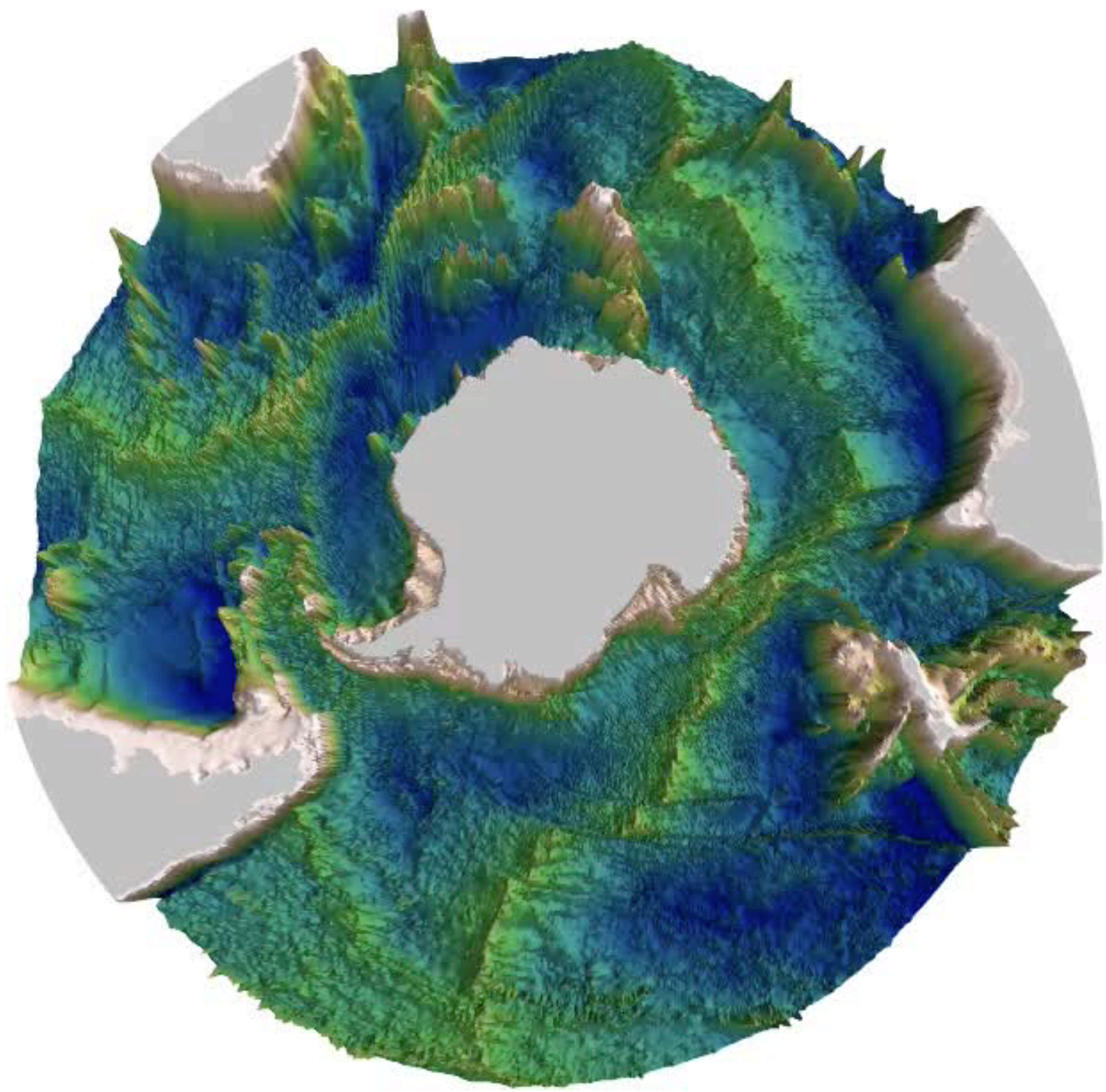
Eddy saturation in a barotropic model

Navid Constantinou
Scripps Institution of Oceanography

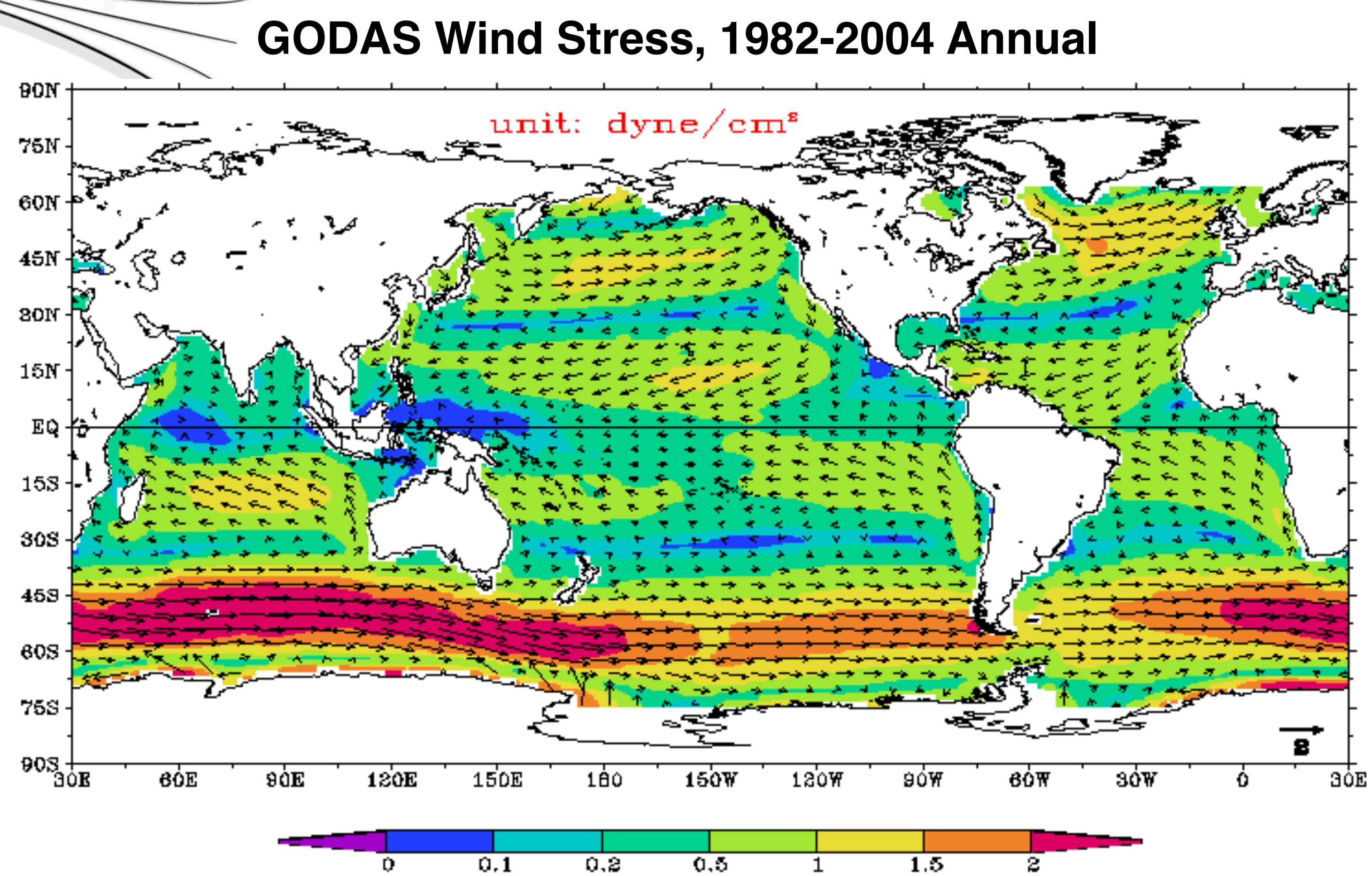


Animated view of the Southern Ocean topography by V. Tamsitt.

Based on 1/4-degree resolution topography (Smith & Sandwell). Most small-scale ocean topography is unknown.

what drives the Antarctic Circumpolar Current?

Climate Prediction Center



strong westerly winds blow over the Southern Ocean transferring momentum through wind stress at the surface

how is this momentum balanced?

Note on the Dynamics of the Antarctic Circumpolar Current

By W. H. MUNK and E. PALMÉN

Abstract

Unlike all other major ocean currents, the Antarctic Circumpolar Current probably does not have sufficient frictional stress applied at its lateral boundaries to balance the wind stress. The balancing stress is probably applied at the bottom, largely where the major submarine ridges lie in the path of the current. The meridional circulation provides a mechanism for extending the current to a large enough depth to make this possible.

start with the zonal angular momentum equation

$$(\partial_t + u\partial_x + v\partial_y + w\partial_z)\underbrace{\left(u - \int^y f(y')\,\mathrm{d}y'\right)}_{\substack{\stackrel{\text{def}}{=} a \\ \text{angular momentum}}} + p_x = \tau_z$$

vertically integrate, top to bottom

$$\partial_t \int_{-h}^{0} a\,\mathrm{d}z + \partial_x\left[\int_{-h}^{0} ua + p\,\mathrm{d}z\right] + \partial_y \int_{-h}^{0} va\,\mathrm{d}z =$$

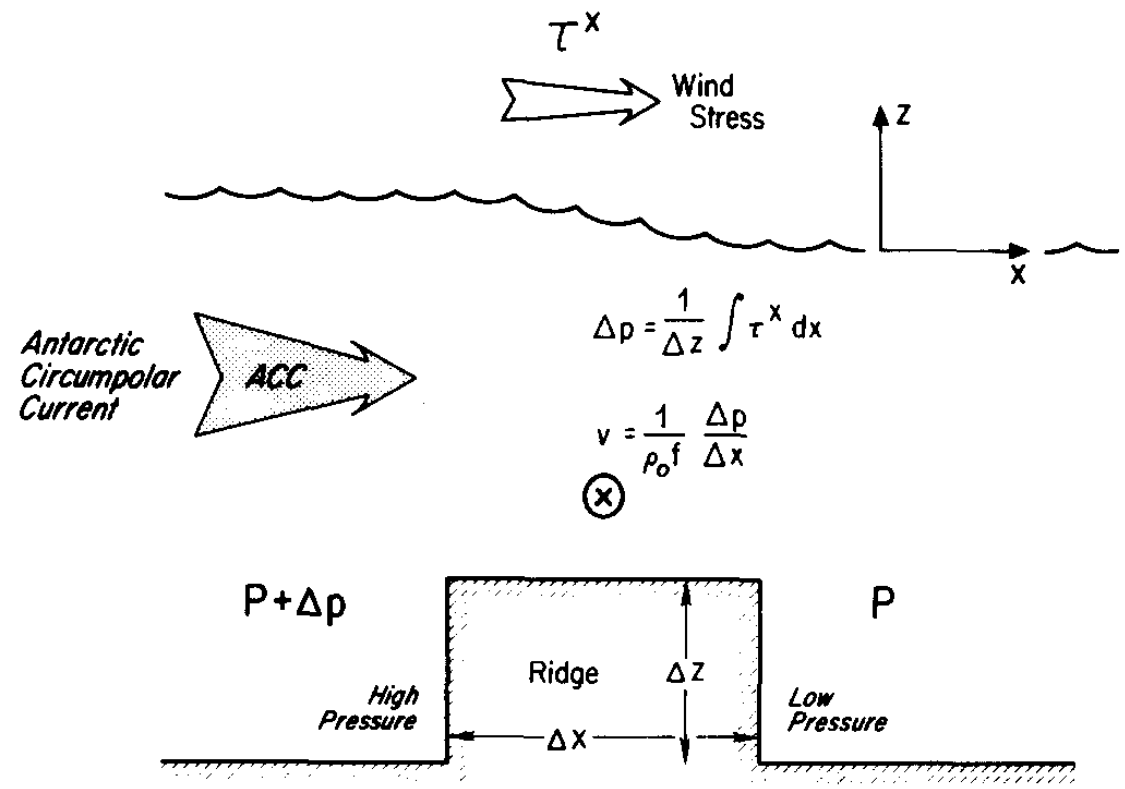
$$= \underbrace{\tau(0)}_{\text{wind stress}} - \underbrace{\tau(-h)}_{\text{bottom drag}} + \underbrace{h_x p(-h)}_{\text{form stress}}$$

we've used integration by parts:

$$\int_{-h}^{0} p_x\,\mathrm{d}z = \partial_x \int_{-h}^{0} p\,\mathrm{d}z - h_x p(-h)$$

topographic form stress

Johnson & Bryden 1989



Schematic presentation of bottom form drag or mountain drag. Wind stress imparted
astward momentum in the water column is removed by the pressure difference across the ridge.

