Statistical state dynamics of jet/wave coexistence in beta-plane turbulence

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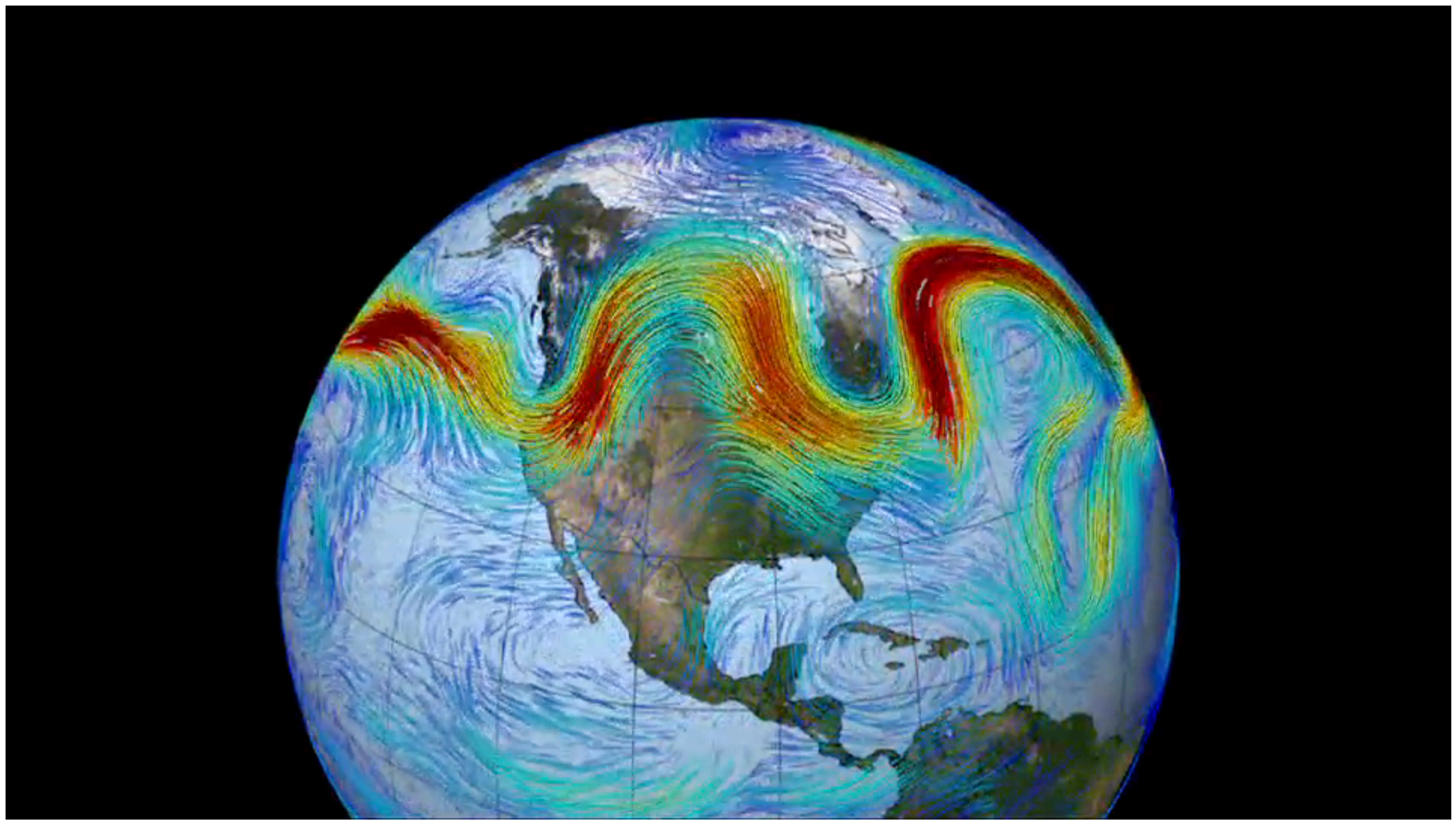
large-scale jets and planetary-scale coherent waves coexist in planetary turbulence



NASA/Goddard Space Flight Center

Question: Can this jet/wave coexistence regime exist merely as a consequence of the underlying dynamics and in the absence of topography?

barotropic vorticity equation on a β-plane

$$\partial_t \zeta + \mathbf{u}\cdot\boldsymbol{\nabla}\zeta + \beta v = \underbrace{-r\zeta + \nu\Delta\zeta}_{\text{dissipation}} + \sqrt{\varepsilon}\,\xi$$

