laminarise without them…



Suppressing modal instabilities of the streaky base flow

$$\mathscr{L}(U(y,z,t)) = \mathscr{U}\begin{pmatrix} \lambda_1 + \mathrm{i}\omega_1 & & & \\ & \lambda_2 + \mathrm{i}\omega_2 & & \\ & & \lambda_3 + \mathrm{i}\omega_3 & \\ & & & \ddots \end{pmatrix}\mathscr{U}^{-1} \qquad \lambda_1 \geq \lambda_2 \geq \cdots$$

@ ***every instance*** we stabilise $\mathscr{L} \implies$ if $\lambda_j > 0$, replace with $-\lambda_j$

E.g., for 2 unstable modes:

$$\widetilde{\mathscr{L}}(U(y,z,t)) = \mathscr{U}\begin{pmatrix} -\lambda_1 + \mathrm{i}\omega_1 & & & \\ & -\lambda_2 + \mathrm{i}\omega_2 & & \\ & & \lambda_3 + \mathrm{i}\omega_3 & \\ & & & \ddots \end{pmatrix}\mathscr{U}^{-1}$$

Modally stable wall-turbulence

$$\frac{\partial \boldsymbol{U}}{\partial t} + \boldsymbol{U} \cdot \nabla \boldsymbol{U} = -\frac{1}{\rho} \nabla P + \nu \nabla^2 \boldsymbol{U} - \langle \boldsymbol{u}' \cdot \nabla \boldsymbol{u}' \rangle \qquad \nabla \cdot \boldsymbol{U} = 0$$

$$\frac{\partial \boldsymbol{u}'}{\partial t} = \widetilde{\mathscr{L}}(U)\, \boldsymbol{u}' + \mathscr{N}(\boldsymbol{u}')$$

stabilized operator
the *only* modification
to Navier-Stokes

fully coupled

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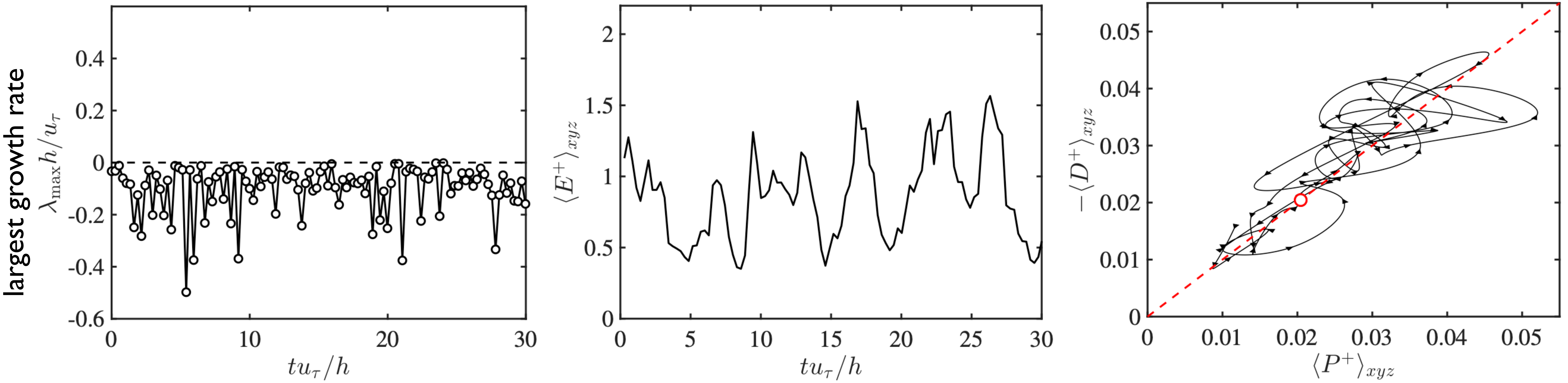

turbulence persists…

[Turbulence also persist if $\mathscr{N}$ is set to 0!]

Farrell & Ioannou (2012)

Modally stable wall-turbulence

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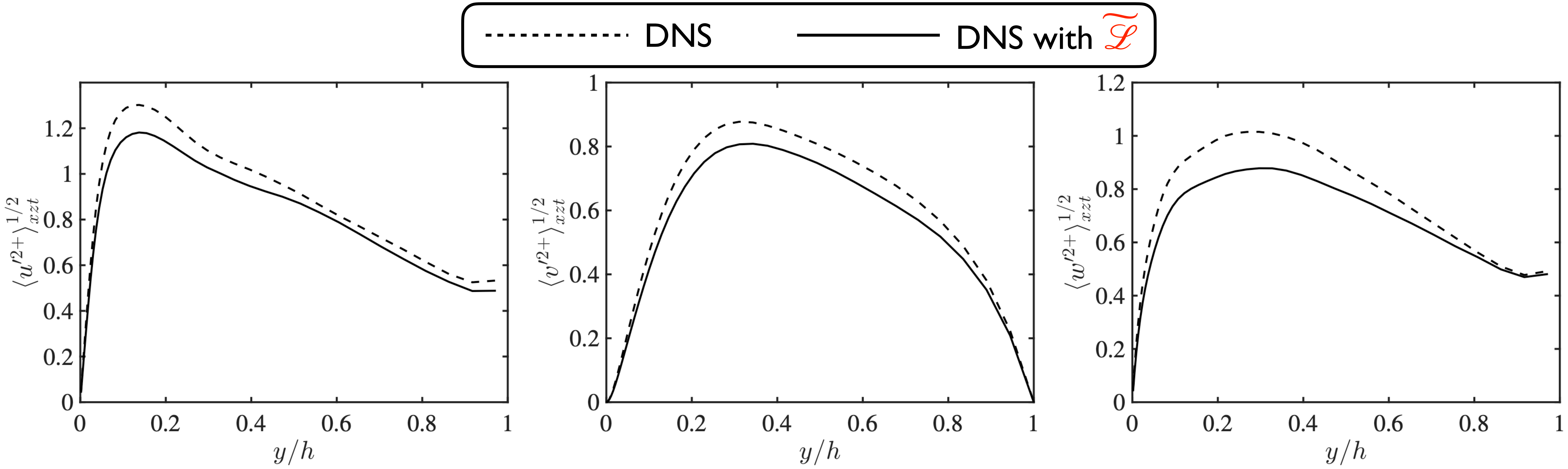
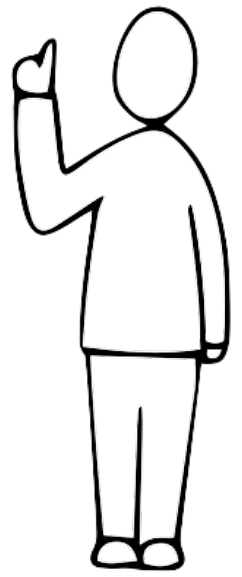