$$\partial_t \int_{-h}^{0} a \,\mathrm{d}z + \partial_x \left[\int_{-h}^{0} ua + p \,\mathrm{d}z \right] + \partial_y \int_{-h}^{0} va \,\mathrm{d}z =$$

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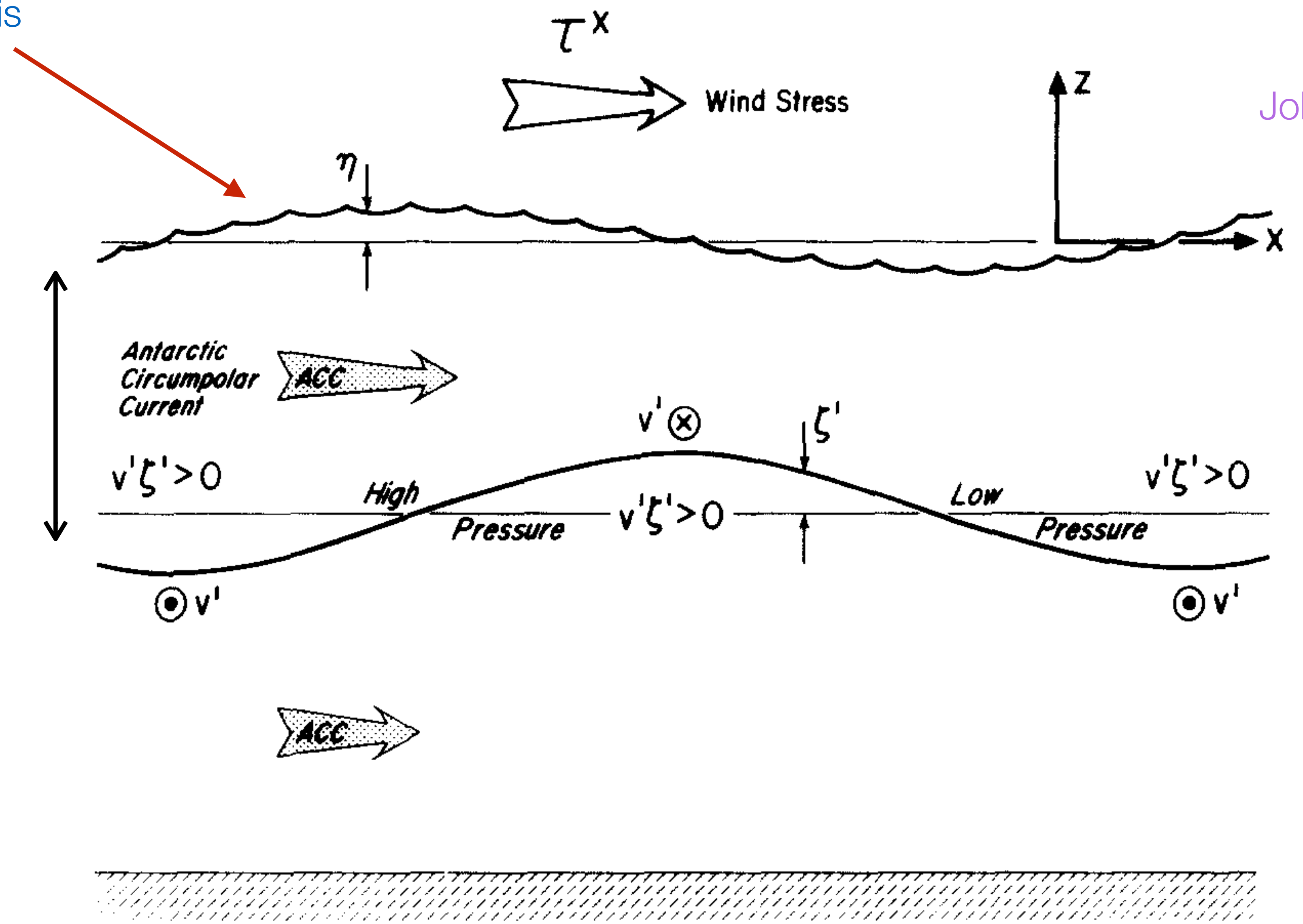
Topographic form stress is a purely **barotropic** process.

interfacial form stress

Note: wind stress is also form stress!

Johnson & Bryden 1989

vertically integrate from the sea-surface down to a moving buoyancy surface



Schematic presentation of interfacial form drag. Correlations of perturbations in the interface height, ζ', and the meridional velocity, V' ($\odot$ indicating poleward flow and $\otimes$ indicating equatorward flow), which are related to pressure perturbations by geostrophy, allow the upper layer to exert an eastward force on the lower layer and the lower layer to exert a westward force on the upper layer; thus effecting a downward flux of zonal momentum.

Interfacial form stress requires **baroclinicity**.

the most popular scenario for the momentum balance

- momentum in imparted at the surface by wind,
- isopycnals slope, creating baroclinic instability,
- momentum is transferred downwards by interfacial eddy form stress
- momentum reaches the bottom where is transferred to the solid Earth by topographic form stress.

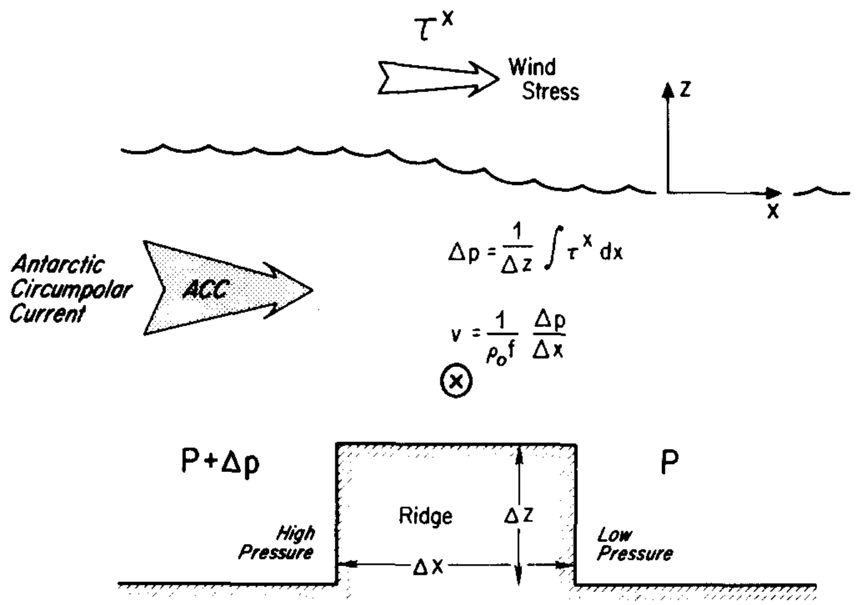
Johnson & Bryden 1989

$$\begin{matrix}\text{isopycnal}\\ \text{slope}\end{matrix} = \left[-\frac{\tau_s}{f\,\kappa}\right]^{1/2}$$

Marshall & Radko 2003

This baroclinic scenario sets up the ACC transport (e.g. the transport through Drake Passage).

but what about barotropic dynamics?



The sea surface pressure gradient can be *directly* communicated to the bottom.

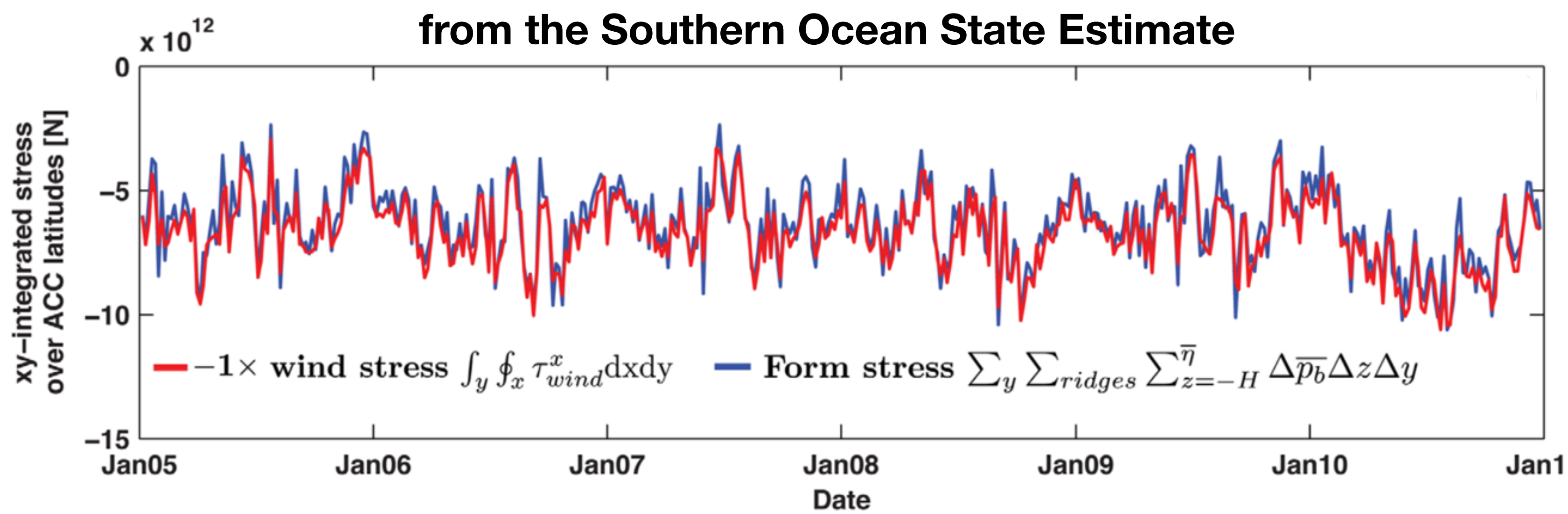
And it will be, unless compensated by internal isopycnal gradients.