dissipation

stochastic forcing

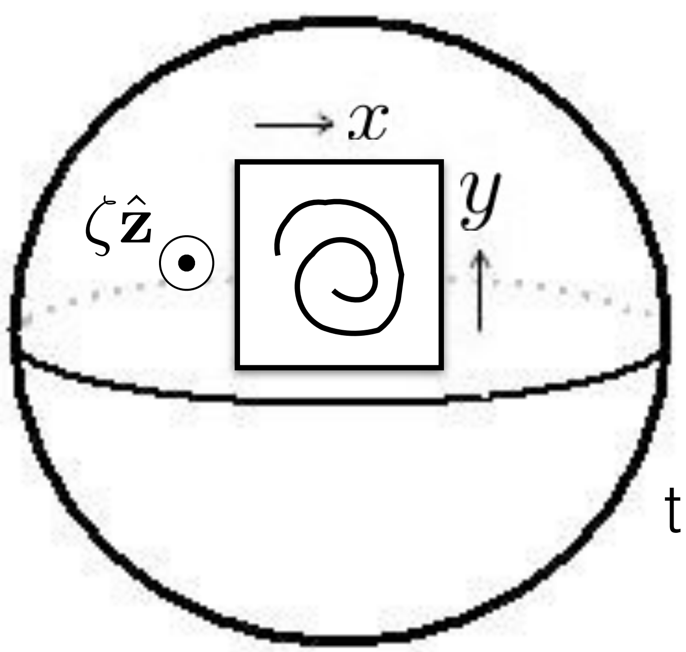
$$\nabla \cdot \mathbf{u} = 0$$

$$\mathbf{u} = (u, v) = (-\partial_y \psi, \partial_x \psi)$$

$$\zeta = (\nabla \times \mathbf{u}) \cdot \hat{\mathbf{z}} = \Delta \psi$$

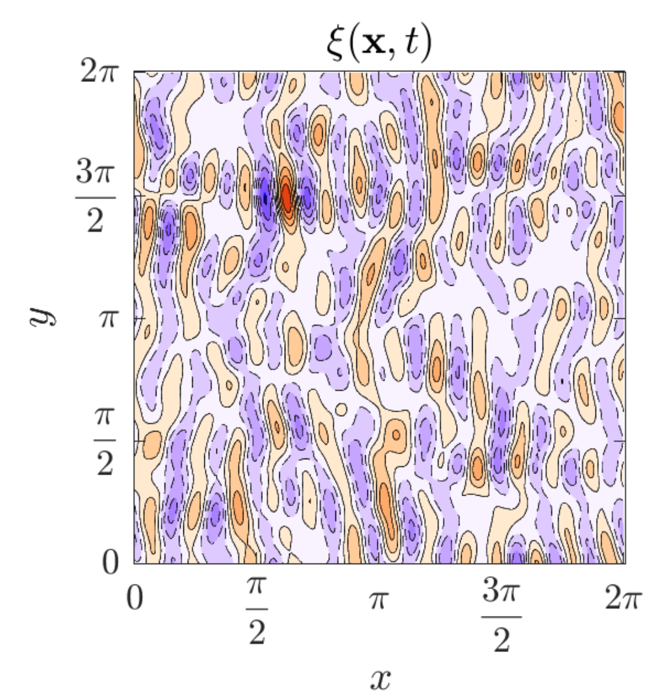
ξ injects energy at rate ε at typical scales $\ll$ domain size