… and it's not that different from the DNS — turbulent intensities only drop by ~10%

So, modal instabilities are ***not crucial***

for the self-sustaining cycle.



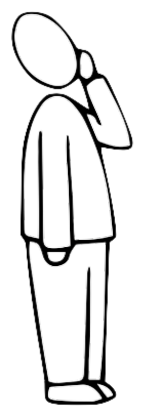
Non-modal transient growth

Since $\int \boldsymbol{u}' \cdot \mathcal{N}(\boldsymbol{u}')\,\mathrm{d}V = 0$, turbulent energy is governed by linear processes

$$\frac{\partial \boldsymbol{u}'}{\partial t} = \mathcal{L}(t)\,\boldsymbol{u}'$$
$$\boldsymbol{u}'(t = t_0) = \boldsymbol{u}'_0$$

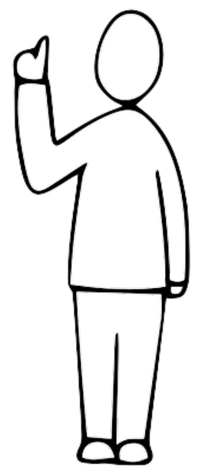
$$\longrightarrow \quad \underbrace{G_{\max}\left(t_0, T\right)}_{\text{maximum energy gain}} = \sup_{\boldsymbol{u}'(t_0)} \frac{\int |\,\boldsymbol{u}'(t_0 + T)\,|^2\,\mathrm{d}V}{\int |\,\boldsymbol{u}'(t_0)\,|^2\,\mathrm{d}V}$$

[Farrell & Ioannou (1996), Schmid (2007)]



How we can disentangle energy growth due to transient growth and exponential instabilities?

We can use the stabilised operator $\widetilde{\mathcal{L}}(U)$.



Non-modal transient growth
frozen base flow $U(y, z, t_0)$

$$\underbrace{\widetilde{G}_{\max}(t_0, T)}_{\text{maximum energy gain}}$$

due to the stabilized $\widetilde{\mathscr{L}}$
linear dynamics

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frozen base flow $U(y, z, t_0)$

