Eddy saturation in a barotropic model

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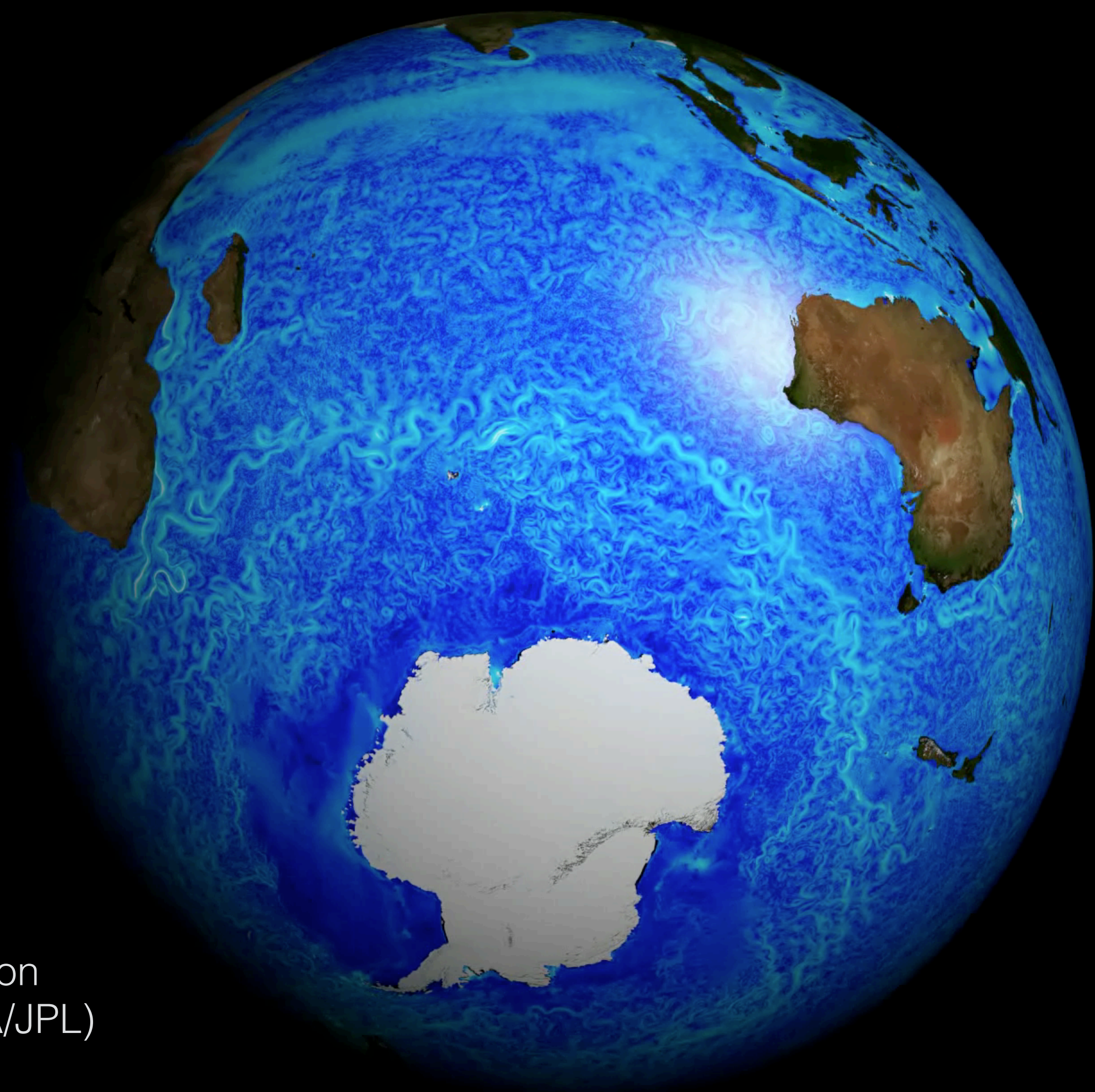
Acknowledgements: Bill Young, NOAA C&GC fellowship

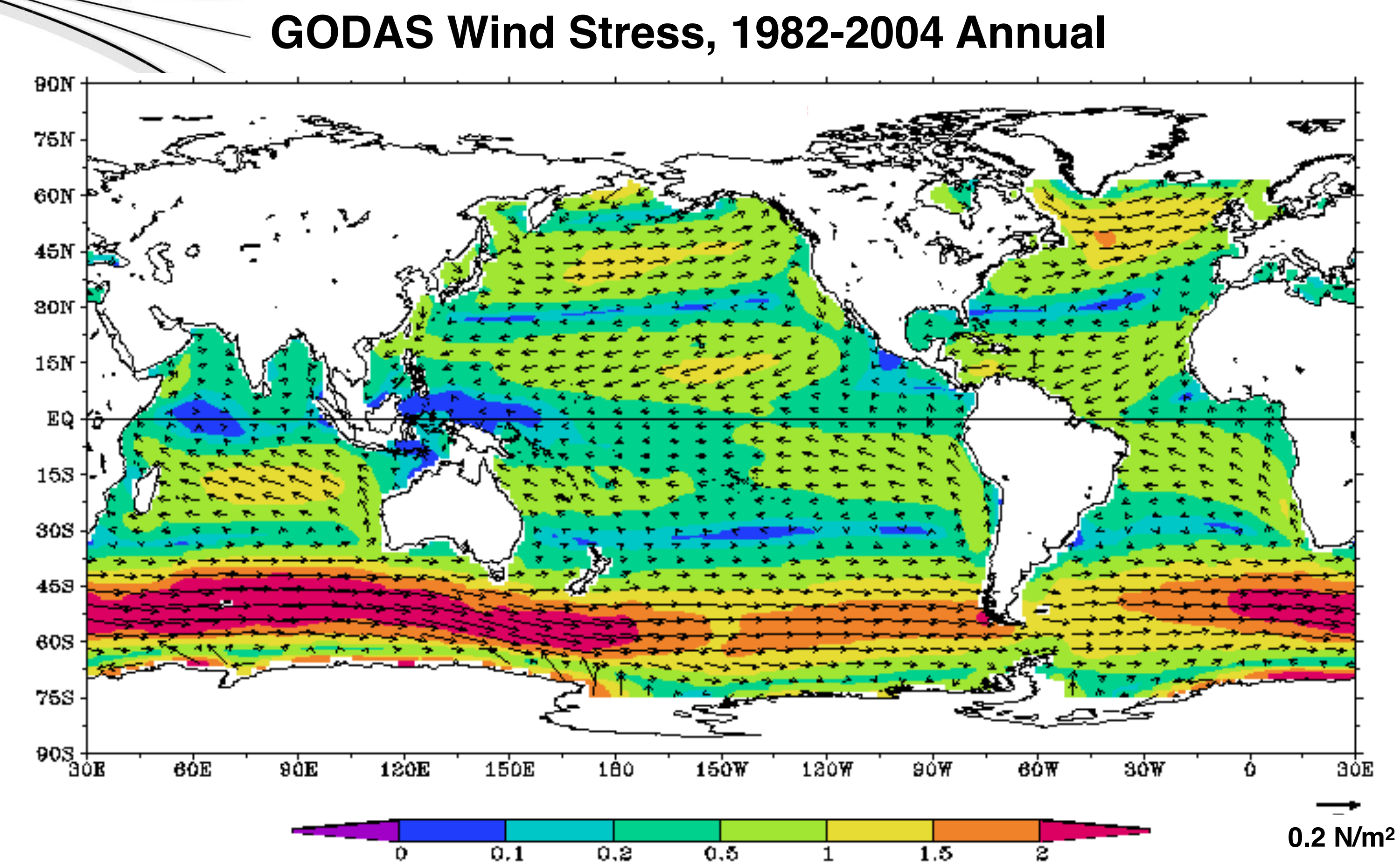


the Antarctic Circumpolar Current

LLC4320 sea surface speed animation
by C. Henze and D. Menemenlis (NASA/JPL)
1/48th degree, 90 vertical levels
MITgcm spun up from ECCO v4 state estimate

what drives the Antarctic Circumpolar Current?