zero mean
statistically homogeneous in space
white in time

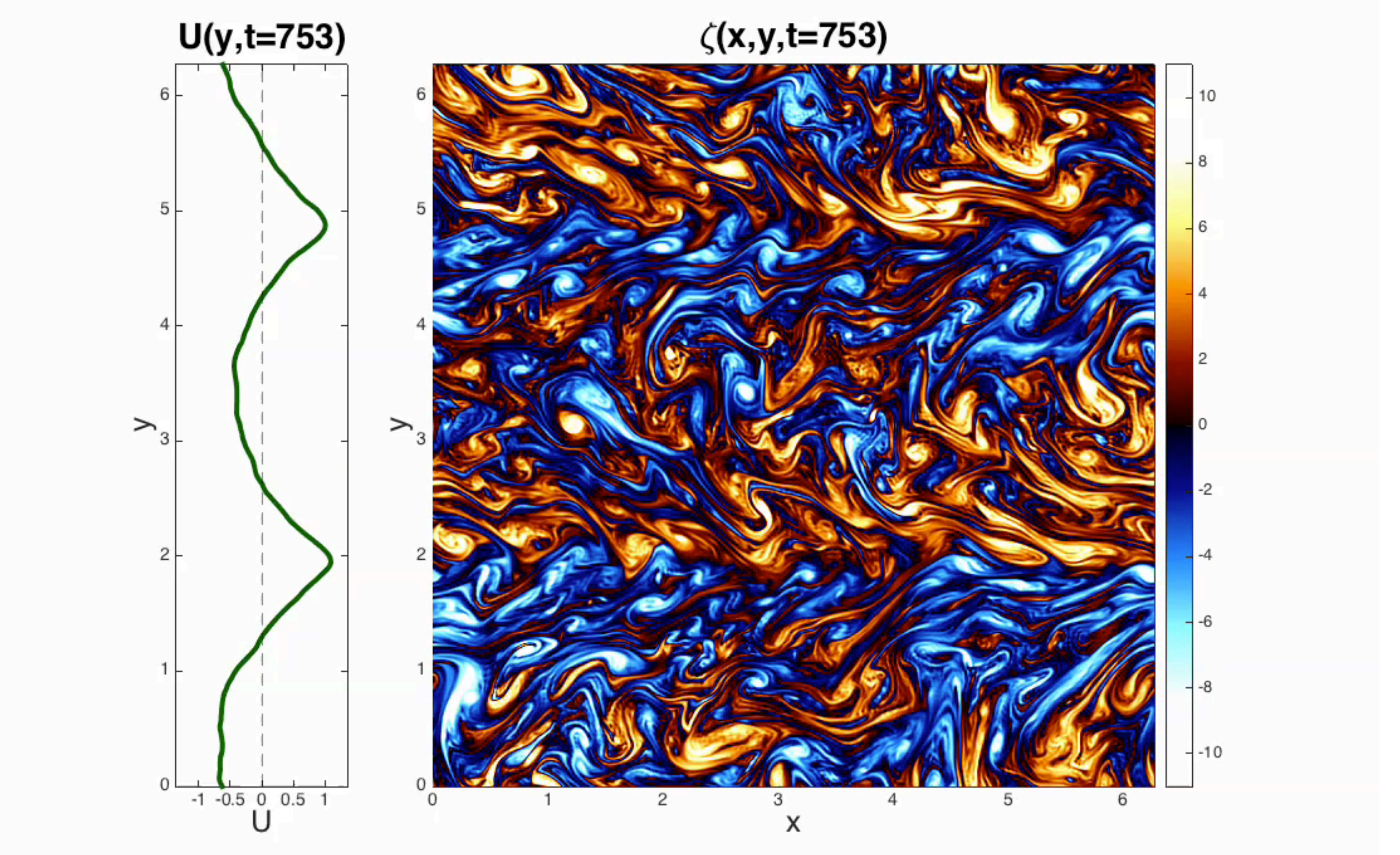


$$\boldsymbol{\beta} = (0, \beta)$$

β is the gradient of the planetary vorticity



this model exhibits large-scale structures



but it is a nonlinear stochastic system…

to try to understand it we will construct
a dynamics that governs its statistics
(statistical state dynamics)

Statistical State Dynamics
(2nd order closure — S3T)

$$\zeta(\mathbf{x},t) \;\to\; Z(\mathbf{x},t) \stackrel{\text{def}}{=} P_K\left[\zeta(\mathbf{x},t)\right] = \sum_{\substack{|k_x|\le K\\ k_y}} \hat{\zeta}(\mathbf{k},t)\,\mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{x}}$$

coherent mean flow
low zonal wavenumber
bandpass filter, e.g. $|k_x| = 0, 1$

$$\zeta(\mathbf{x},t) \;\to\; \zeta'(\mathbf{x},t) \stackrel{\text{def}}{=} (1-P_K)\left[\zeta(\mathbf{x},t)\right] = \sum_{\substack{|k_x|> K\\ k_y}} \hat{\zeta}(\mathbf{k},t)\,\mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{x}}$$

incoherent eddy motions

hierarchy of same-time *n*-point cumulants:

$$C(\mathbf{x}_a,\mathbf{x}_b,t) \stackrel{\text{def}}{=} \langle \zeta'(\mathbf{x}_a,t)\zeta'(\mathbf{x}_b,t)\rangle \;,\quad \Gamma(\mathbf{x}_a,\mathbf{x}_b,\mathbf{x}_b,t) \stackrel{\text{def}}{=} \langle \zeta'(\mathbf{x}_a,t)\zeta'(\mathbf{x}_b,t)\zeta'(\mathbf{x}_c,t)\rangle \;,\quad \cdots$$

neglecting cumulants 3rd order and above we get a ***closed***, ***autonomous***, ***deterministic*** system for the evolution of 1st and 2nd cumulants

$$\begin{aligned}
\partial_t Z &= -\beta V - rZ + \nu\Delta Z - P_K[\mathbf{U}\cdot\boldsymbol{\nabla} Z + \mathcal{R}(C)]\\
\partial_t C &= (I - P_{Ka})\mathcal{A}_a\, C + (I - P_{Kb})\mathcal{A}_b\, C + \varepsilon\, Q
\end{aligned}$$

S3T

where

$$\langle \xi(\mathbf{x}_a,t)\xi(\mathbf{x}_b,t')\rangle = Q(\mathbf{x}_a-\mathbf{x}_b)\,\delta(t-t')$$

$$\mathcal{R}(C) \stackrel{\text{def}}{=} \boldsymbol{\nabla}\cdot\left[\frac{\hat{\mathbf{z}}}{2}\times\left(\boldsymbol{\nabla}_a\Delta_a^{-1}+\boldsymbol{\nabla}_b\Delta_b^{-1}\right)C\right]_{\mathbf{x}_a=\mathbf{x}_b} = \boldsymbol{\nabla}\cdot\langle \mathbf{u}'\zeta'\rangle$$

$$\mathcal{A} \stackrel{\text{def}}{=} -\mathbf{U}\cdot\boldsymbol{\nabla} - [\beta\partial_x - (\Delta\mathbf{U})\cdot\boldsymbol{\nabla}]\Delta^{-1} - r + \nu\Delta$$

S3T turbulent fixed states

$$\mathbf{U}^e = 0 \quad , \quad C^e(\mathbf{x}_a - \mathbf{x}_b)$$

extensively studied in the literature

zero mean flow + non-zero homogeneous 2nd-order eddy statistics

$$\mathbf{U}^e(\mathbf{x}) = \Big(U^e(y), 0\Big) \quad , \quad C^e(x_a - x_b, y_a, y_b)$$

the focus of this work

zonal jet mean flow + non-zero zonally homogeneous 2nd-order eddy statistics

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stability of turbulent states

perturbations ($\delta Z, \ \delta C$) about S3T turbulent equilibrium state obey the linearized S3T equations:

we linearized about a turbulent state!