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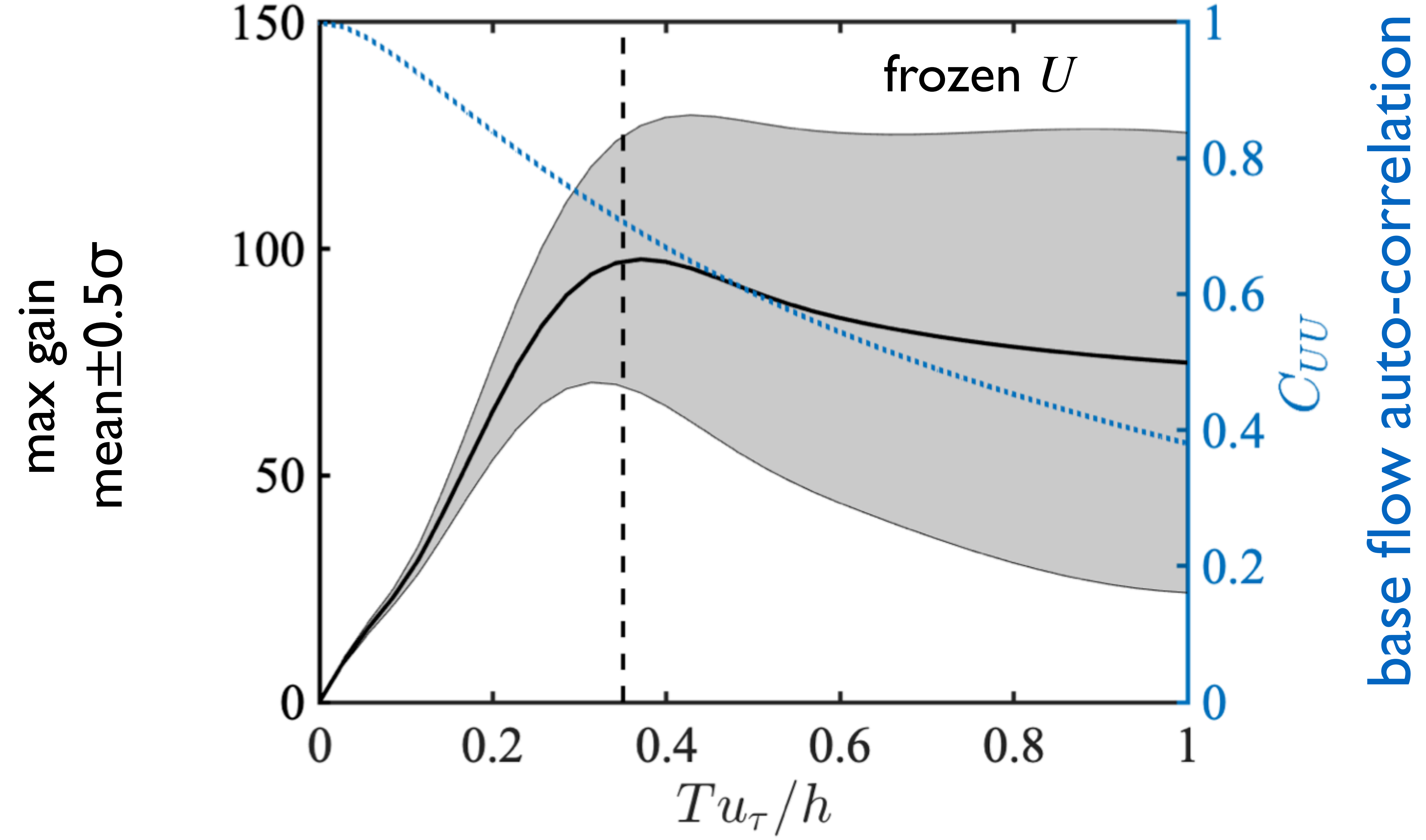
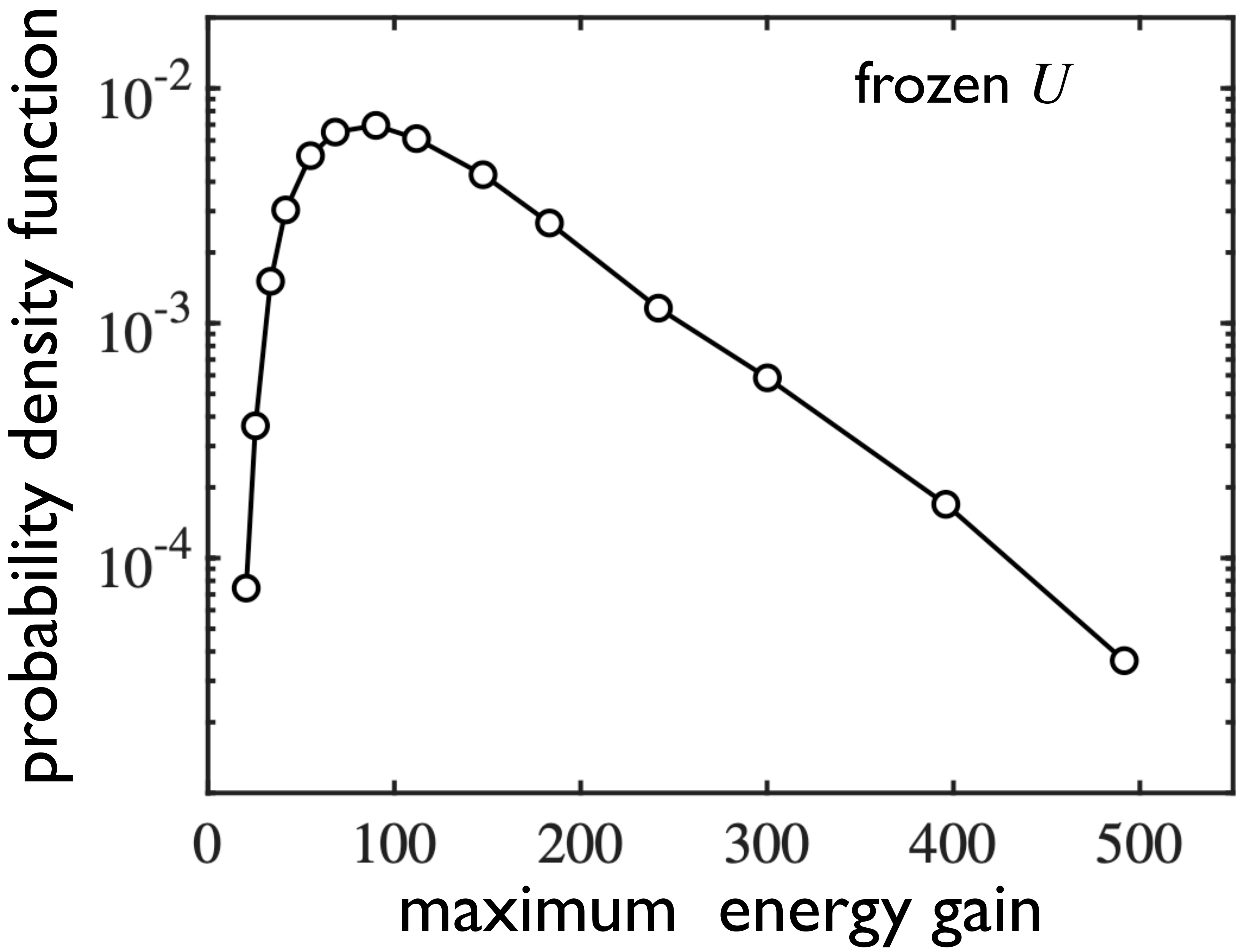