Climate Prediction Center



strong westerly winds blow over the Southern Ocean transferring momentum through wind stress at the surface

how is this momentum balanced? bottom drag?

Note on the Dynamics of the Antarctic Circumpolar Current

W.H. Munk
(100th bday on Oct 19th, 2017)

By W. H. MUNK and E. PALMÉN

1951

Abstract

Unlike all other major ocean currents, the Antarctic Circumpolar Current probably does not have sufficient frictional stress applied at its lateral boundaries to balance the wind stress. The balancing stress is probably applied at the bottom, largely where the major submarine ridges lie in the path of the current. The meridional circulation provides a mechanism for extending the current to a large enough depth to make this possible.

start with the zonal angular momentum equation

$$(\partial_t + u\partial_x + v\partial_y + w\partial_z)\underbrace{\left(u - \int^y f(y')\,\mathrm{d}y'\right)}_{\overset{\text{def}}{=}a} + p_x = \tau_z$$

angular momentum

$f(y)$ is the Coriolis parameter

$f = 2\Omega \sin\vartheta$

vertically integrate, top $z=0$ to bottom $z=-h(x,y)$

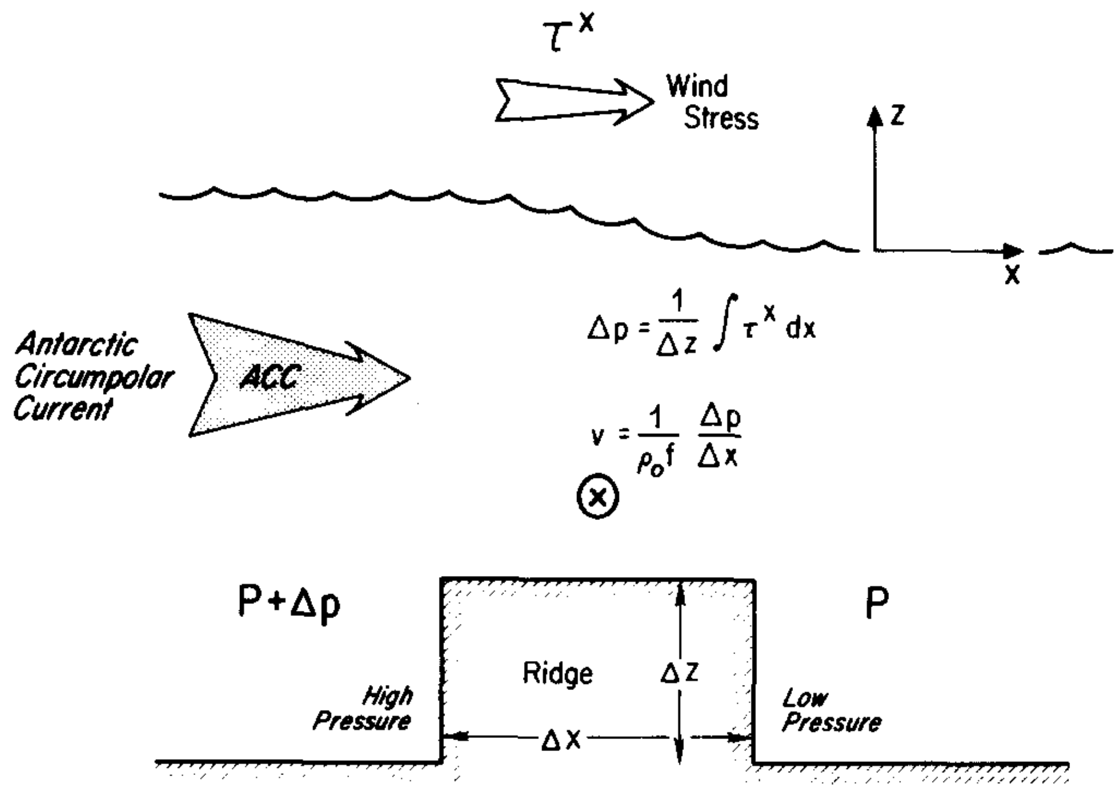
$$\partial_t \int_{-h}^{0} a\,\mathrm{d}z + \partial_x \left[\int_{-h}^{0} ua + p\,\mathrm{d}z\right] + \partial_y \int_{-h}^{0} va\,\mathrm{d}z =$$

$$= \underbrace{\tau(0)}_{\text{wind stress}} - \underbrace{\tau(-h)}_{\text{bottom drag}} + \underbrace{h_x p(-h)}_{\text{form stress}}$$

we've used integration by parts:

$$\int_{-h}^{0} p_x\,\mathrm{d}z = \partial_x \int_{-h}^{0} p\,\mathrm{d}z - h_x p(-h)$$

topographic form stress



Johnson & Bryden 1989

Schematic presentation of bottom form drag or mountain drag. Wind stress imparted eastward momentum in the water column is removed by the pressure difference across the ridge.

$$\partial_t \int_{-h}^{0} a \,\mathrm{d}z + \partial_x \left[\int_{-h}^{0} ua + p \,\mathrm{d}z \right] + \partial_y \int_{-h}^{0} va \,\mathrm{d}z =$$

$$= \underbrace{\tau(0)}_{\text{wind stress}} - \underbrace{\tau(-h)}_{\text{bottom drag}} + \underbrace{h_x p(-h)}_{\text{form stress}}$$