$$\partial_t \, \delta Z = \mathcal{A}(\mathbf{U}^e) \, \delta Z + P_K \left[\mathcal{R}(\delta C)\right]$$

$$\partial_t \, \delta C = (I - P_{Ka}) \left[\mathcal{A}_a(\mathbf{U}^e)\delta C + \delta \mathcal{A}_a C^e\right] + (I - P_{Kb}) \left[\mathcal{A}_b(\mathbf{U}^e)\delta C + \delta \mathcal{A}_b C^e\right]$$

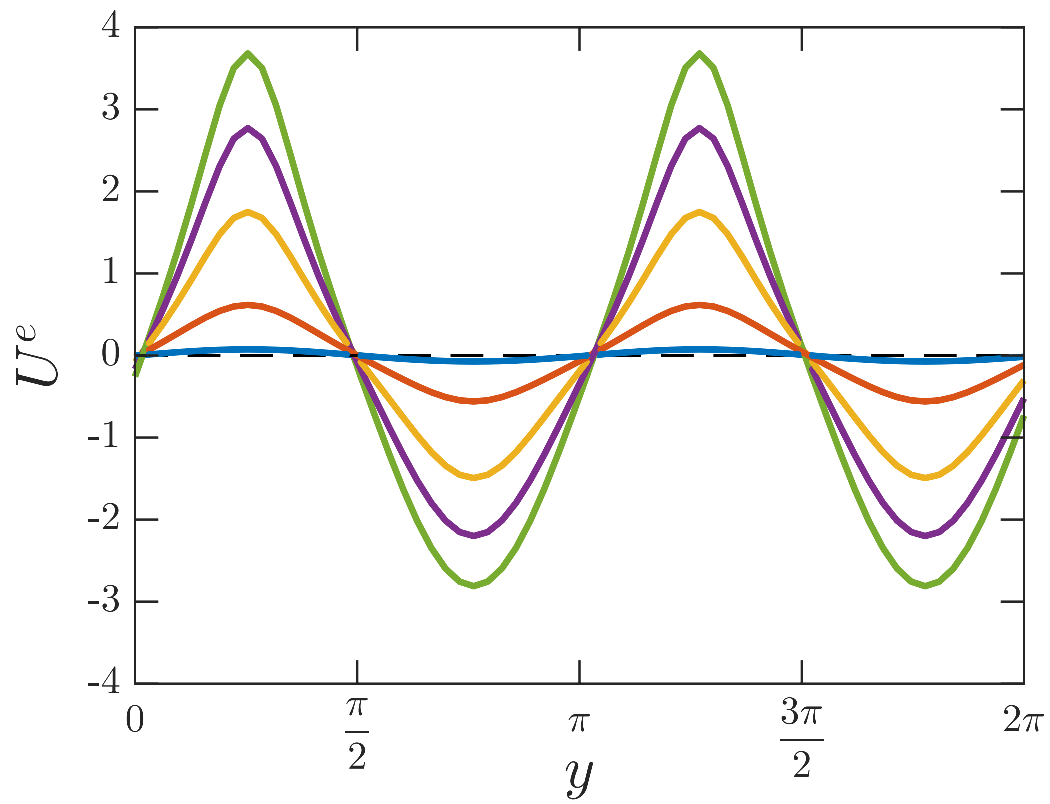
solve the eigenvalue problem (non-trivial — very high dimensionality)

zonal jet S3T equilibria

for the specified forcing structure and $\beta = 10$, $r = 0.15$, $\nu = 10^{-2}$ (nondimensional Earth-like values)