Isn't barotropic "communication" much simpler?

wind stress is *rapidly* communicated to the bottom through barotropic processes

Barotropic processes are fast (~days).

Baroclinic processes are much slower (~years).



Masich, Chereskin, and Mazloff 2015

~90% of variance in the topographic form stress signal is explained by the **0-day** time lag.

Similar statements also made by:
Straub 1993, Ward & Hogg 2011, Rintoul et al. 2014, Peña Molino et al. 2014, Donohue et al. 2016.

topographic form stress $= \quad h_x \, p(-h)$

"I don't know why I don't care about the bottom of the ocean, but I don't."

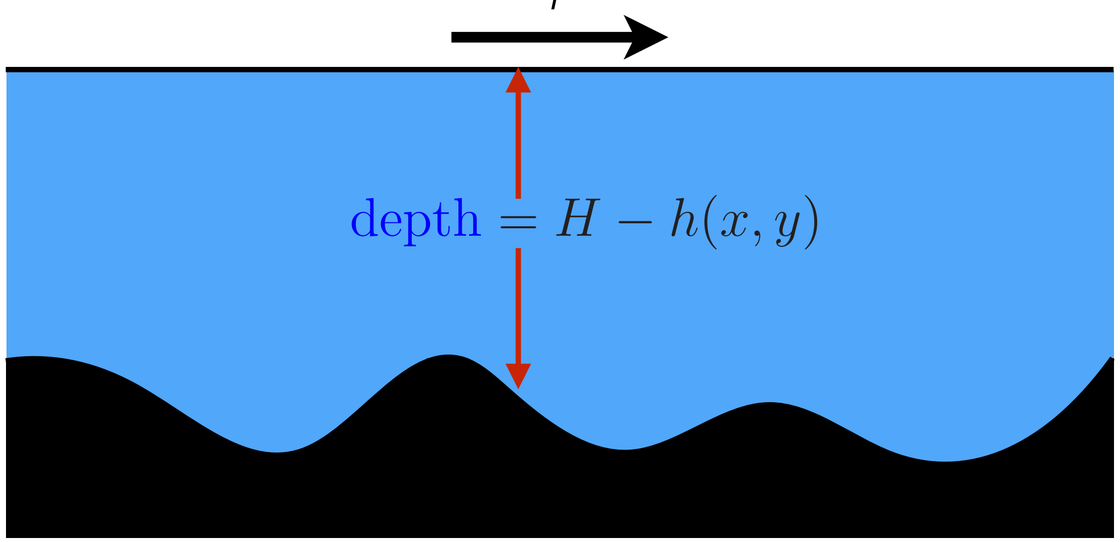
"My dear, if you are interested in ACC transport then, despite whether the ***baroclinic*** *or* ***barotropic*** *scenario is pertinent, you* ***should*** *care for the bottom of the ocean."*

the plan

Revisit an old barotropic QG model on a beta-plane.

(Hart 1979, Davey 1980, Bretherton & Haidvogel 1976, Holloway 1987, Carnevale & Fredericksen 1987)

steady mean zonal wind stress
F



A distinctive feature of this model is a "large-scale barotropic flow" *U*(*t*).

this is the ACC

topographic PV $\eta = \dfrac{f_0 h}{H}$

QGPV $q = \underbrace{\psi_{xx} + \psi_{yy}}_{\zeta} + \eta + \beta y$

total streamfunction $-U(t)y + \psi(x,y,t)$

Study how momentum is balanced by topographic form stress and investigate the requirements for eddy saturation.

a barotropic QG model for mid-ocean region

total streamfunction $-U(t)y + \psi(x, y, t)$

QGPV $$q = \underbrace{\psi_{xx} + \psi_{yy}}_{\zeta} + \eta + \beta y$$

Material conservation of QGPV

$$(\zeta + \eta)_t + U(\zeta + \eta)_x + J(\psi, \zeta + \eta) \\ + \beta\psi_x = -\mu\zeta + \text{hyper visc.}$$

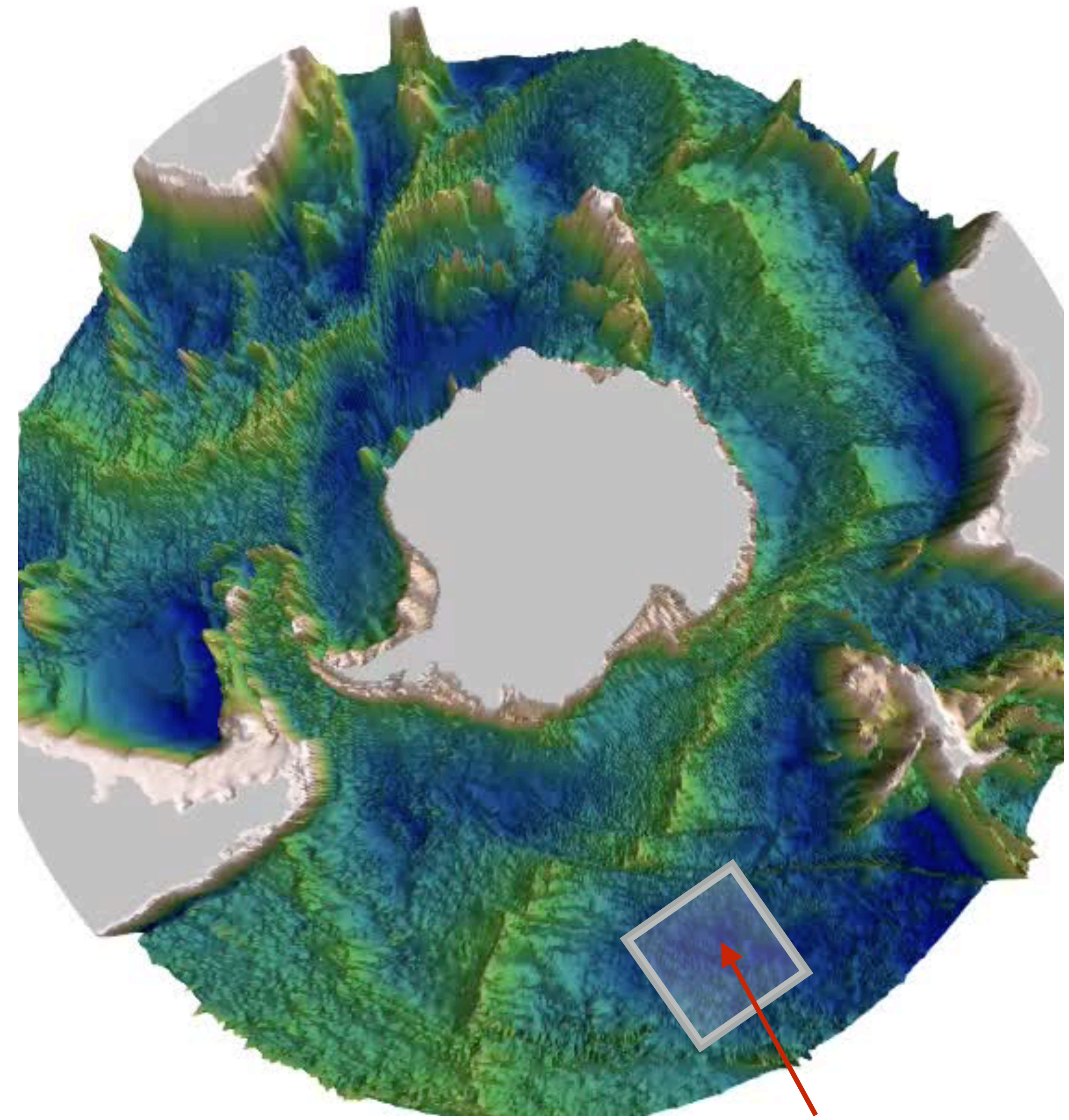
Large-scale zonal momentum

$$U_t = F - \mu U - \langle \psi \eta_x \rangle$$

topographic form stress

$\langle\ \rangle$ is domain average; $F = \dfrac{\tau_{\mathrm{s}}}{\rho_0 H}$ wind stress forcing

periodic boundary conditions



a mid-ocean region size $2\pi L$ x $2\pi L$

(Hart 1979, Davey 1980, Bretherton & Haidvogel 1976, Holloway 1987, Carnevale & Fredericksen 1987)

the large-scale flow equation: $U_t = F - \mu U - \langle \psi \eta_x \rangle$

zonal angular momentum density: $a(x,y,z,t) = u(x,y,z,t) - \int^y f(y')\,\mathrm{d}y'$

vertically integrated zonal angular momentum equation

$$\partial_t \int_{-h}^{0} a\,\mathrm{d}z + \partial_x \left[\int_{-h}^{0} ua + p\,\mathrm{d}z\right] + \partial_y \int_{-h}^{0} va\,\mathrm{d}z =$$

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$$= \underbrace{\tau(0)}_{\text{wind stress}} - \underbrace{\tau(-h)}_{\text{bottom drag}} + \underbrace{h_x p(-h)}_{\text{form stress}}$$

horizontally integrate, drop the boundary fluxes, and divide by the volume

$$U_t = F - \mu U - \langle \psi \eta_x \rangle$$

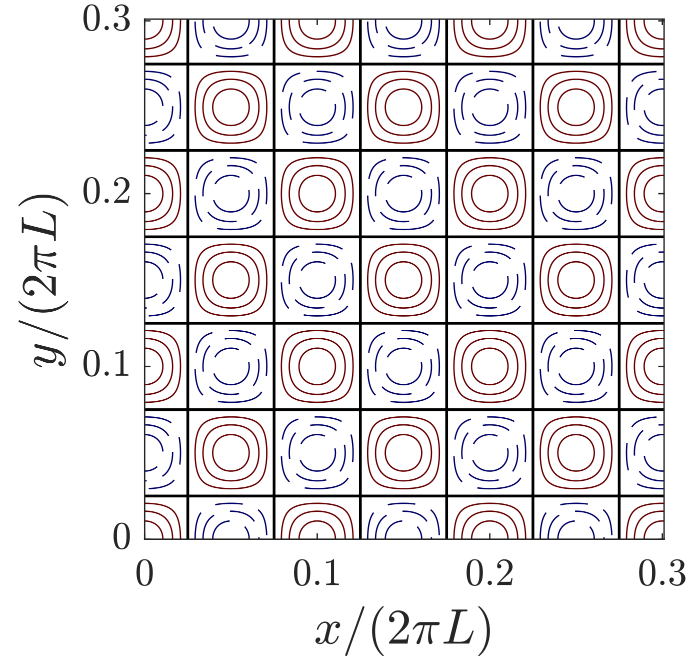
$$U(t) \overset{\mathrm{def}}{=} V^{-1} \iiint a(x,y,z,t)\,\mathrm{d}V$$

vertical & horizontal integral over a mid-ocean region (**not** a zonal average)

Let’s see some solutions.