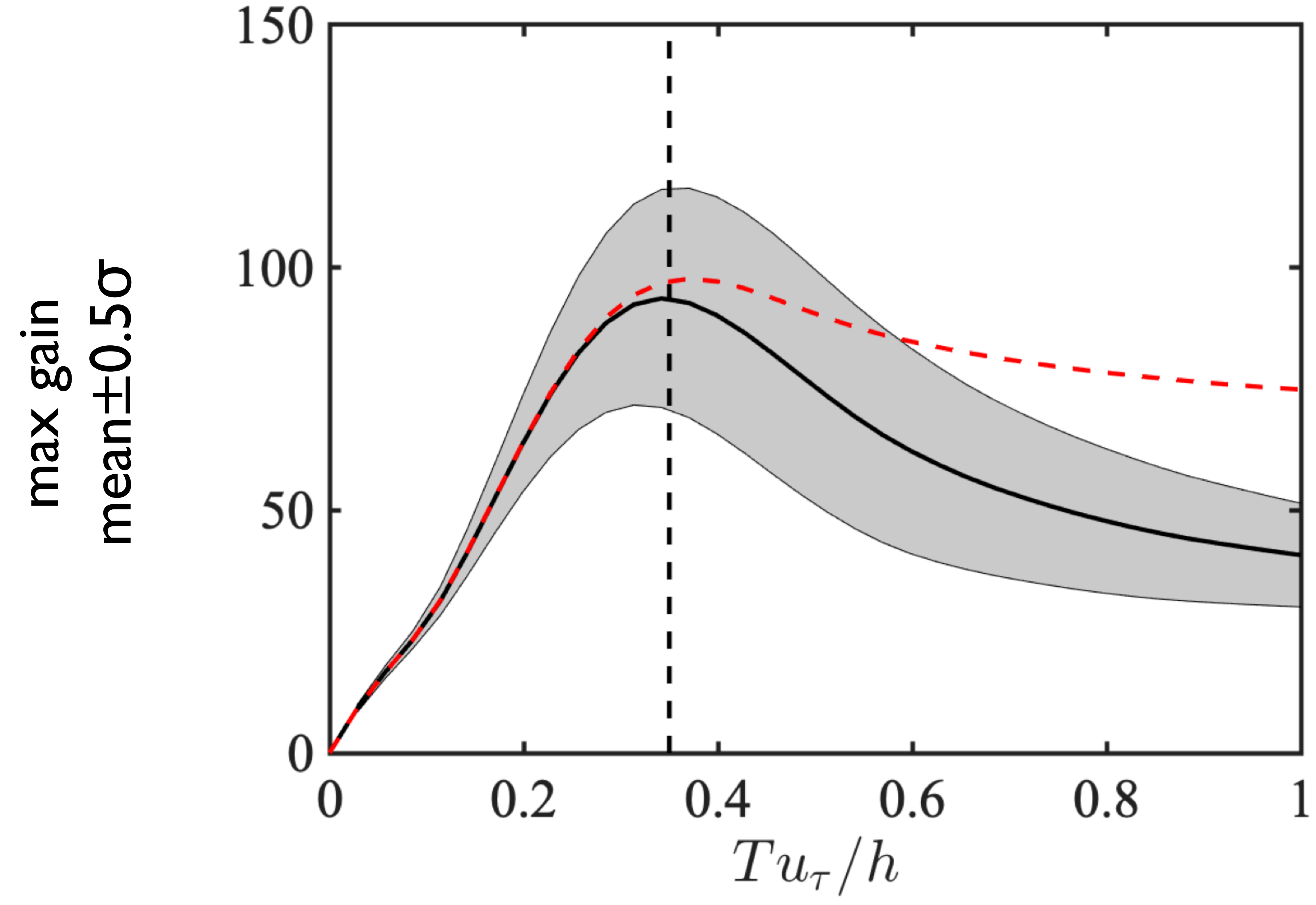
[Note that streaky base flow $U(y, z, t_0)$ gives gains $O(100)$. Base flows $U(y)$ induce gain $O(10)$.]