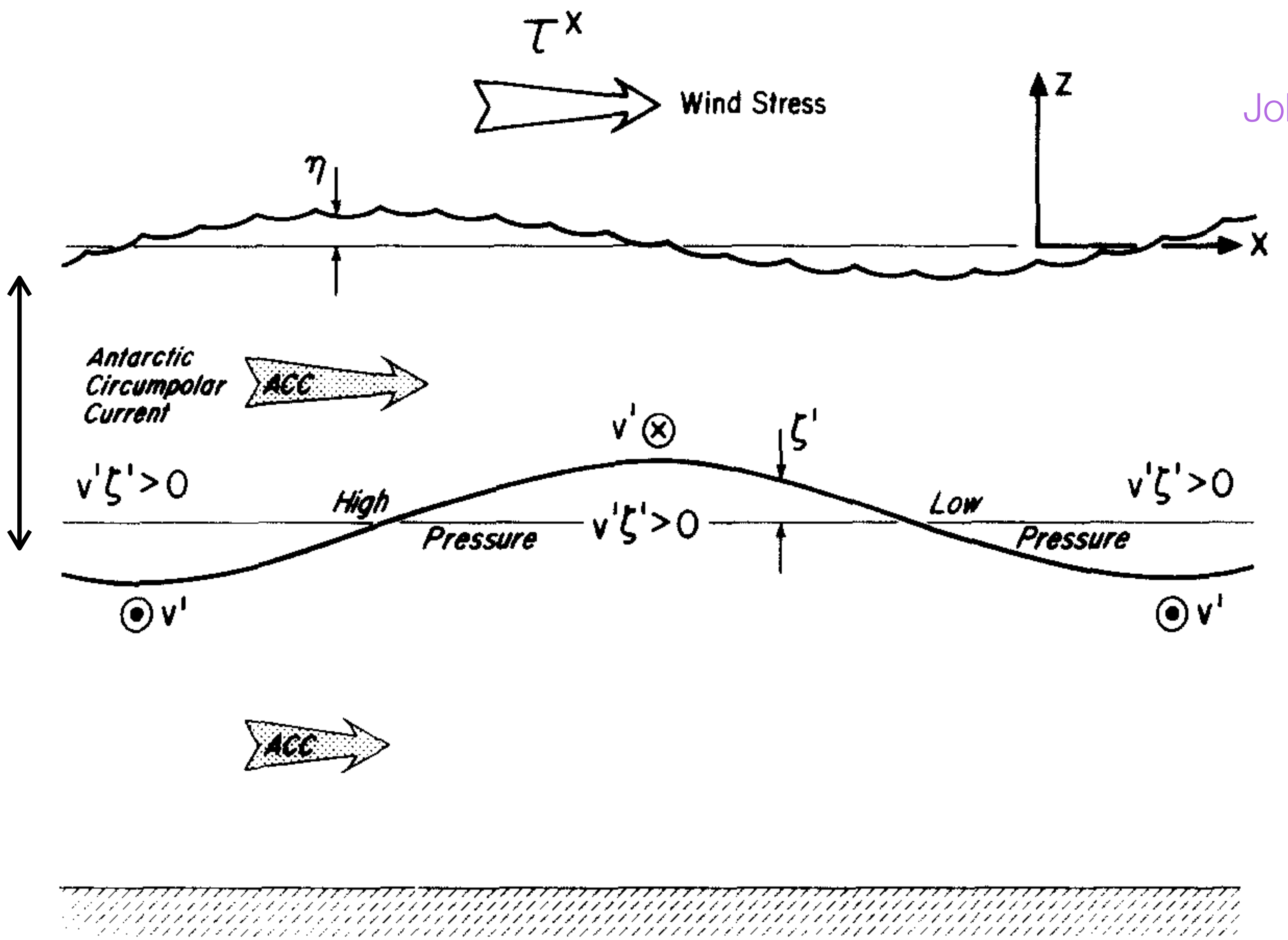
Topographic form stress is a purely **barotropic** process.

interfacial form stress

Johnson & Bryden 1989

vertically integrate
from the sea-surface
down to a moving
buoyancy surface

(i.e., integrate within
a layer of constant density)



Schematic presentation of interfacial form drag. Correlations of perturbations in the interface height, ζ', and the meridional velocity, V' (⊙ indicating poleward flow and ⊗ indicating equatorward flow), which are related to pressure perturbations by geostrophy, allow the upper layer to exert an eastward force on the lower layer and the lower layer to exert a westward force on the upper layer; thus effecting a downward flux of zonal momentum.

Interfacial form stress requires **baroclinicity**.

the most popular scenario for the momentum balance

- momentum in imparted at the surface by wind,
- isopycnals slope, creating baroclinic instability,
- momentum is transferred downwards by interfacial eddy form stress
- momentum reaches the bottom where is transferred to the solid Earth by topographic form stress.

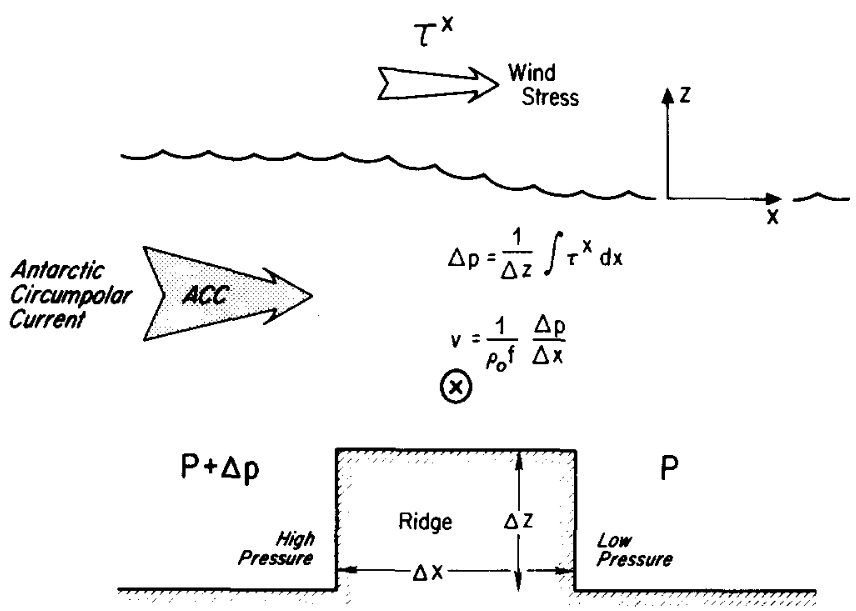
Johnson & Bryden 1989

$$\begin{matrix}\text{isopycnal}\\ \text{slope}\end{matrix} = \left[-\frac{\tau_s}{f\,\kappa} \right]^{1/2}$$

Marshall & Radko 2003

This baroclinic scenario sets up the ACC transport (e.g. the transport through Drake Passage).

but what about barotropic dynamics?



The sea surface pressure gradient can be *directly* communicated to the bottom.

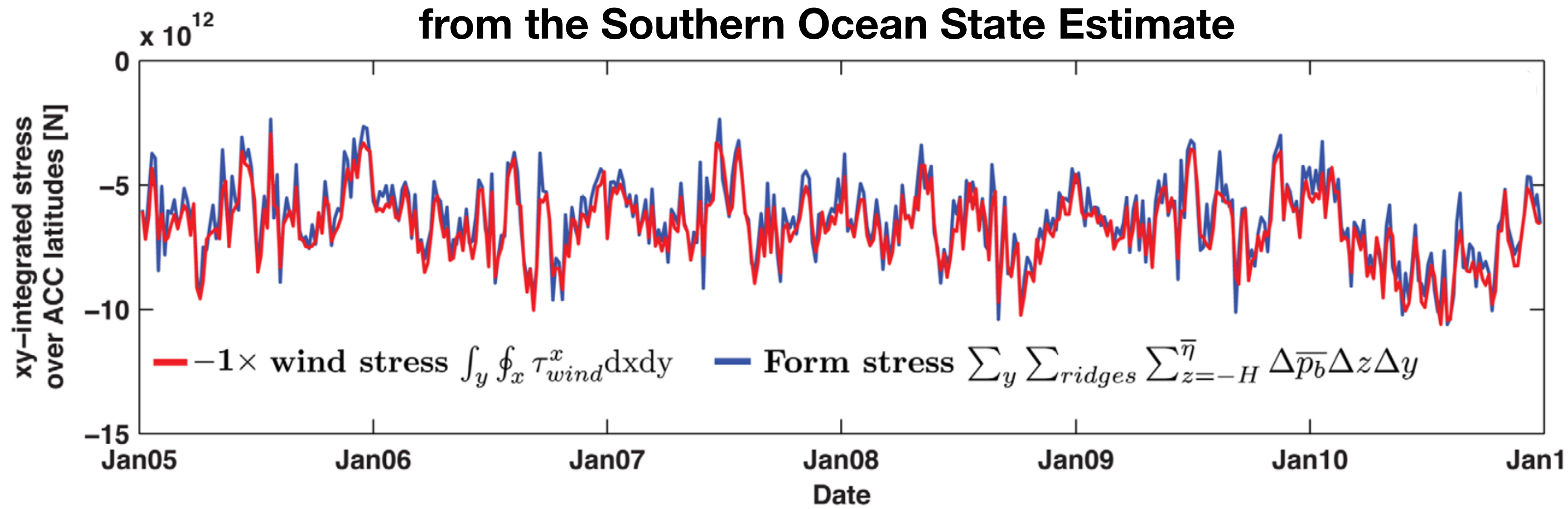
And it will be, unless compensated by internal isopycnal gradients.