by varying ε we find a series of zonal jet S3T statistical equilibria

$$\mathbf{U}^e(\mathbf{x}) = \Big(U^e(y), 0\Big)$$

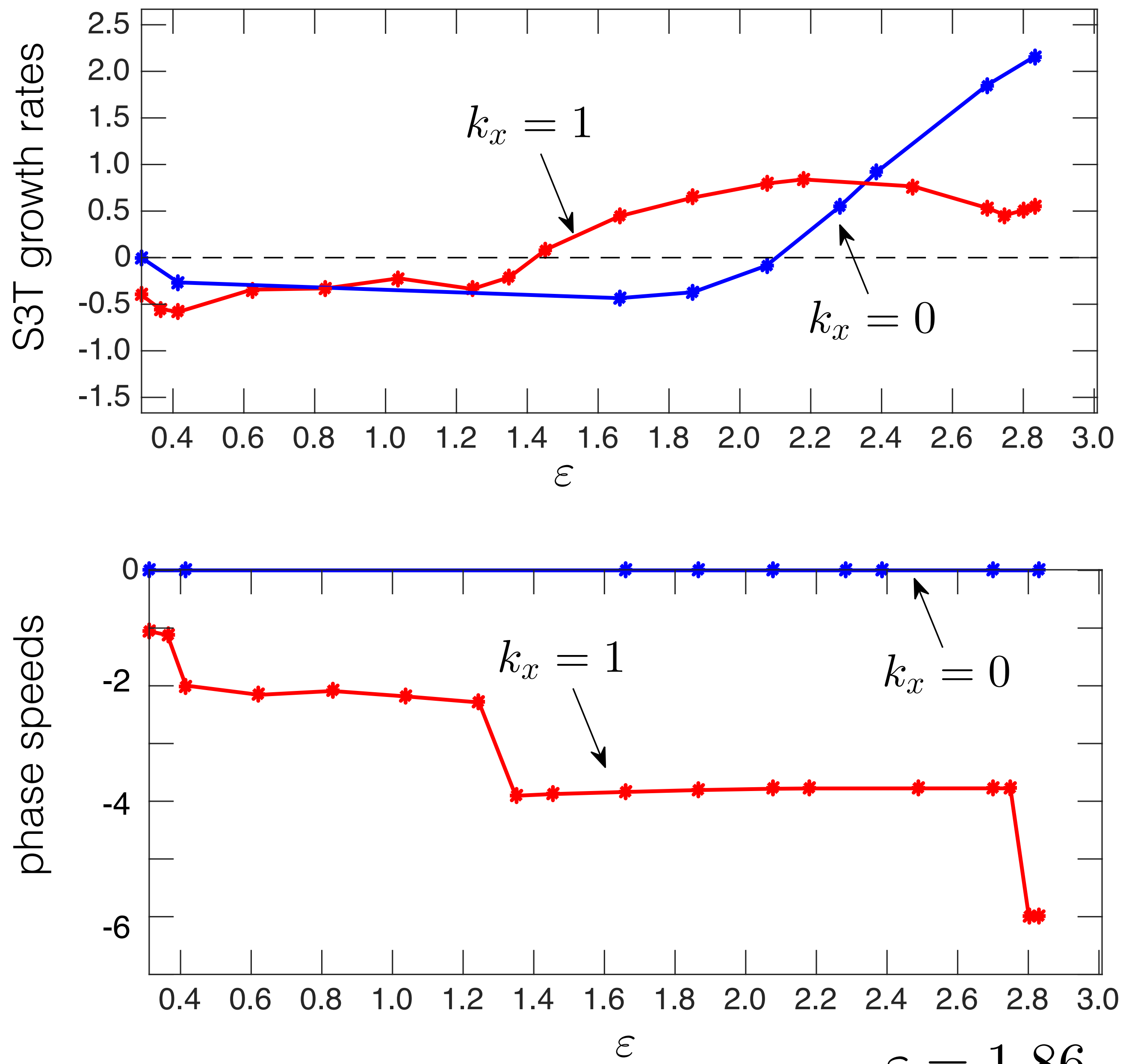


+

$$C^e(x_a - x_b, y_a, y_b)$$

for each of them

S3T stability predicted for the 2-jet S3T equilibria



k_x : zonal wavenumber

$\varepsilon = 1.86$