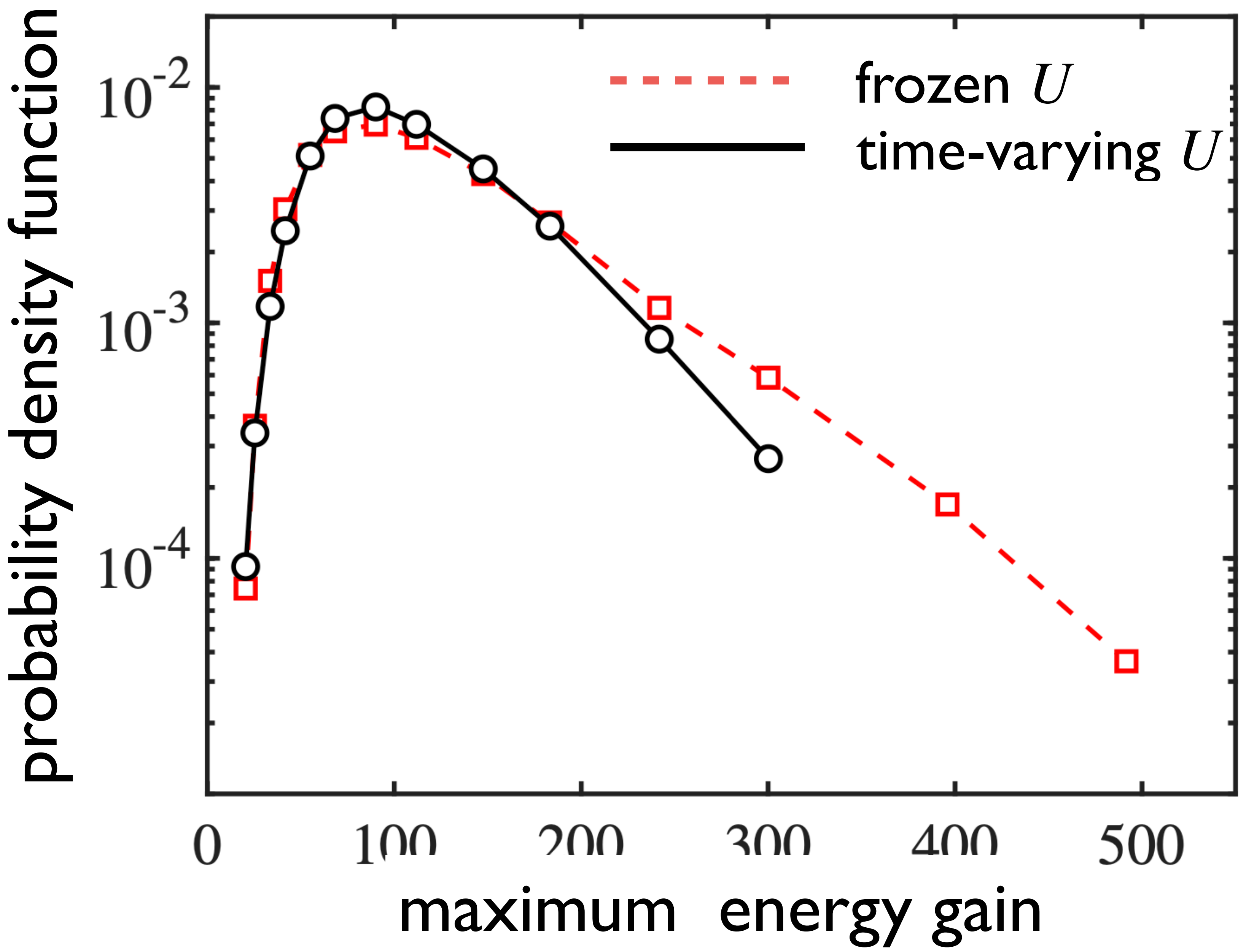
Del Alamo & Jiménez 2006; Pujals et al. 2009; Cossu et al. 2009

Non-modal transient growth
frozen & time-varying base flows

$$\underbrace{\widetilde{G}_{\max}(t_0, T)}_{\text{maximum energy gain}}$$

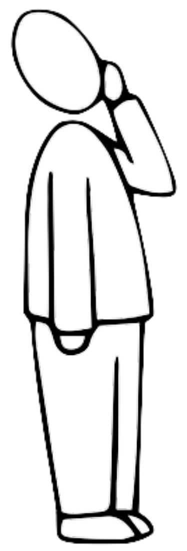
due to the stabilized $\widetilde{\mathscr{L}}$ linear dynamics





Time-variability of the base flow $U(y, z, t)$ does not enhance energy transfer to fluctuations for short times.

Is transient growth sufficient to sustain turbulence?



Turbulence with *only* transient growth operable

500 simulations

$$\frac{\partial \boldsymbol{u}'}{\partial t} = \widetilde{\mathscr{L}}(U(y,z,t_i))\,\boldsymbol{u}' + \mathscr{N}(\boldsymbol{u}') \qquad i = 1,2,\ldots,500$$