Isn't barotropic "communication" much simpler?

wind stress is *rapidly* communicated to the bottom through barotropic processes

Barotropic processes are fast (~days).

Baroclinic processes are much slower (~years).



Masich, Chereskin, and Mazloff 2015

~90% of variance in the topographic form stress signal is explained by the **0-day** time lag.

Similar statements also made by:
Straub 1993, Ward & Hogg 2011, Rintoul et al. 2014, Peña Molino et al. 2014, Donohue et al. 2016.

topographic form stress $= \quad h_x \, p(-h)$

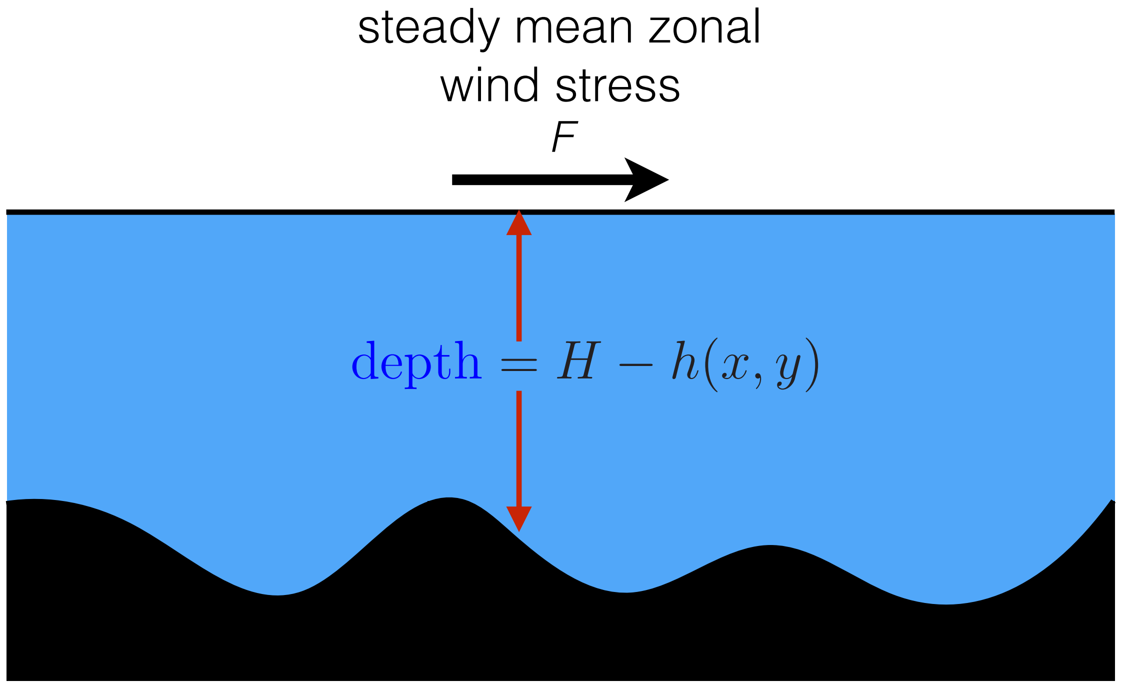
"I don't know why I don't care about the bottom of the ocean, but I don't."

"My dear, if you are interested in the ACC transport then, despite whether the ***baroclinic*** *or* ***barotropic*** *scenario is pertinent, you* ***should*** *care for the bottom of the ocean."*

the plan

Revisit an old barotropic quasigeostrophic (QG) model on a beta-plane.

(Hart 1979, Davey 1980, Bretherton & Haidvogel 1976, Holloway 1987, Carnevale & Fredericksen 1987)



A distinctive feature of this model is a "large-scale barotropic flow" *U*(*t*).

this is the ACC

topographic potential vorticity (PV) $\eta = \dfrac{f_0 h}{H}$

QGPV $\nabla^2\psi + \eta + \beta y$

total streamfunction $-U(t)y + \psi(x, y, t)$

Study how momentum is balanced by topographic form stress and investigate the requirements for eddy saturation.

a barotropic QG model for a mid-ocean region

total streamfunction $-U(t)y + \psi(x, y, t)$

QGPV $\nabla^2\psi + \eta + \beta y$

Material conservation of QGPV

$$\nabla^2\psi_t + U(\nabla^2\psi + \eta)_x + \mathsf{J}(\psi, \nabla^2\psi + \eta) + \beta\psi_x = -\mu\,\nabla^2\psi + \text{hyper visc.}$$