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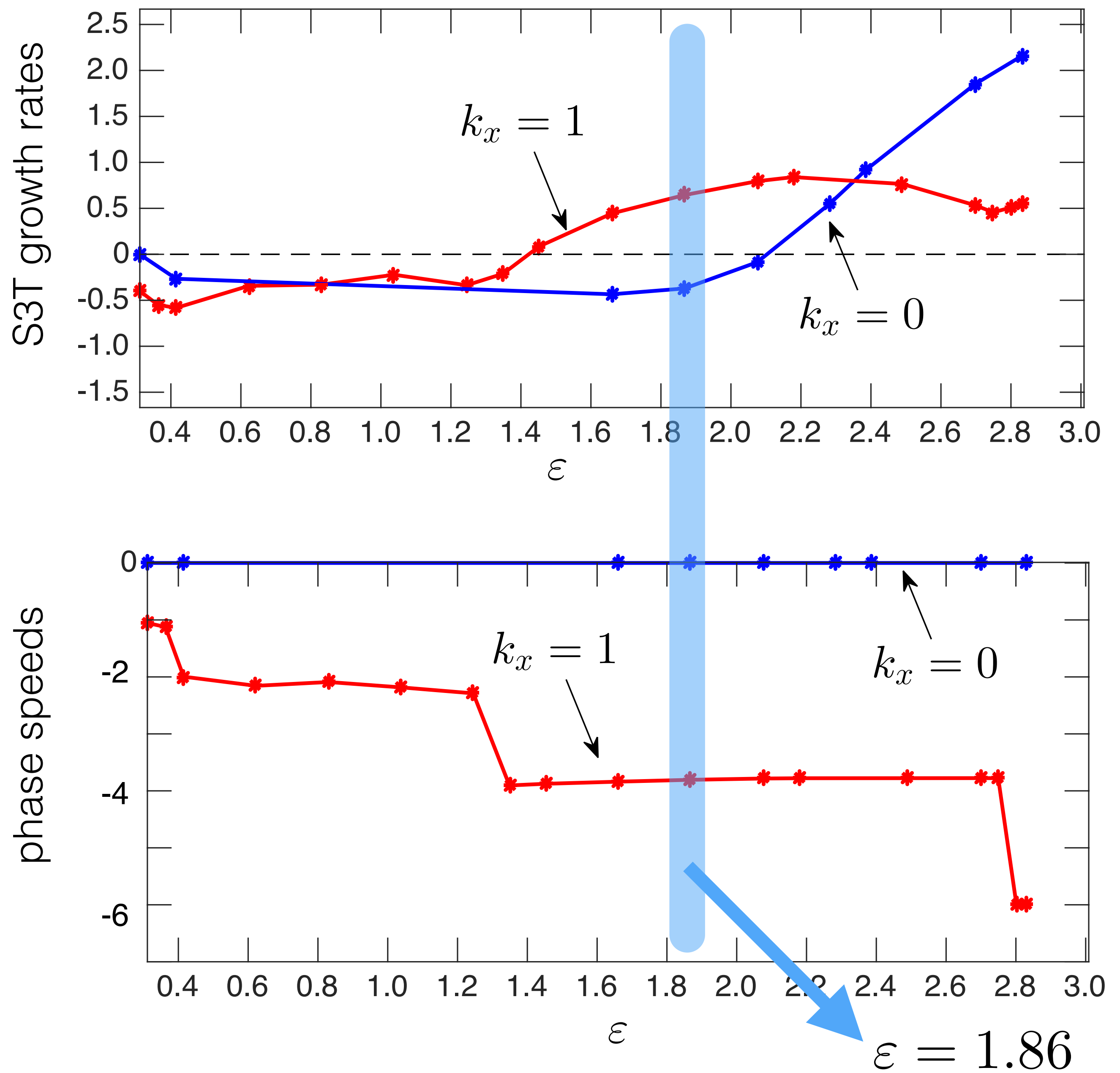
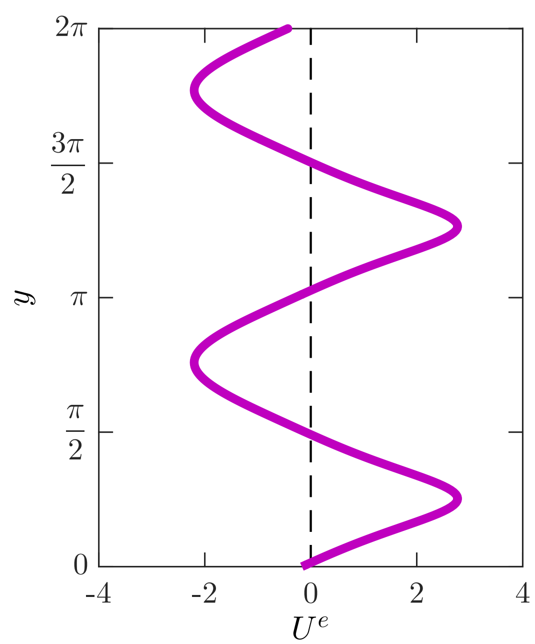
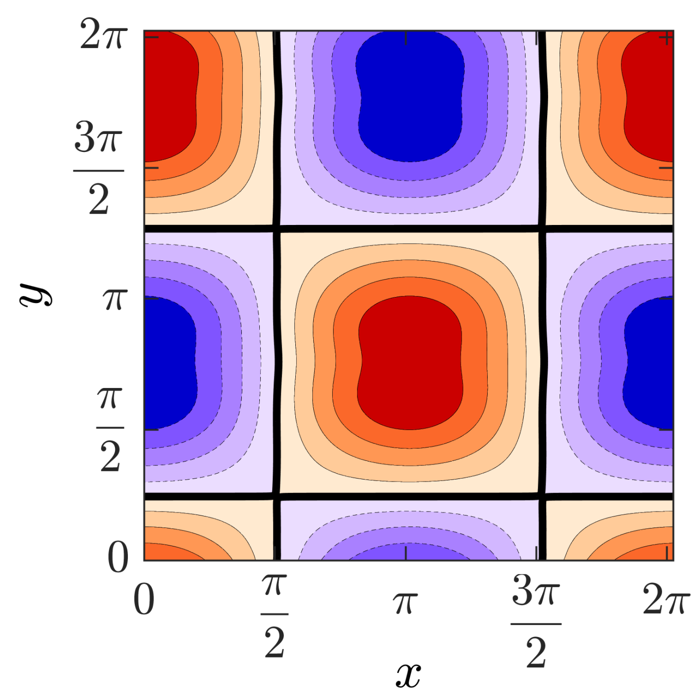


k_x : zonal wavenumber

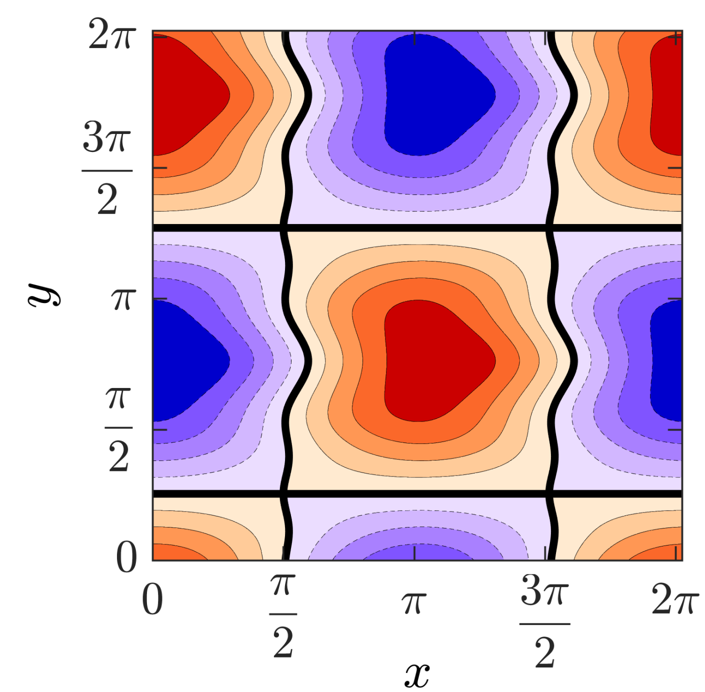
hydrodynamic stability Vs S3T stability of the jet at $\varepsilon = 1.86$