with a ***frozen*** snapshot $U(y,z,t_j)$ from DNS

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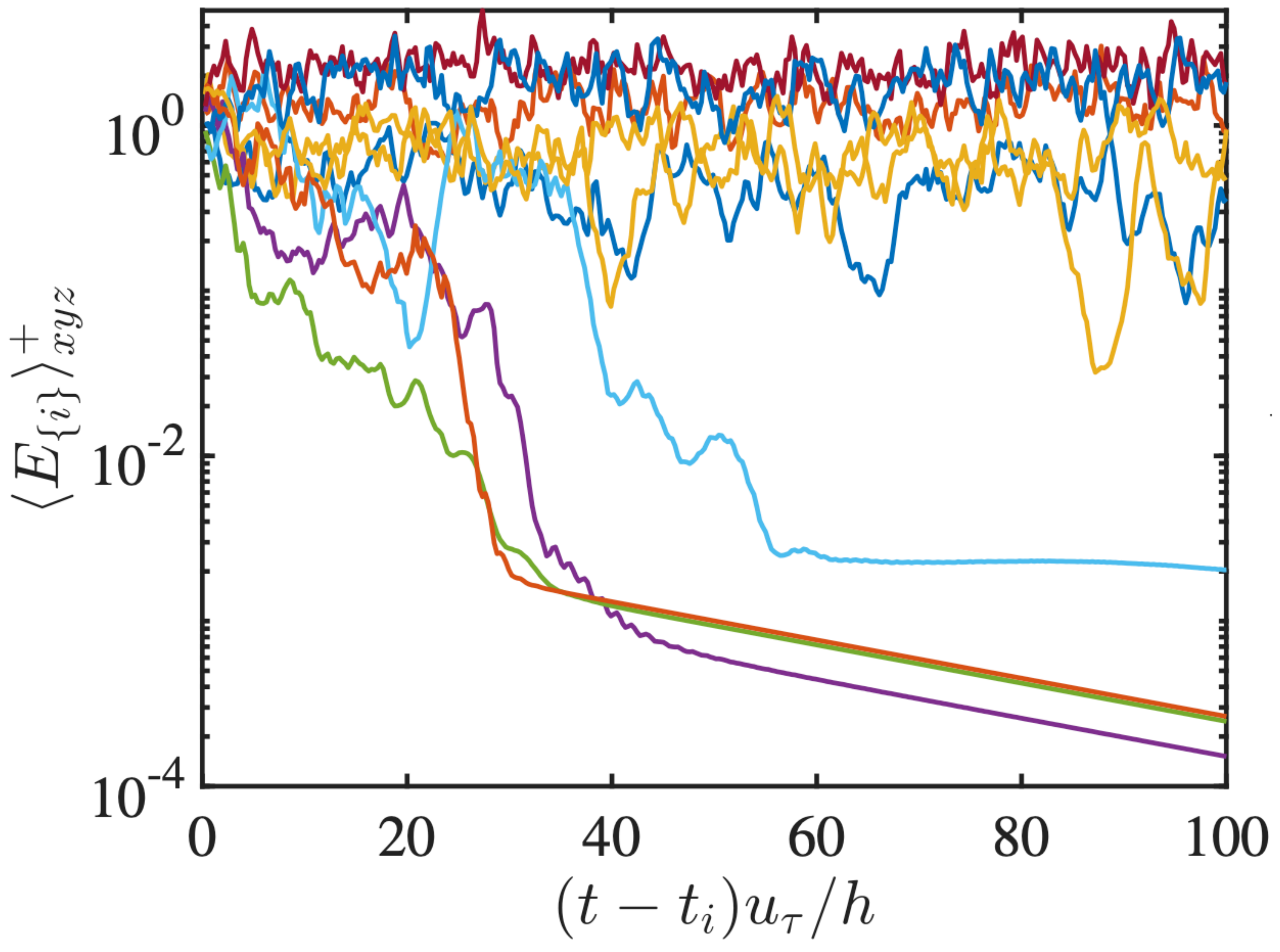
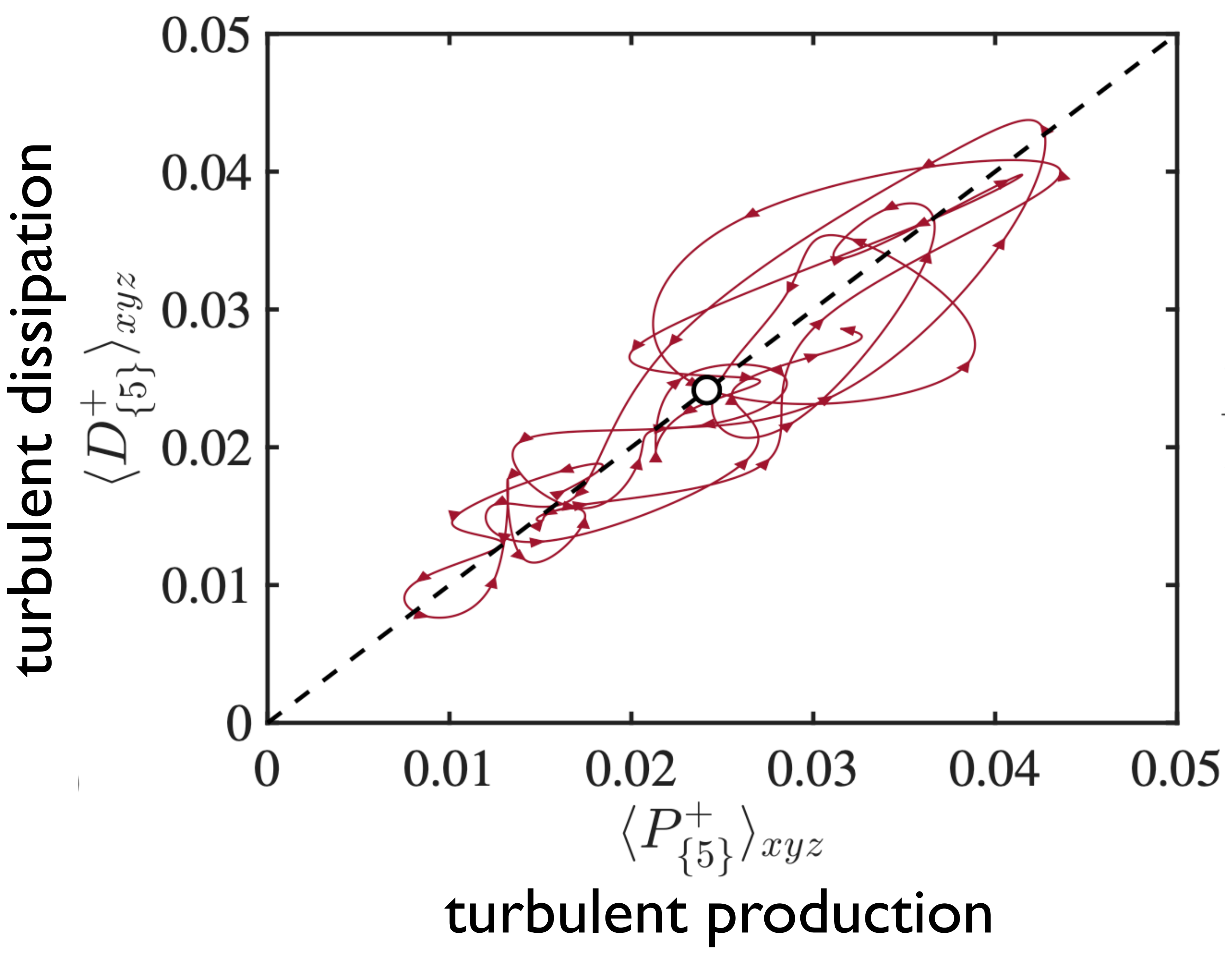
with a ***frozen*** snapshot $U(y,z,t_j)$ from DNS