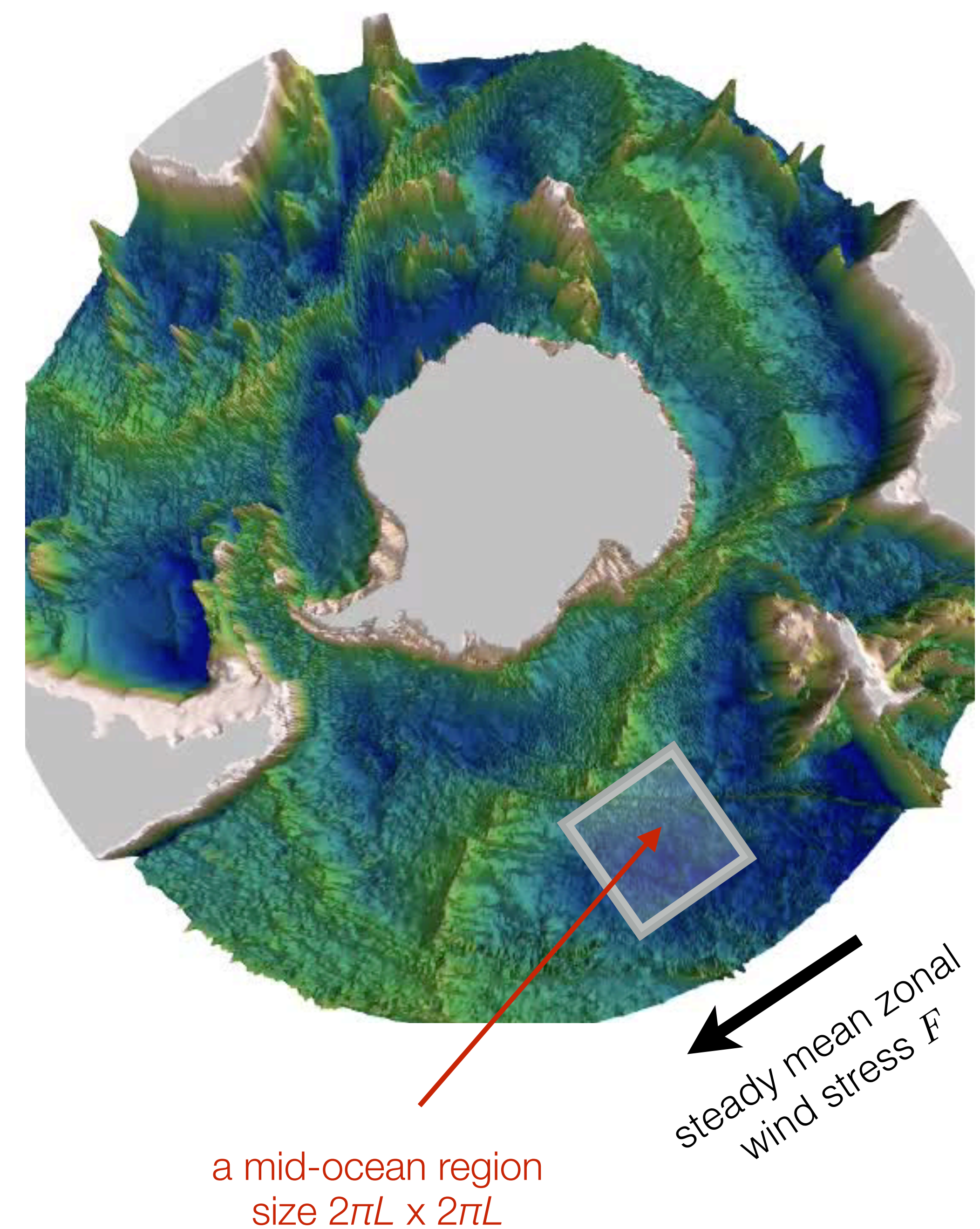
Large-scale zonal momentum

$$U_t = F - \mu U - \langle\psi\eta_x\rangle$$

topographic form stress

$\langle\ \rangle$ is domain average ; $F = \dfrac{\tau_s}{\rho_0 H}$ wind stress forcing

periodic boundary conditions



(Hart 1979, Davey 1980, Bretherton & Haidvogel 1976, Holloway 1987, Carnevale & Fredericksen 1987)

the large-scale flow equation: $U_t = F - \mu U - \langle \psi \eta_x \rangle$

zonal angular momentum density: $a(x,y,z,t) = u(x,y,z,t) - \int^y f(y')\,\mathrm{d}y'$

vertically integrated zonal angular momentum equation

$$\partial_t \int_{-h}^{0} a\,\mathrm{d}z + \partial_x \left[\int_{-h}^{0} ua + p\,\mathrm{d}z \right] + \partial_y \int_{-h}^{0} va\,\mathrm{d}z =$$

$$= \underbrace{\tau(0)}_{\text{wind stress}} - \underbrace{\tau(-h)}_{\text{bottom drag}} + \underbrace{h_x p(-h)}_{\text{form stress}}$$

horizontally integrate, drop the boundary fluxes, and divide by the volume

$$U_t = F - \mu U - \langle \psi \eta_x \rangle$$

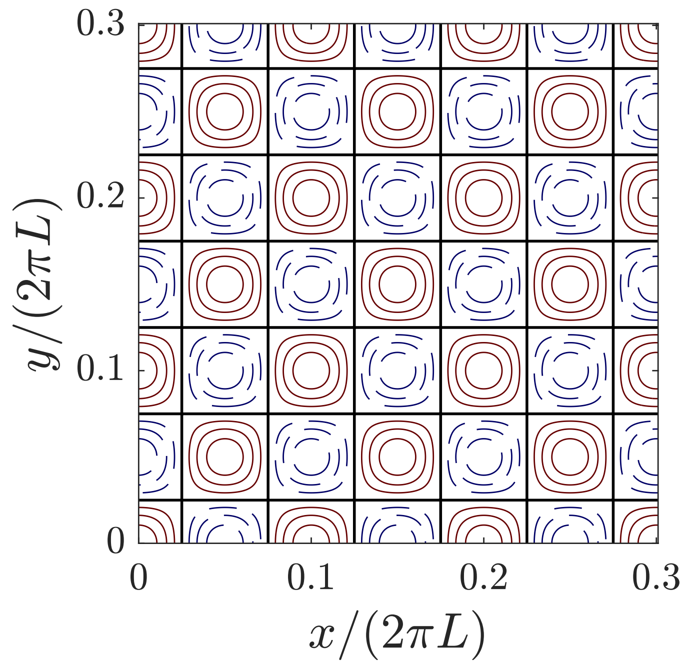
$$U(t) \stackrel{\text{def}}{=} V^{-1} \iiint u(x,y,z,t)\,\mathrm{d}V$$

vertical & horizontal integral over a mid-ocean region (**not** a zonal average)

Let’s see some solutions.

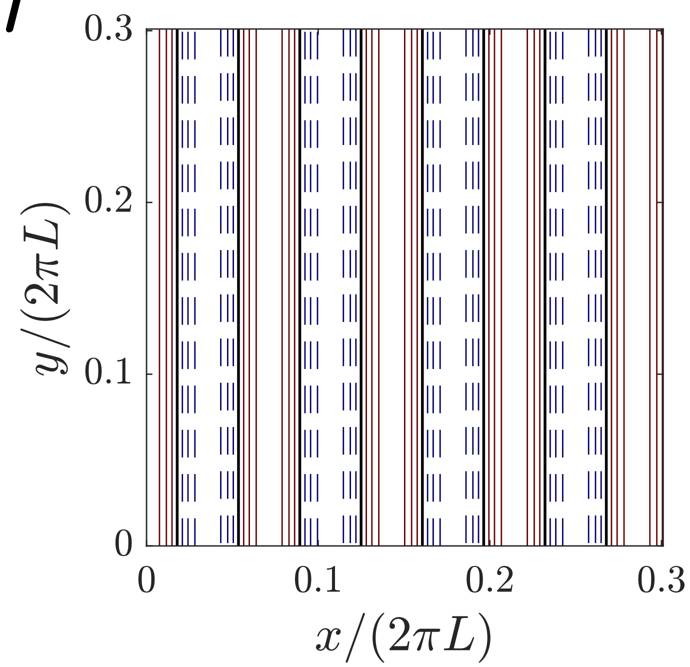
let's use these two topographies

$\eta^{\times}$ $2\eta_{\rm rms}\cos(10x/L)\cos(10y/L)$



1/9 of the periodic domain

$\eta^{\parallel}$ $\sqrt{2}\eta_{\rm rms}\cos(14x/L)$



(both topographies imply the same length-scale: $\ell_\eta = \sqrt{\frac{\langle \eta^2 \rangle}{\langle |\boldsymbol{\nabla}\eta|^2 \rangle}} = 0.07L$)

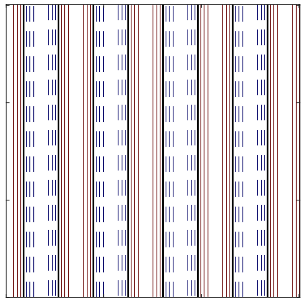
let's put some "quasi-realistic" numbers for the Southern Ocean

$$L = 775\ \text{km} \quad H = 4\,\text{km} \quad \rho_0 = 1035\ \text{kg m}^{-3}$$