least stable mode
growth rate: $-r$

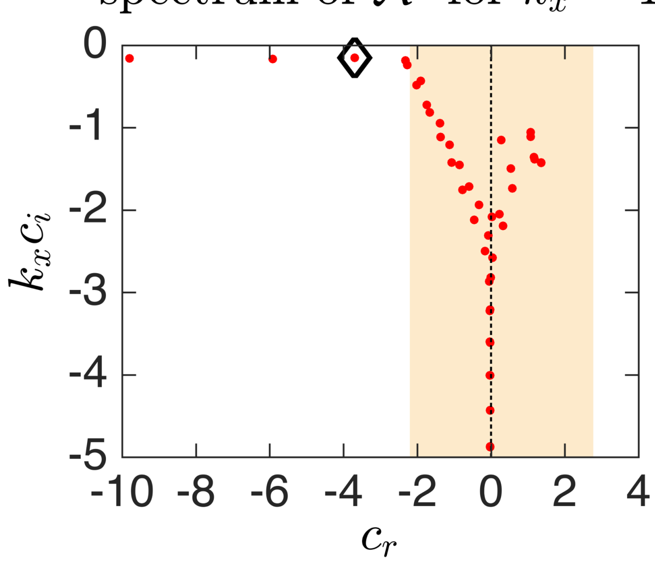


most unstable S3T mode
growth rate: $0.66r$



spectrum of $\mathcal{A}^e$ for $k_x = 1$

growth rates



phase speeds

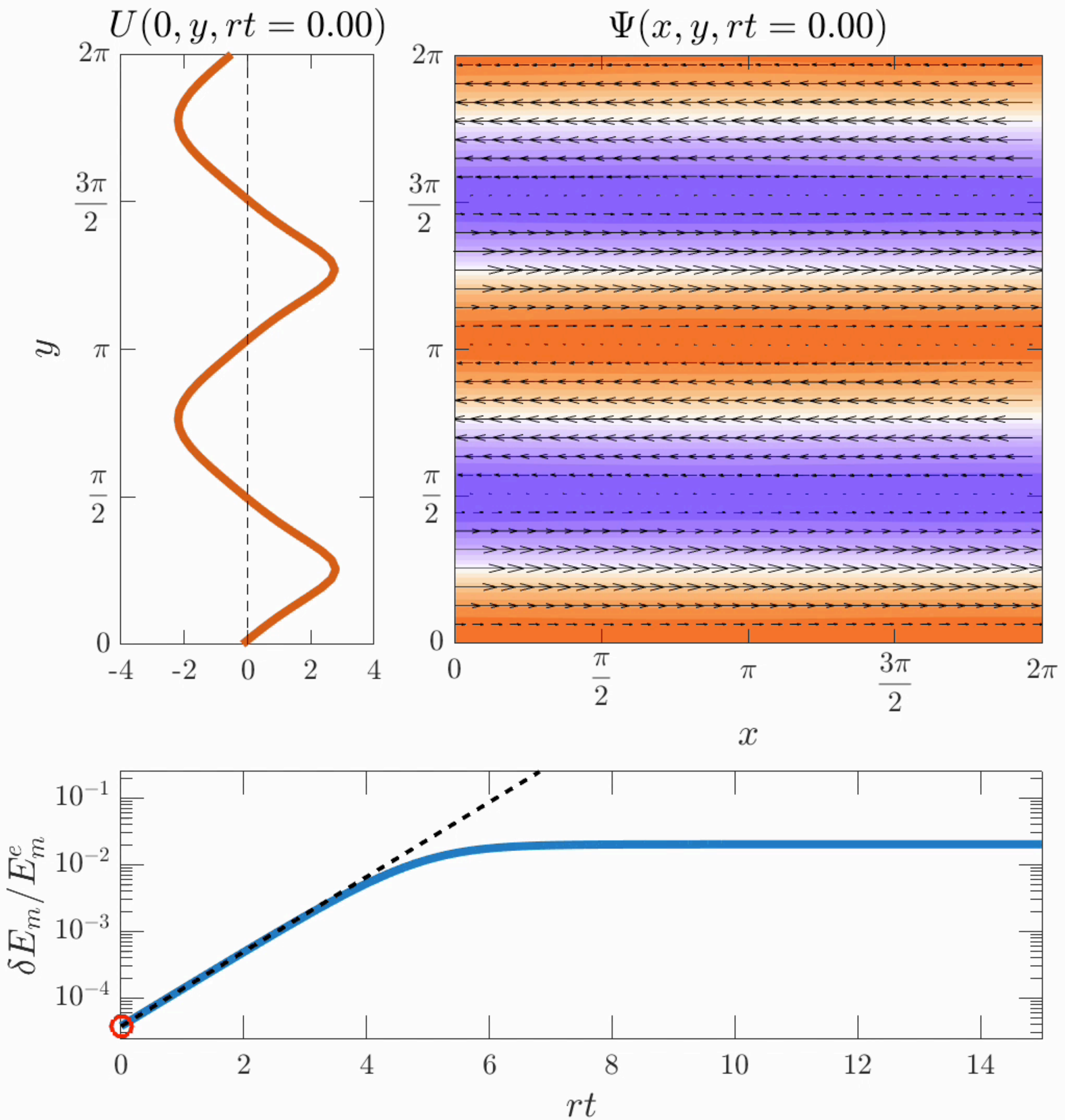
hydrodynamic stability of the laminar jet