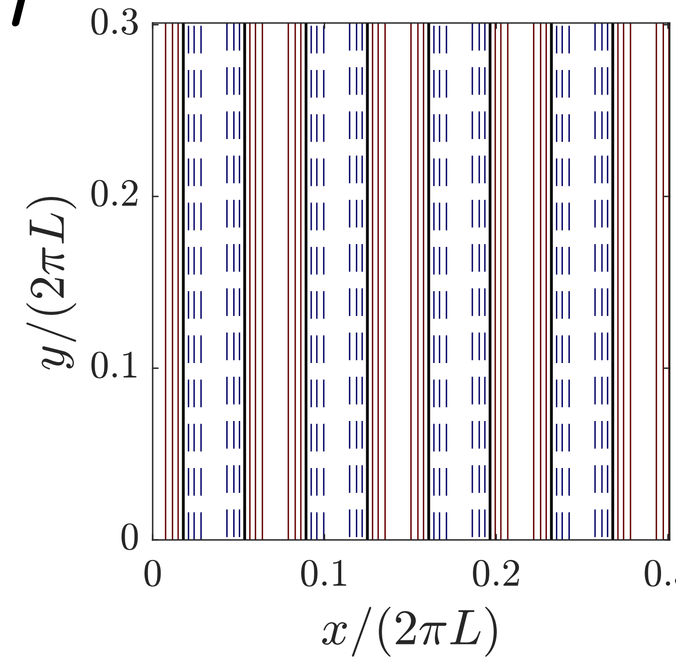
let's use these two topographies

$\eta^{\times}$ $\quad 2\eta_{\mathrm{rms}}\cos(10x/L)\cos(10y/L)$



1/9 of the periodic domain

$\eta^{\parallel}$ $\quad \sqrt{2}\eta_{\mathrm{rms}}\cos(14x/L)$



Both topographies imply the same length-scale: $\ell_\eta = \sqrt{\dfrac{\langle \eta^2 \rangle}{\langle |\boldsymbol{\nabla}\eta|^2 \rangle}} = 0.07L$

let's put some "quasi-realistic" numbers

$$L = 775\ \mathrm{km} \quad H = 4\,\mathrm{km} \quad \rho_0 = 1035\ \mathrm{kg\ m^{-3}}$$

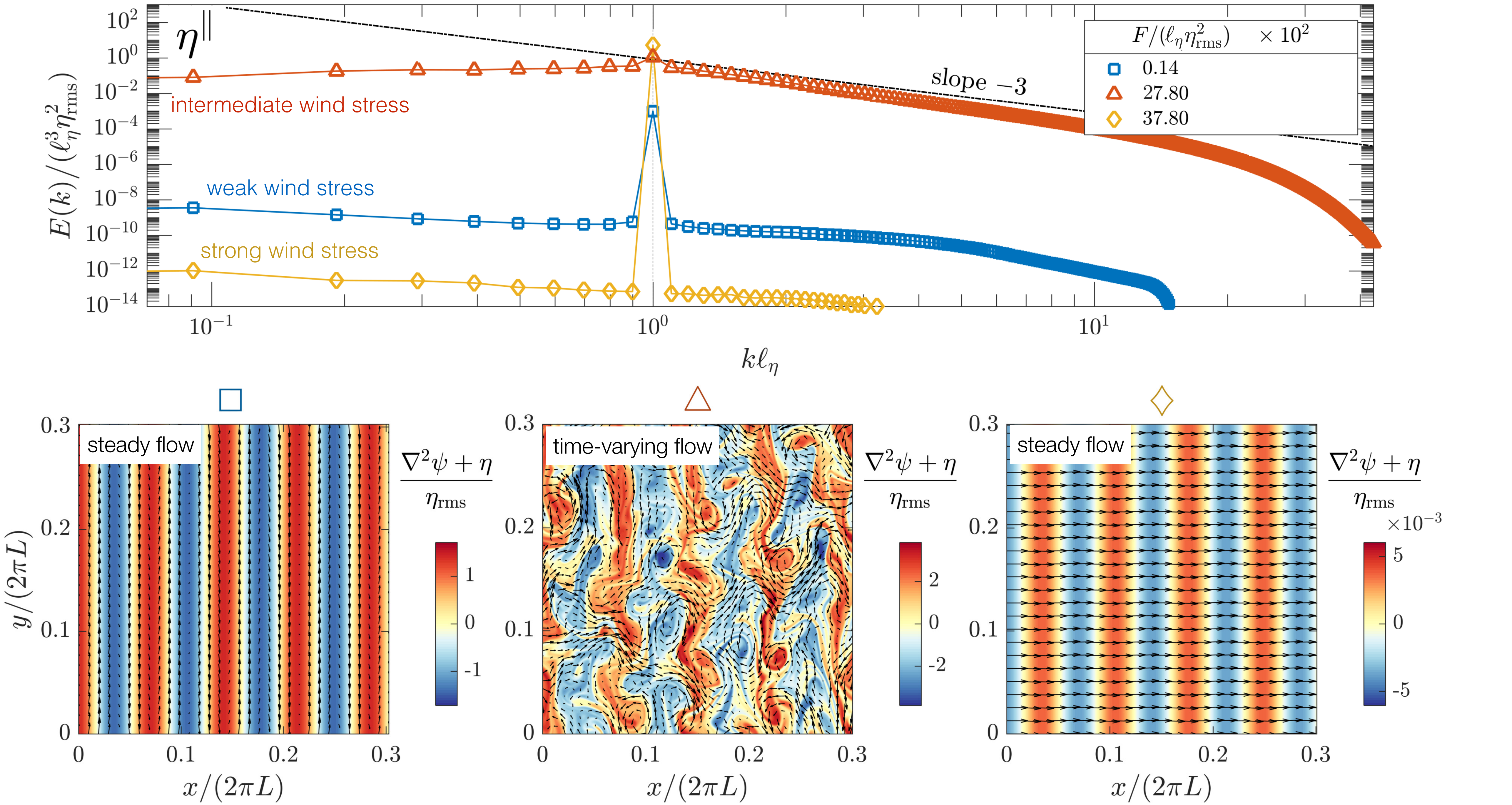
$$f_0 \quad \& \quad \beta \quad \text{for } 60^\circ\mathrm{S}$$

$$\mu = (180\ \text{days})^{-1}$$

$$h_{\mathrm{rms}} = 200\ \mathrm{m} \Rightarrow \eta_{\mathrm{rms}} = (1.8\ \text{days})^{-1}$$

thus, a typical wind stress forcing $\quad \tau = 0.2\,\mathrm{N\,m^{-2}} \Leftrightarrow \dfrac{F}{\ell_\eta \eta_{\mathrm{rms}}^2} \approx 0.02$

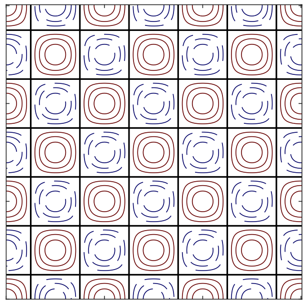
energy spectra & flow snapshots



weak
wind stress

intermediate
("realistic")
wind stress

strong
wind stress



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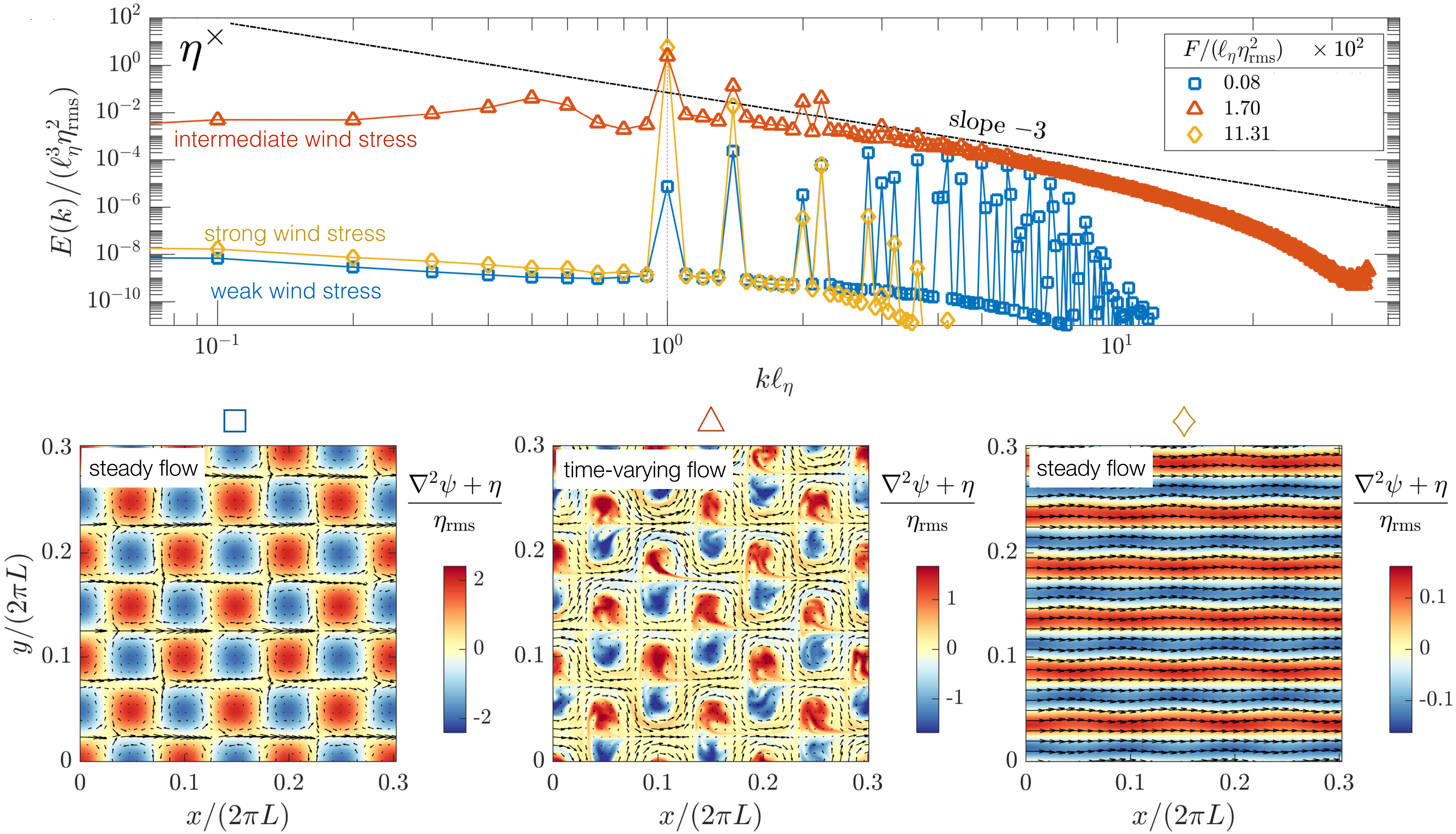