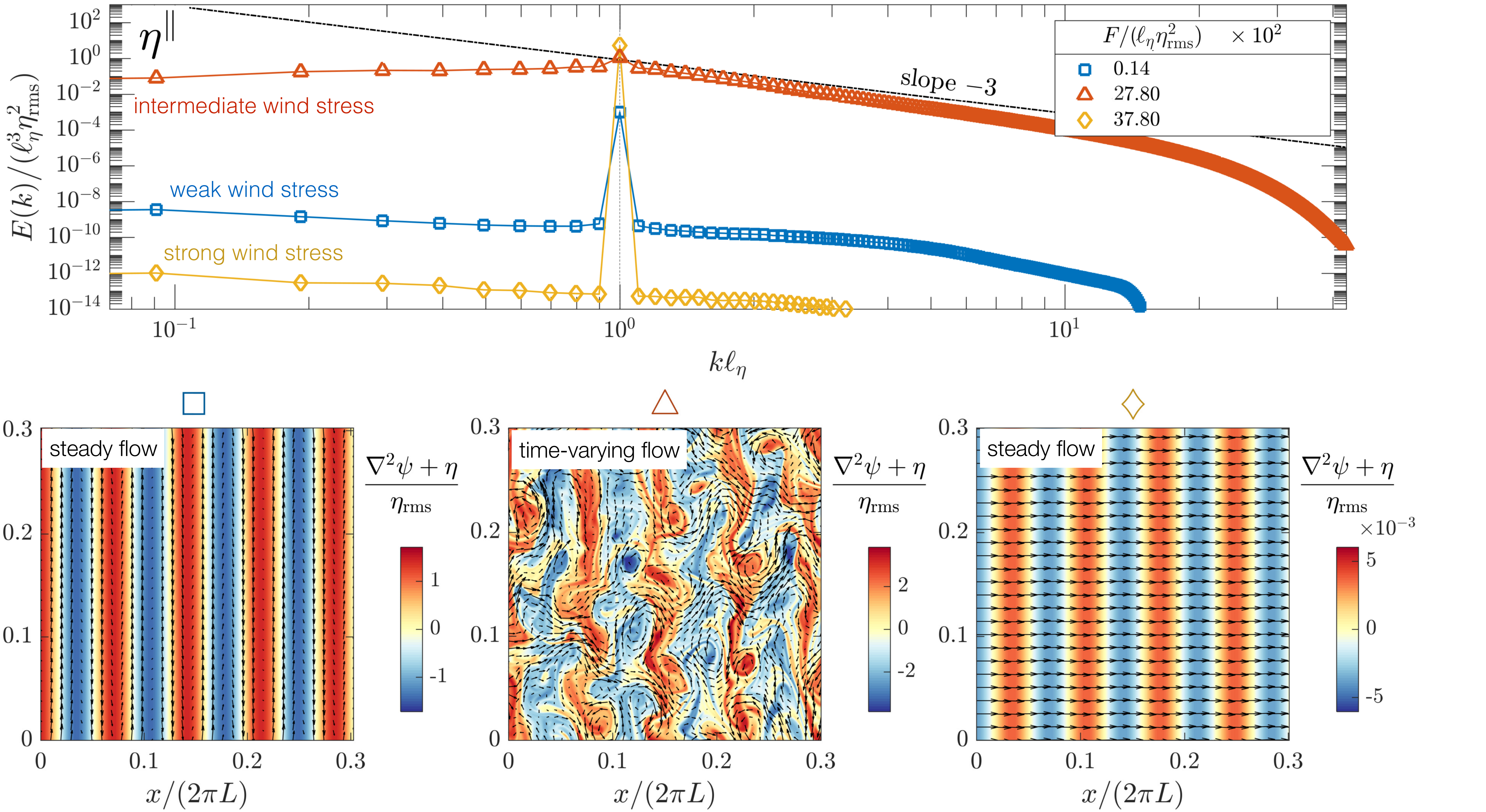
$$f_0 \quad \& \quad \beta \quad \text{for } 60^\circ\text{S}$$

$$\mu = (180\ \text{days})^{-1}$$

$$h_{\rm rms} = 200\ \text{m} \Rightarrow \eta_{\rm rms} = (1.8\ \text{days})^{-1}$$



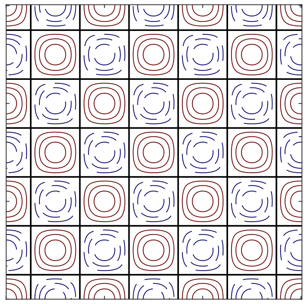
energy spectra & flow snapshots $\eta^{\parallel}$