Turbulence persists in ≈80% of the simulations.

TKE for 10 of the simulations





a case that sustains

Turbulence with *only* transient growth operable

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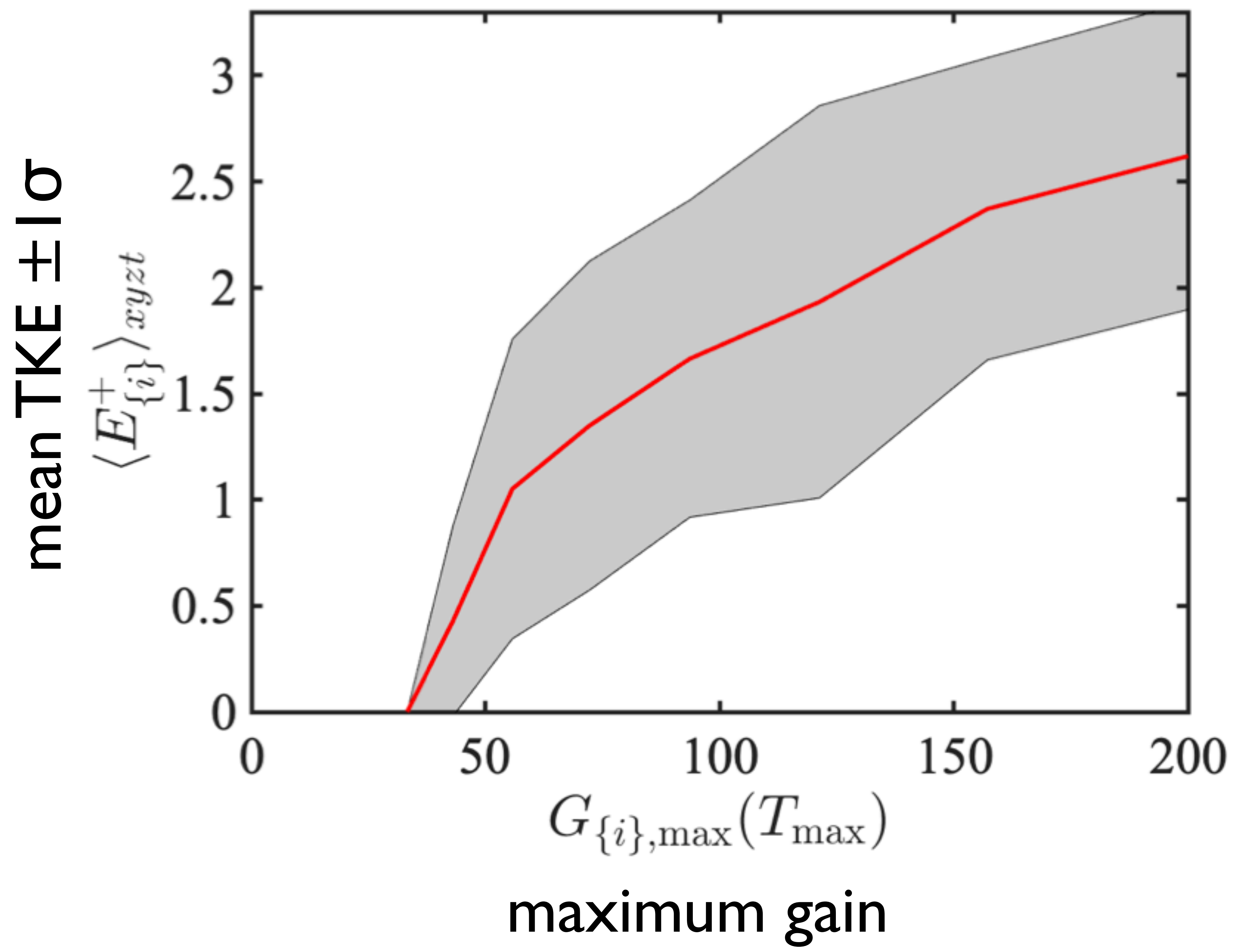
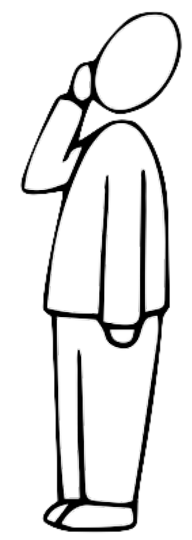


frozen base flows $U(y,z,t_i)$
with gain $\gtrapprox$40
sustain turbulence

(for $Re_\tau = 180$)