weak
wind stress

intermediate
("realistic")
wind stress

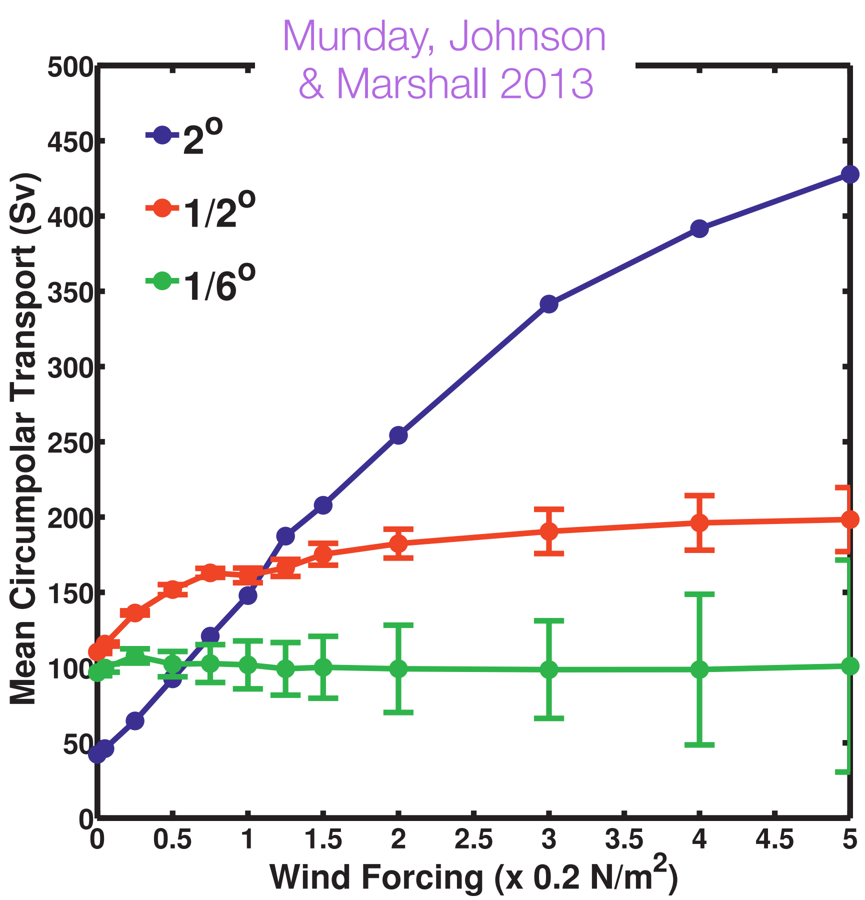
strong
wind stress

Question:

Does this barotropic QG model show eddy saturation?

but first, what is “eddy saturation”?

When the total ACC volume transport
is ***insensitive*** to wind stress increase
we say there is “eddy saturation”.



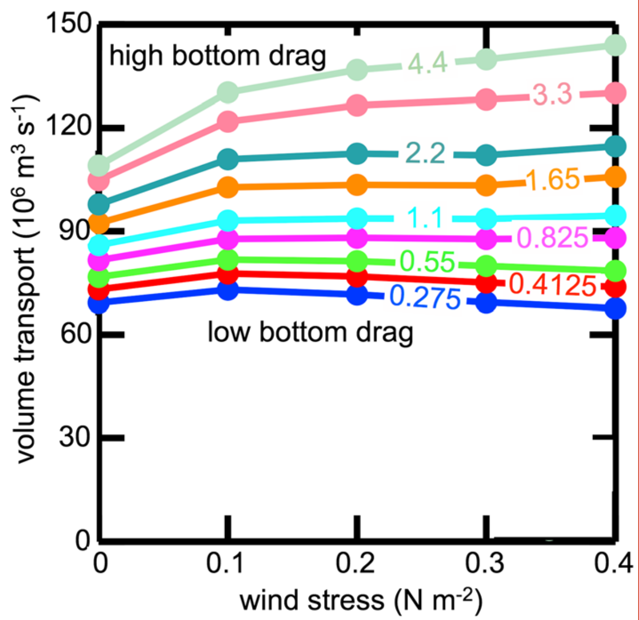
As resolution increases eddies are resolved
and eddy saturation “occurs”.

Eddy saturation means that ACC transport is
insensitive to wind stress — even with no wind!

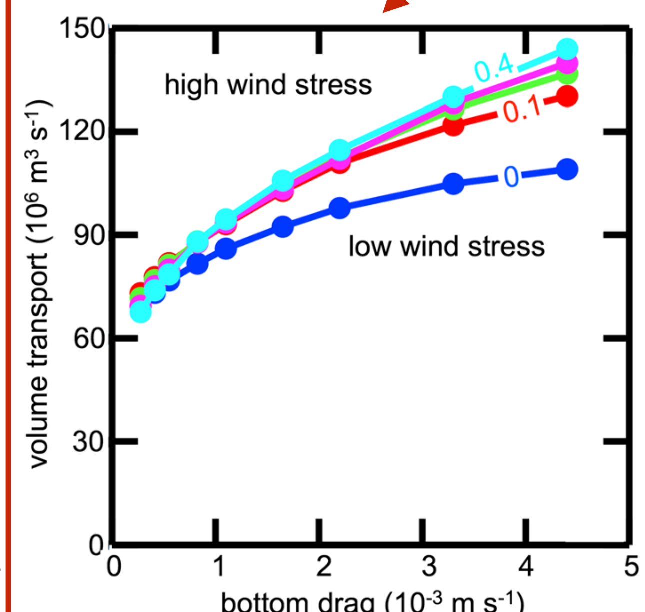
[There are many other examples: Hallberg & Gnanadesikan 2001, Tansley & Marshall 2001, Hallberg & Gnanadesikan 2006, Hogg et al. 2008, Nadeau & Straub 2009, Farneti et al. 2010, Nadeau & Straub 2012, Meredith et al. 2012, Morisson & Hogg 2013, Abernathey & Cessi 2014, Farneti et al. 2015, Nadeau & Ferrari 2015.]

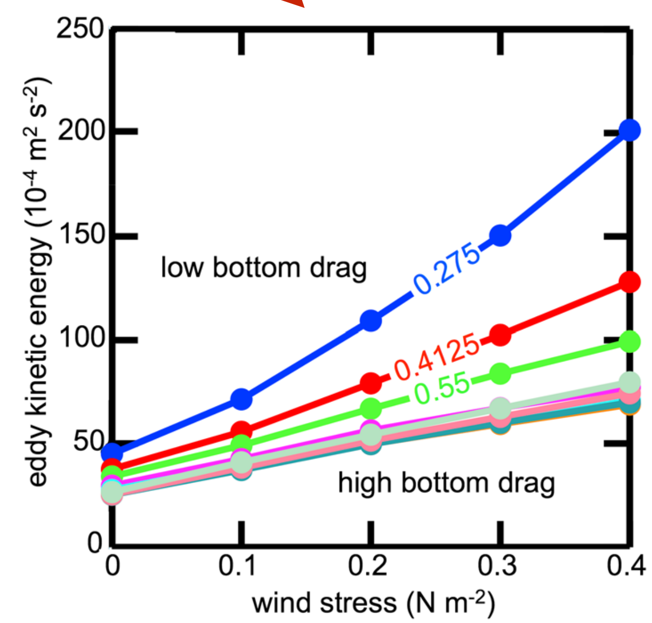
yet more eddy saturation

Marshall Ambaum, Maddison, Munday & Novak 2016



further “symptoms” of eddy saturation





increasing the bottom drag increases the transport

EKE varies linearly with wind stress

remarks on eddy saturation

Eddy saturation is seen in eddy-resolving ocean models.
It's not observed in nature — we can't turn the "wind stress knob".

Eddy saturation was theoretically predicted by Straub (1993).
Straub's argument, though, is *entirely* based on the baroclinic scenario.

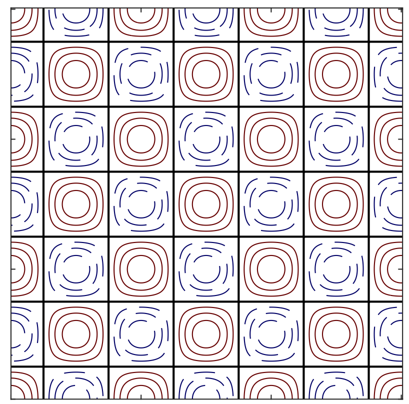
Eddies need to be resolved to see eddy saturation.
Models with parametrized eddies don't show eddy saturation.

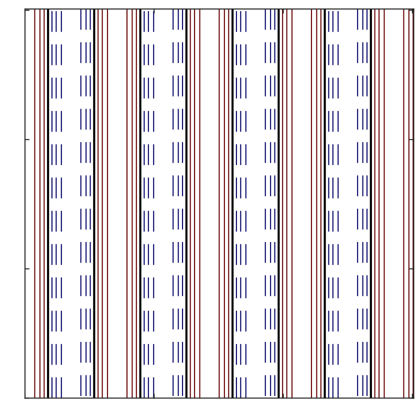
(Recent exceptions are: Mak et al. 2017, Constantinou & Young 2017)

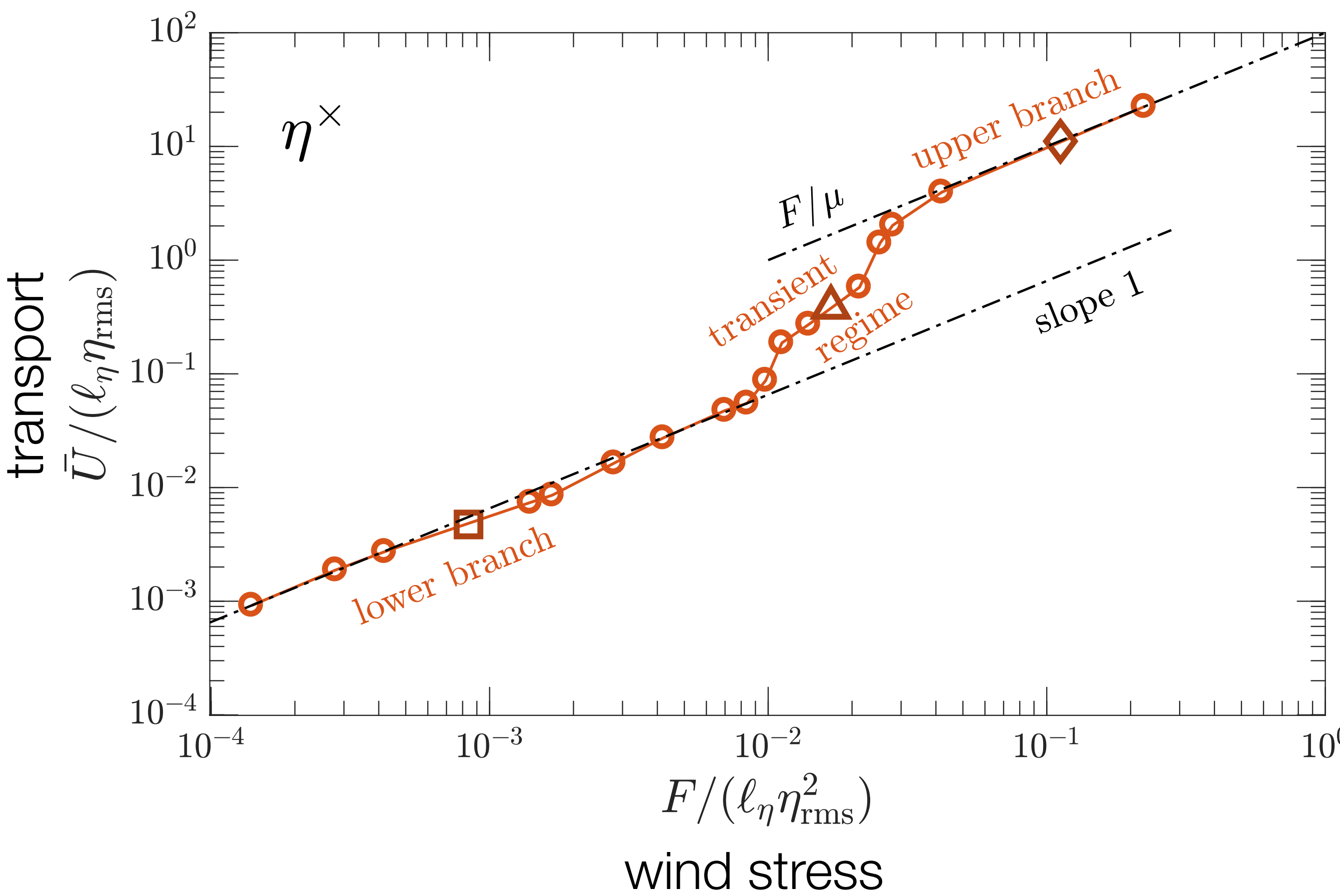
Question:

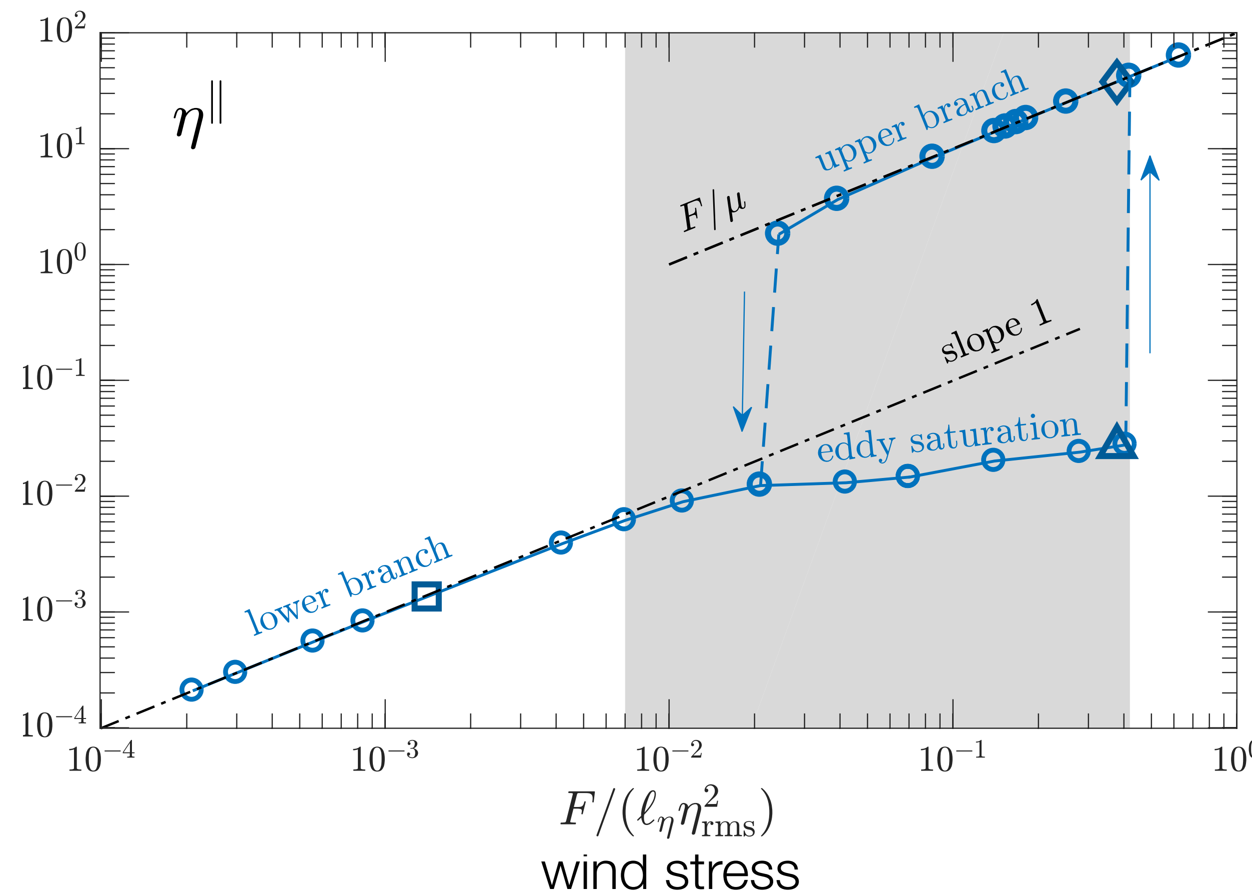
So, does this barotropic QG model show eddy saturation or not?

how does the transport vary with wind stress in our barotropic QG model?









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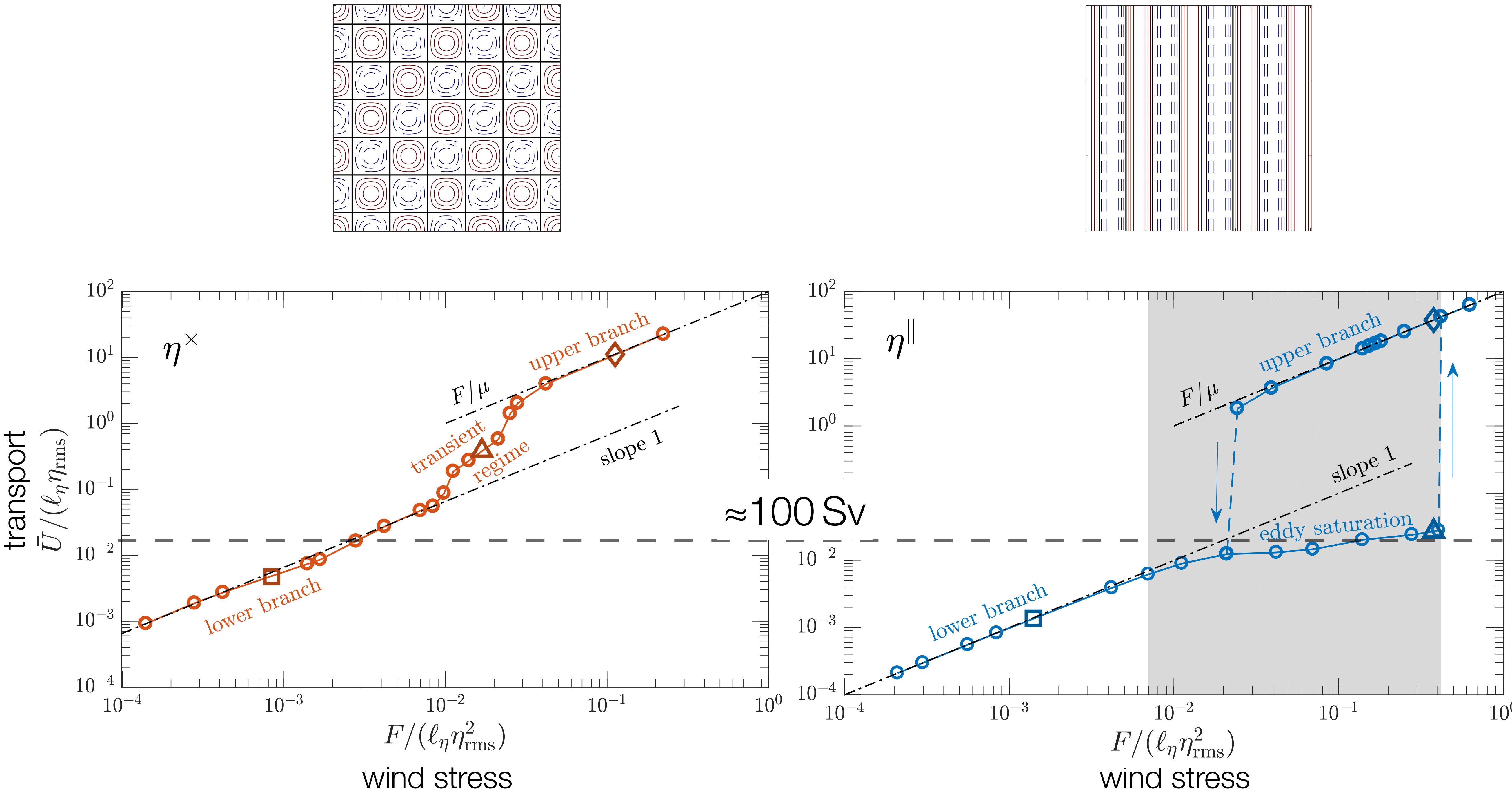