weak
wind stress

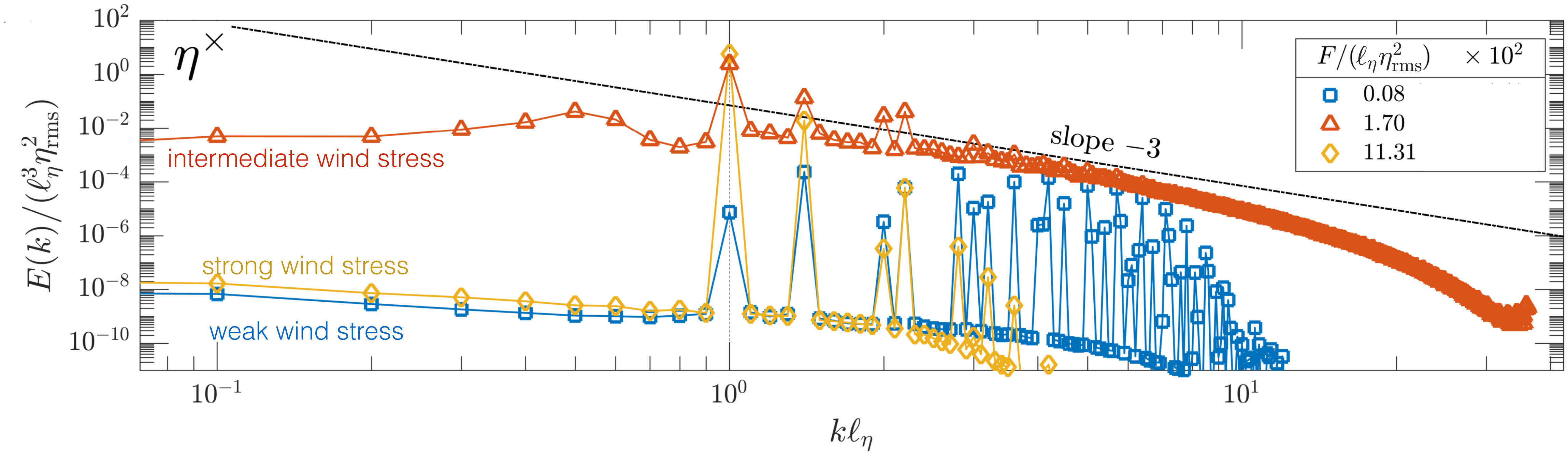
intermediate
("realistic")
wind stress

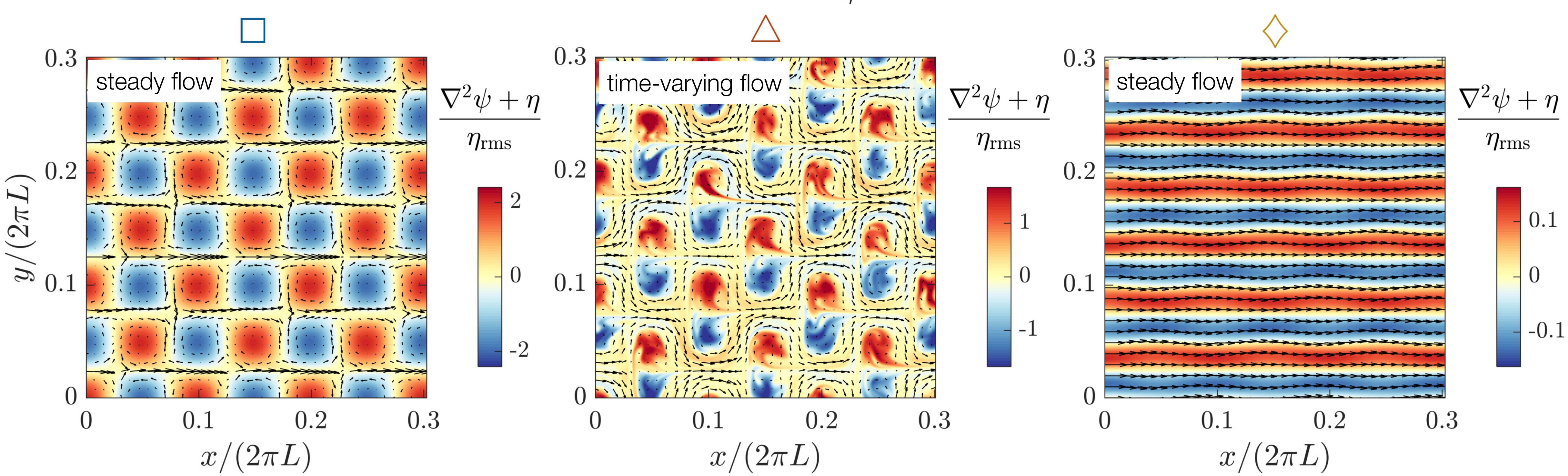
strong
wind stress



energy spectra & flow snapshots

$\eta^{\times}$





weak
wind stress

intermediate
(“realistic”)
wind stress

strong
wind stress

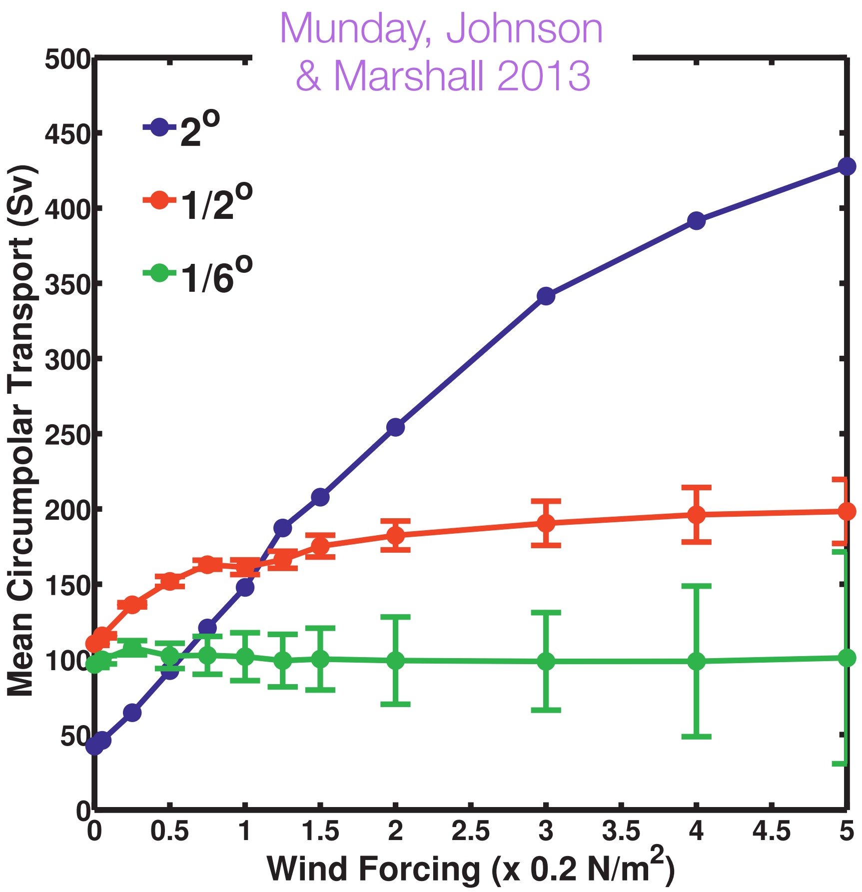
Question:

Does this barotropic QG model show eddy saturation?

Do we need baroclinicity?
Do we even need channel walls?

but first, what is "eddy saturation"?

The *insensitivity* of the total ACC volume transport
to wind stress increase.



Eddy saturation is seen in eddy-resolving ocean models.

Higher resolution ⟶ eddy saturation "occurs"

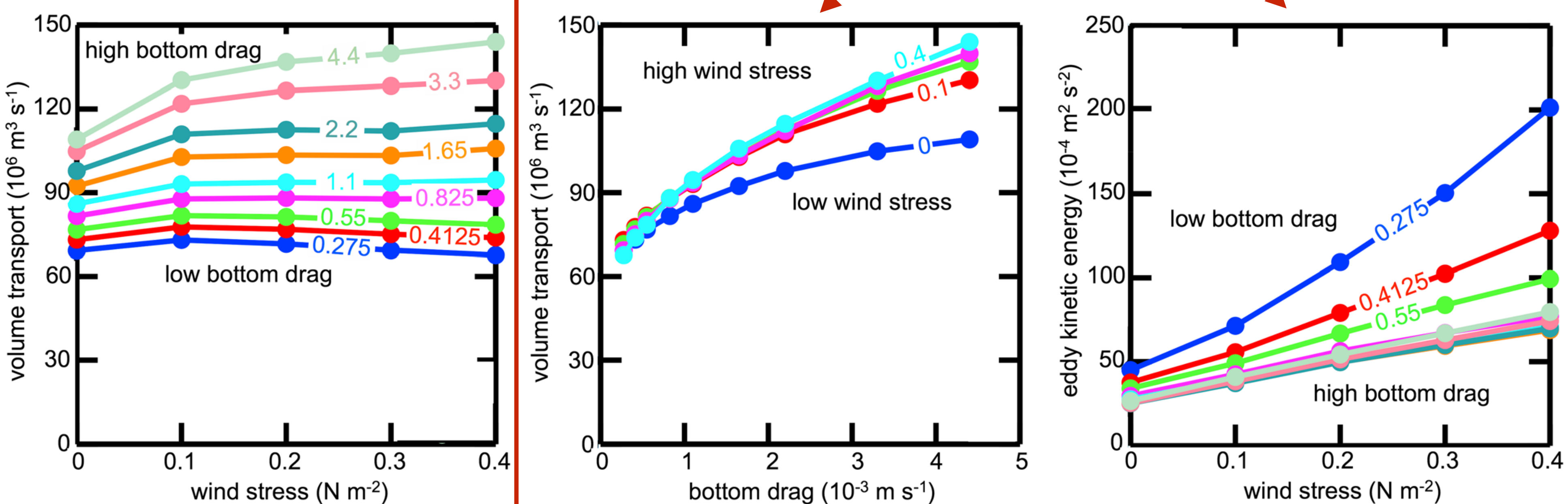
Eddy saturation was theoretically predicted by Straub (1993)
but with an *entirely* baroclinic argument.
(based on vertical momentum transfer interfacial eddy form stress)

[There are many other examples: Hallberg & Gnanadesikan 2001, Tansley & Marshall 2001, Hallberg & Gnanadesikan 2006, Hogg et al. 2008, Nadeau & Straub 2009, Farneti et al. 2010, Nadeau & Straub 2012, Meredith et al. 2012, Morisson & Hogg 2013, Abernathey & Cessi 2014, Farneti et al. 2015, Nadeau & Ferrari 2015, Marshall et al. 2016.]

yet more eddy saturation

Marshall, Ambaum, Maddison, Munday & Novak 2016

further "symptoms" of eddy saturation



increasing the bottom drag increases the transport

EKE varies linearly with wind stress

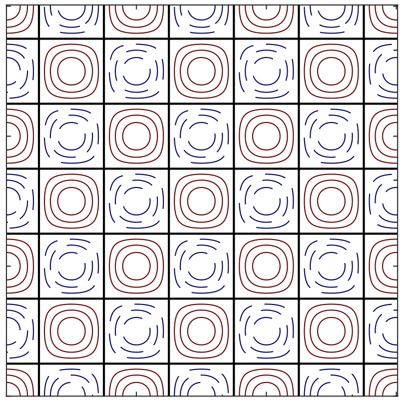
[See also: Hogg & Blundell 2006, Nadeau & Straub 2012, and Nadeau & Ferrari 2015.]