S3T stability of the turbulent jet

STABLE Vs UNSTABLE

growth and equilibration of S3T wave instability

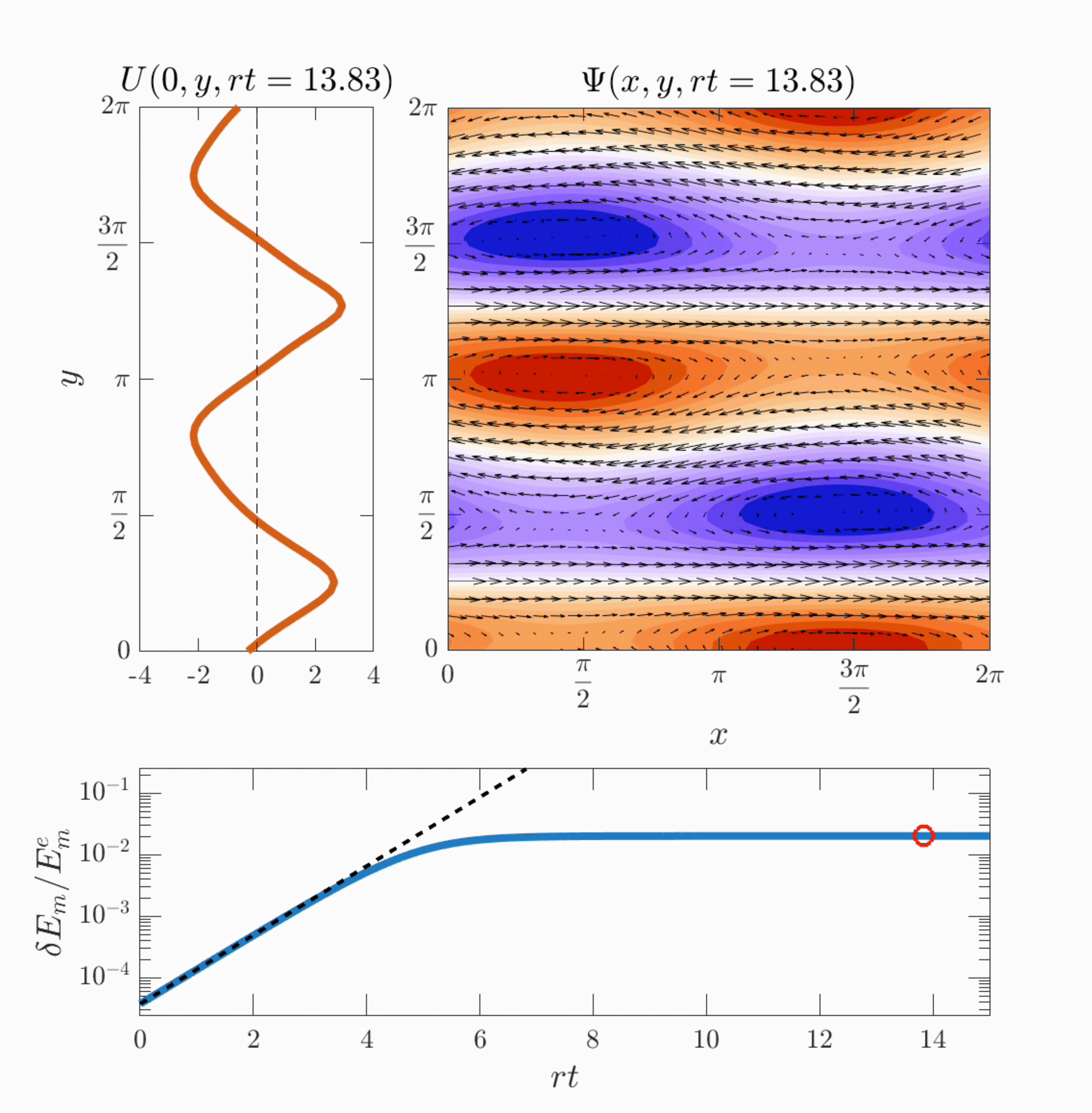
$\varepsilon = 1.86$



energy of the deviation from zonal jet equilibrium over energy of the zonal jet

growth and equilibration of S3T wave instability

$\varepsilon = 1.86$



energy of the deviation from zonal jet equilibrium over energy of the zonal jet

conclusions

- Planetary turbulence may *bifurcate* to a state in which coherent large-scale waves coexist with jets
- These large-scale waves are equilibrated external Rossby waves destabilized by the turbulence
- This work provides a new mechanism for understanding planetary scale waves in the atmosphere and may even provide explanation for the existence of the ovals that are embedded in the turbulent jets of the outer planets (e.g. Jupiter)

Constantinou, Farrell & Ioannou (2016) Statistical state dynamics of jet/wave coexistence in barotropic beta-plane turbulence, *J. Atmos. Sci.*, doi:`10.1175/JAS-D-15-0288.1`, in press.

thanks