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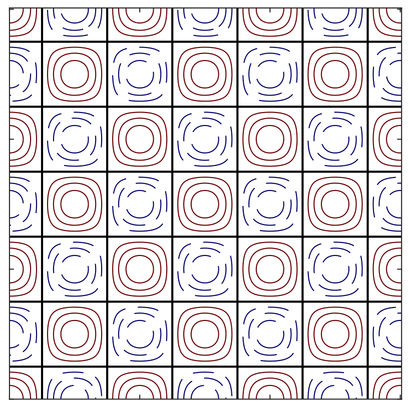
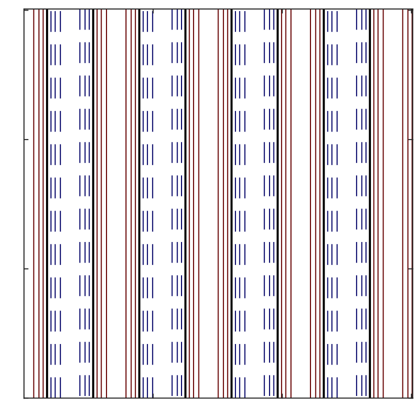
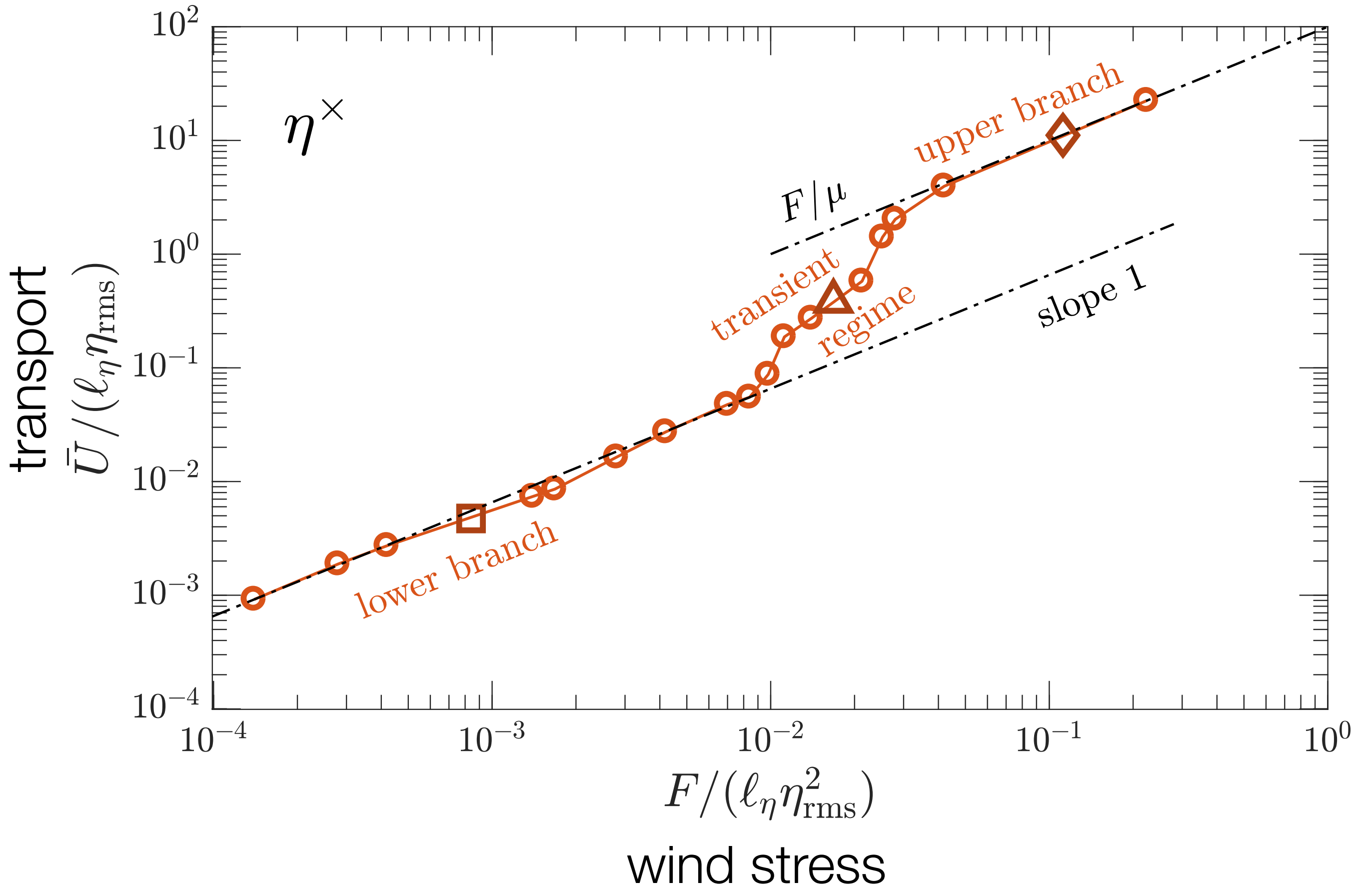
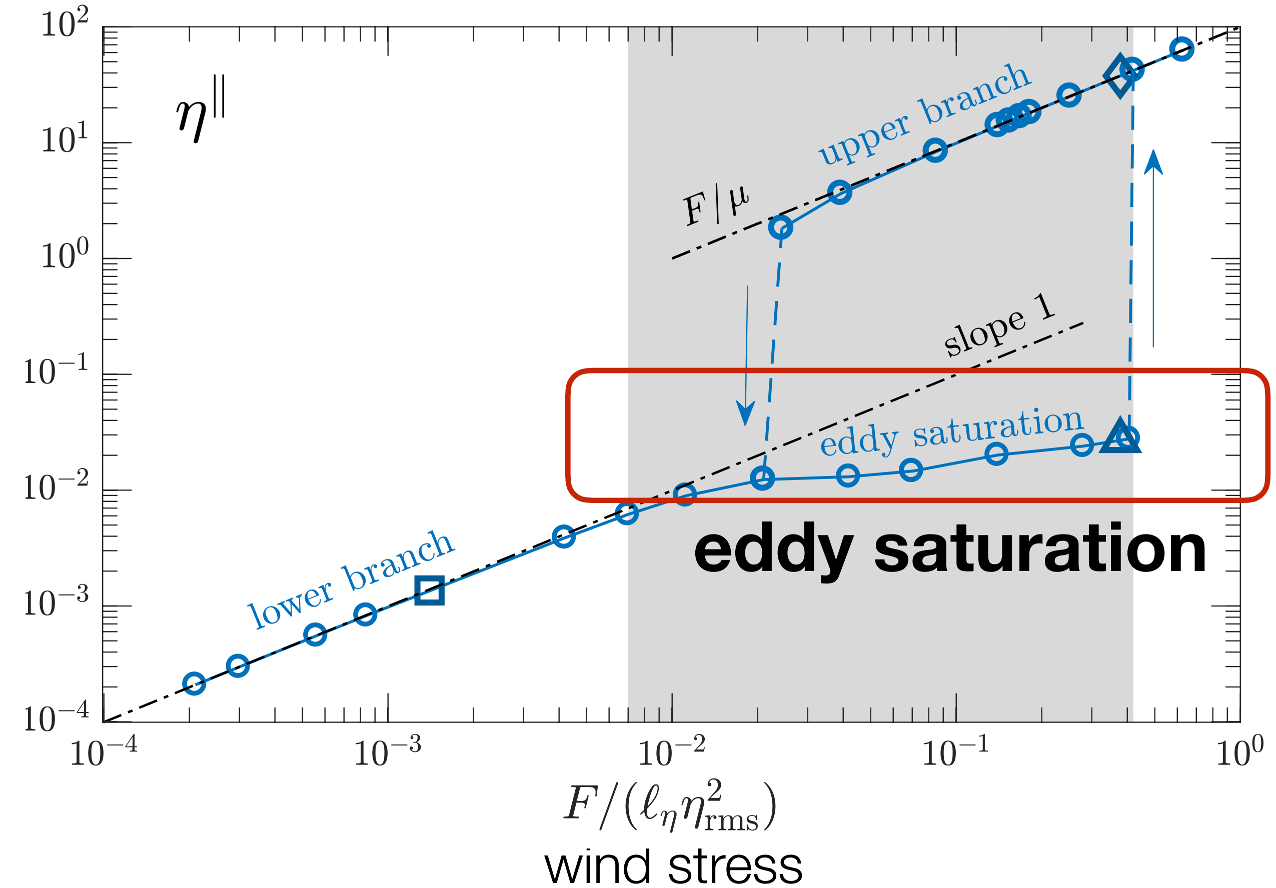








do we understand why?

geostrophic contours $\beta y + \eta(x, y)$

this is $f/(H+h)$
for small Rossby number

main take-home messages

The *main control* parameter for whether eddy saturation occurs is the structure of the geostrophic contours.

Eddy saturation occurs when
the geostrophic contours are "open",
that is, when the geostrophic contours
span the domain in the zonal direction.

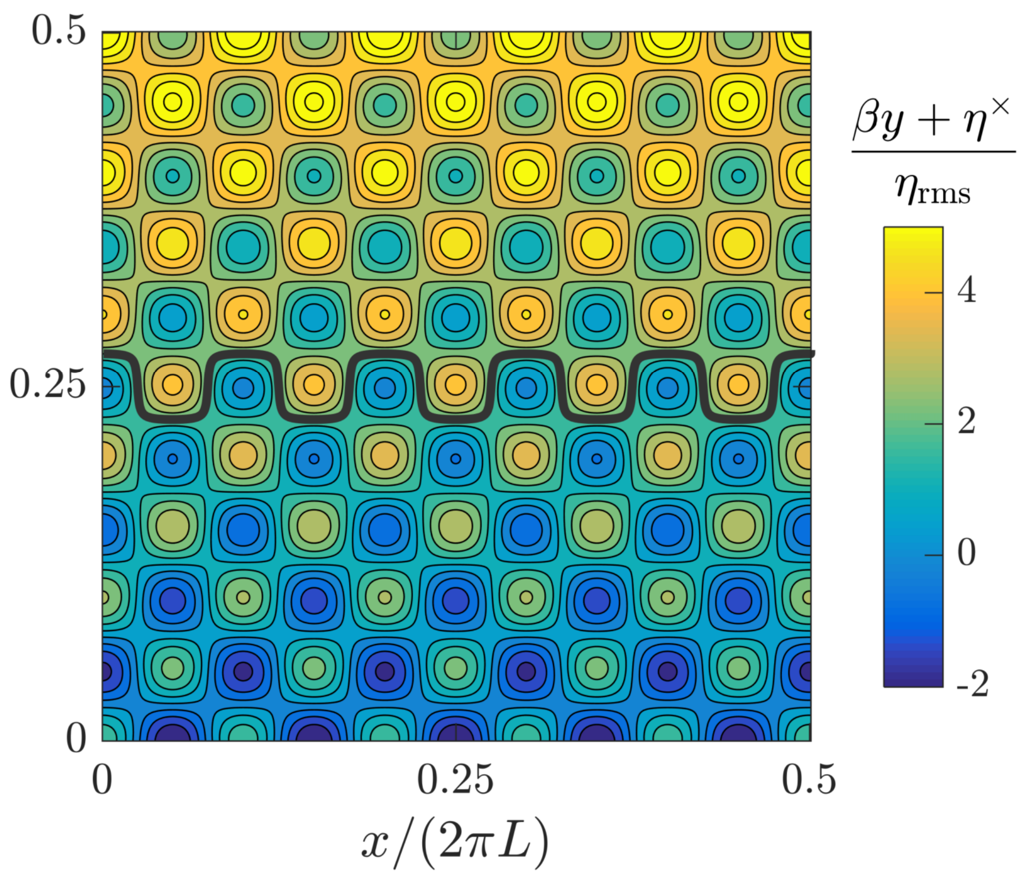
this is a general result
we've seen it in various cases
whatever the topography

Eddy saturation can occur *without* baroclinicity!

What's the structure of the geostrophic contours
for the two simple topographies we've used?