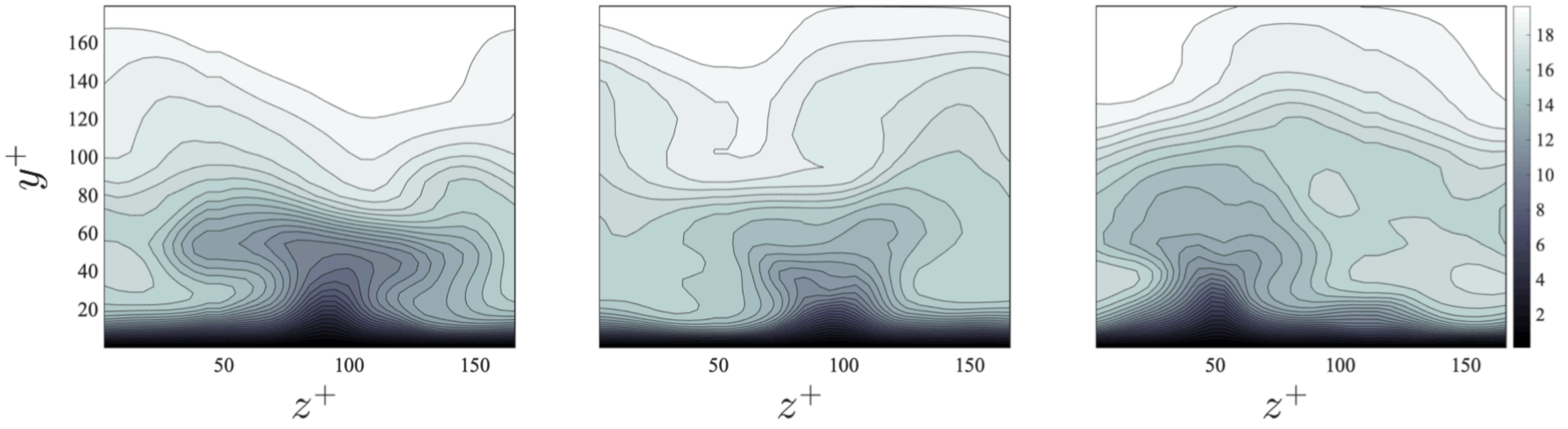
What differentiates the ***frozen*** base flows $U(y, z, t_i)$

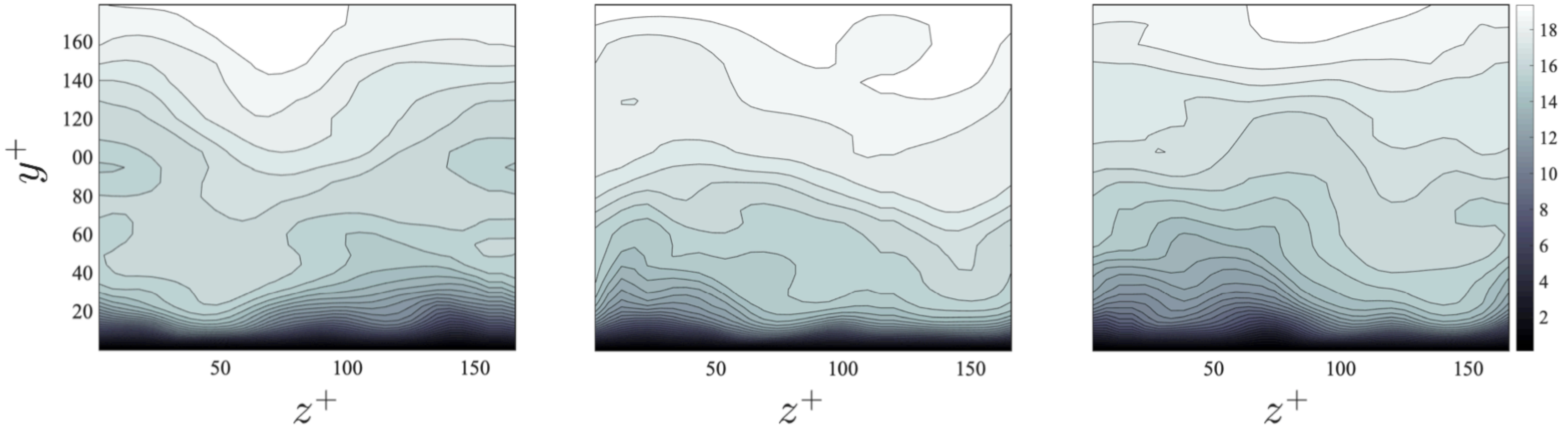
that sustain turbulence from those which laminarise?



Spanwise streaky structure turns out *crucial* for $U(y, z, t_i)$ to sustain

these $U(y, z, t_i)$ sustain





these $U(y, z, t_i)$ laminarise

Precluding the 'push-over' mechanism due to spanwise base-flow shear leads to laminarization.

[for detailed experiments demonstrating this claim see our paper]

summary

modal instabilities of streaks are ***not crucial***

how does energy go from the mean flow to the perturbations?
simple answer: transient growth

what produces this **transient growth?**
the ***spanwise*** shear of the streak & Orr mechanism
(for thorougher discussion see the paper)

time-variability of the streak does not enhance energy transfer to fluctuations
but allows flow to "sample" independent transient-growth events resulting to the observed statistics
(not discussed here; see the paper)

realistic wall-turbulence can be *exclusively* supported by transient growth

Lozano-Duran et al. (2021) Cause-and-effect of linear mechanisms sustaining wall turbulence, *J. Fluid Mech.* (Accepted; arXiv:2005.05303)