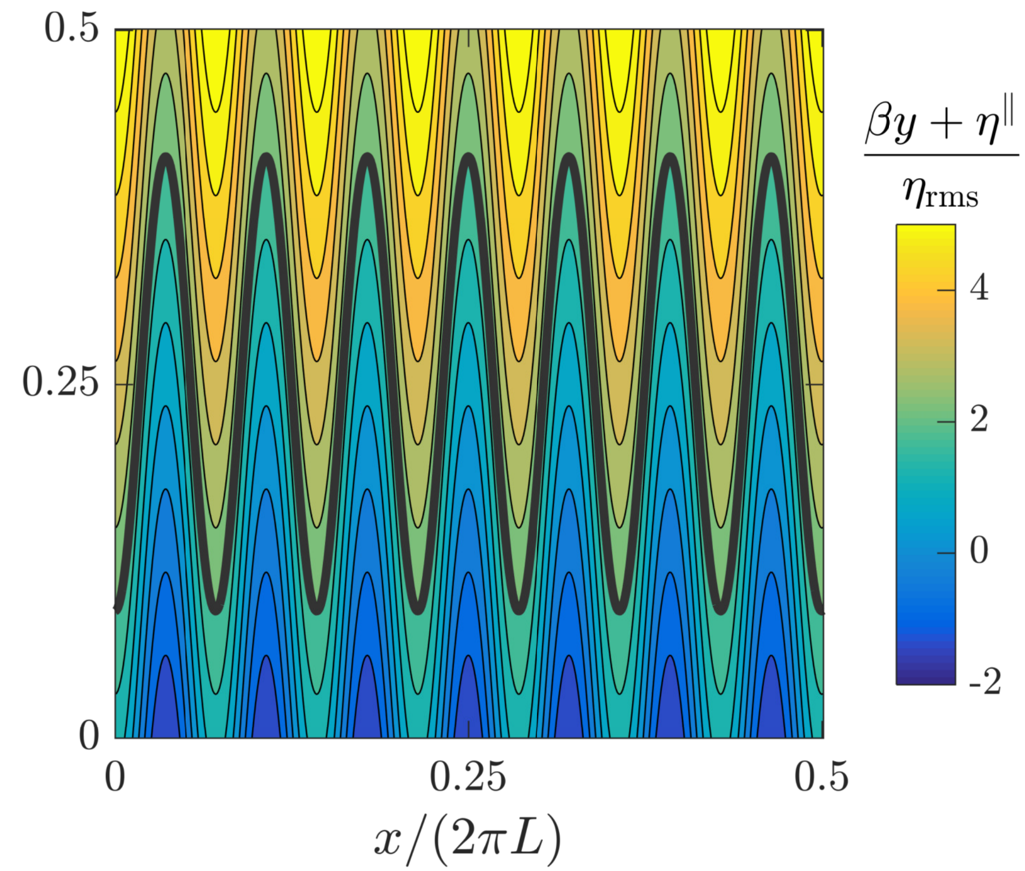
geostrophic contours $\beta y + \eta(x, y)$

$\eta^{\times}$ closed geostrophic contours



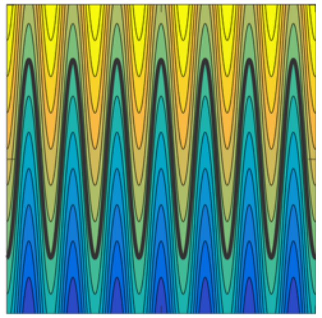
most geostrophic contours
are closed

$\eta^{\parallel}$ open geostrophic contours

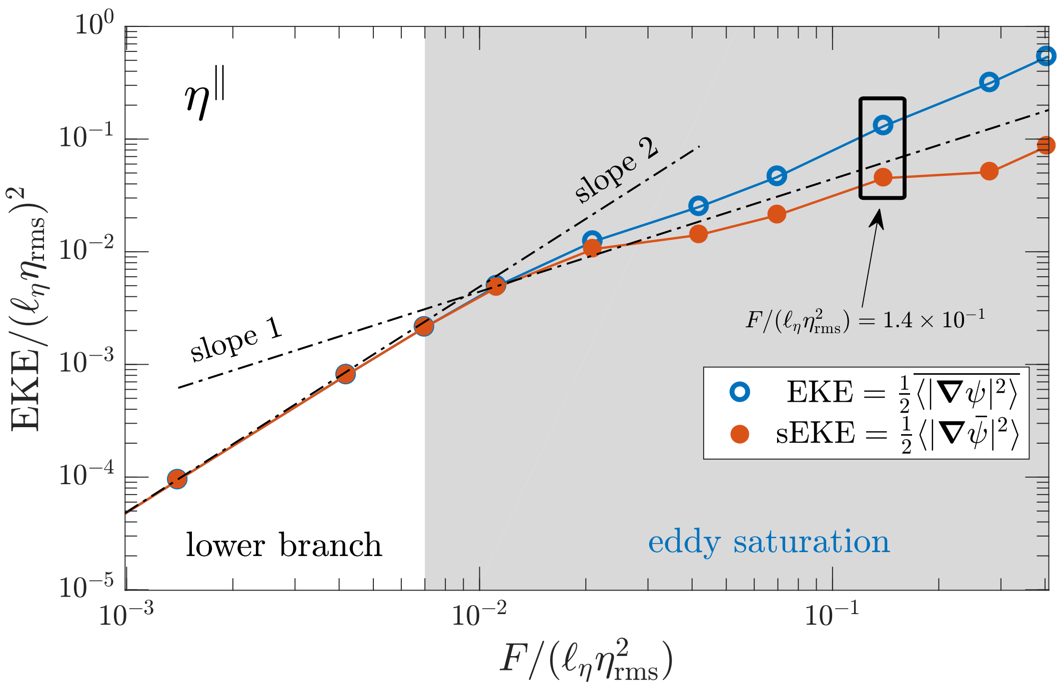


all geostrophic contours
are open

this topography exhibits eddy saturation



further “symptoms” of eddy saturation



wind stress

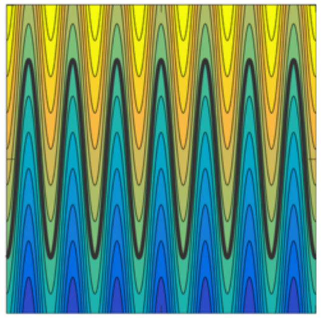
EKE grows roughly linearly with wind stress

large-scale zonal mom. eq.

$$0 = F - \mu\bar U - \langle\bar\psi\eta_x\rangle$$

energy power integral

$$\bar U\underbrace{\langle\bar\psi\eta_x\rangle}_{\sim F} + \cancel{\underbrace{\overline{U'\langle\psi'\eta_x\rangle}}_{\text{negligible}}} = 2\mu\underbrace{\left\langle\tfrac{1}{2}\overline{|\boldsymbol{\nabla}\psi|^2}\right\rangle}_{\overset{\text{def}}{=}\,\text{EKE}} + \cancel{\text{small hyperviscous dissipation}}$$



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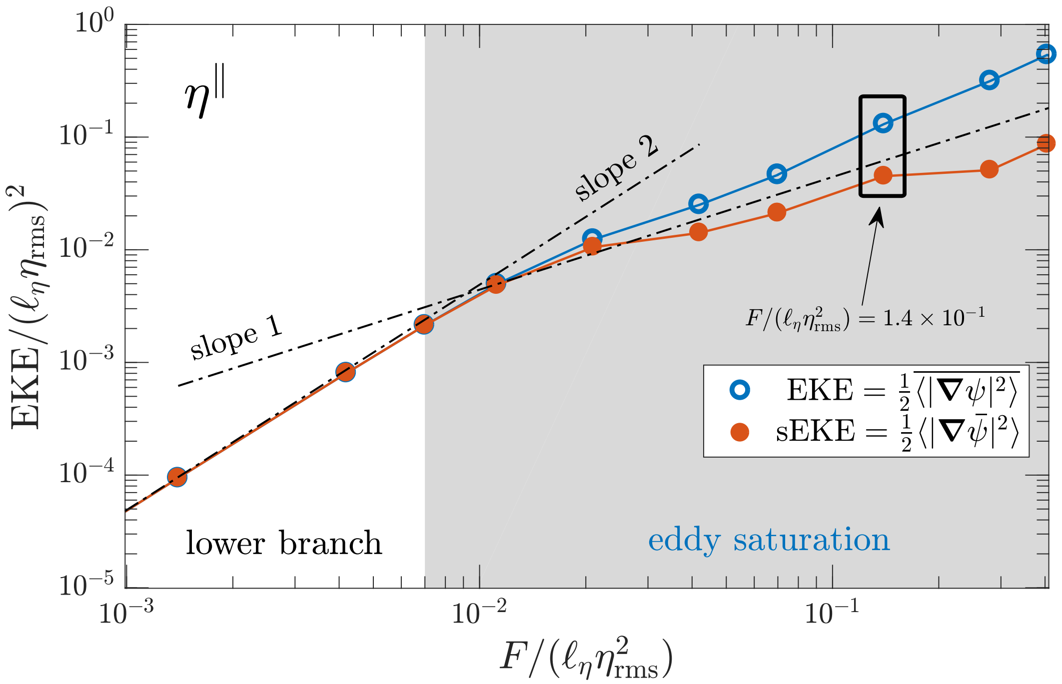

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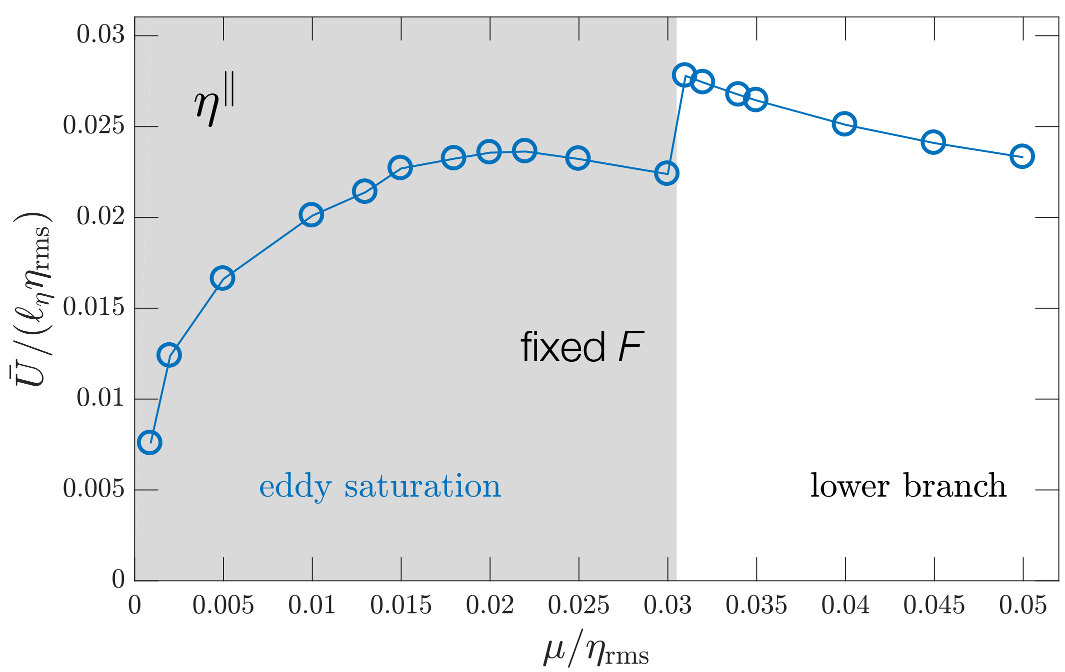

bottom drag

transport grows with increasing bottom drag

Increasing *drag* damps the eddies responsible for form stress.
Thus, *U* increases if the drag is larger.

conclusion and discussion

The barotropic scenario for the momentum balance is viable.

This barotropic QG model shows eddy saturation
when geostrophic contours are open.

This is surprising! All previous arguments were based on baroclinicity.

We need new process models of baroclinic turbulence in which
the mean flow is wind-driven and topography exerts form stress.

(Bill & I are working on this.)

thank you

all these (and more) are found in…

Constantinou (2017). A barotropic model of eddy saturation. *JPO* (submitted, arXiv:1703.06594).

Constantinou and Young (2017). Beta-plane turbulence above monoscale topography. *JFM* (in revision, arXiv:1612.03374)