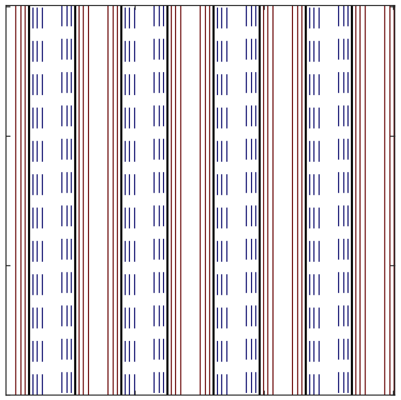
Question:

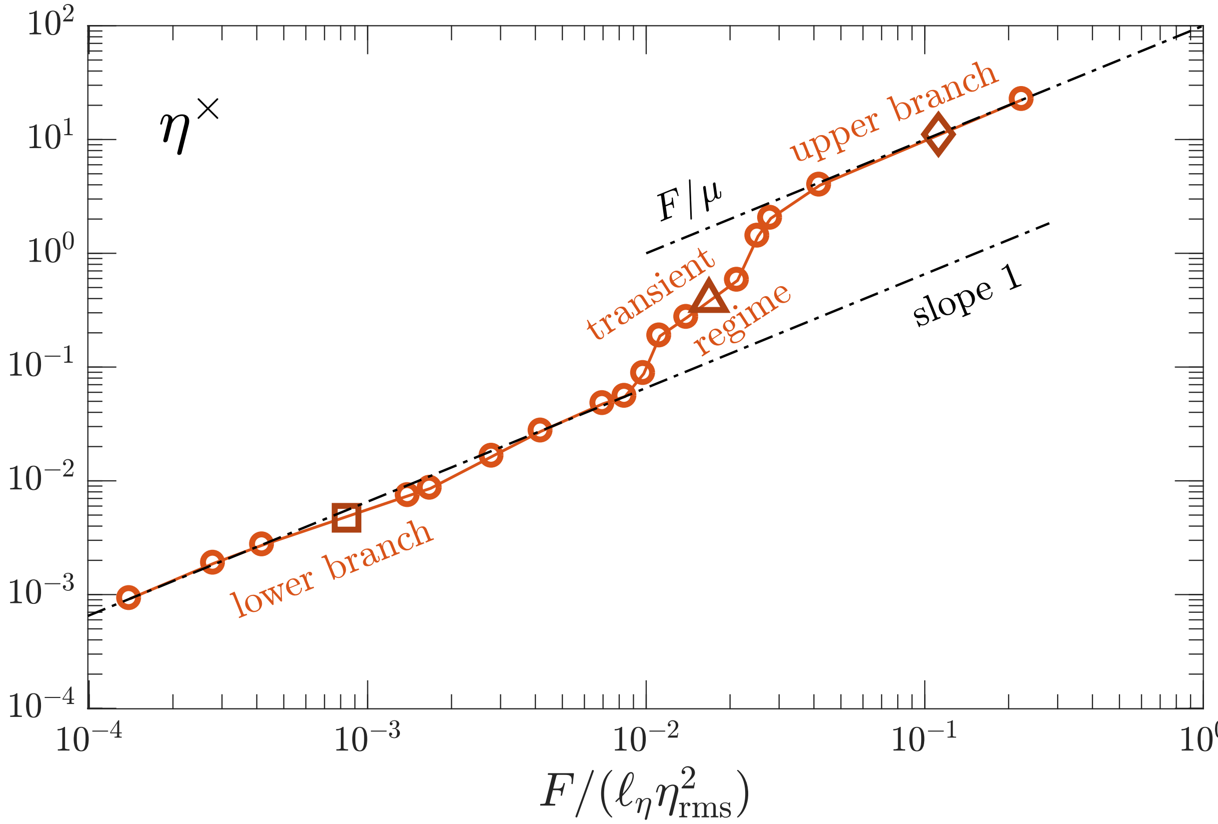
So, does this barotropic QG model show eddy saturation or not?

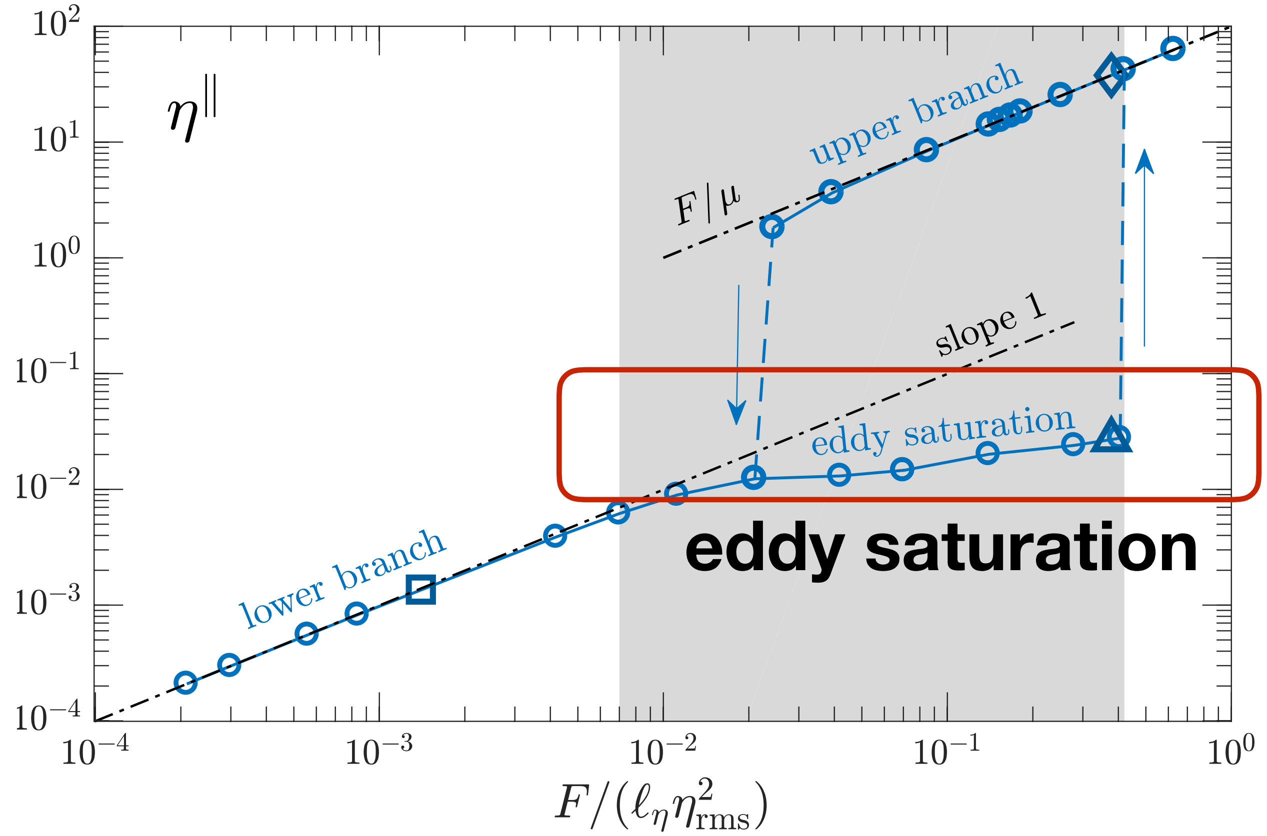
Let’s keep everything fixed and vary the wind stress F.
How does the ACC transport (time-mean of U) respond?

how does the transport vary with wind stress in our barotropic QG model?









do we understand why?

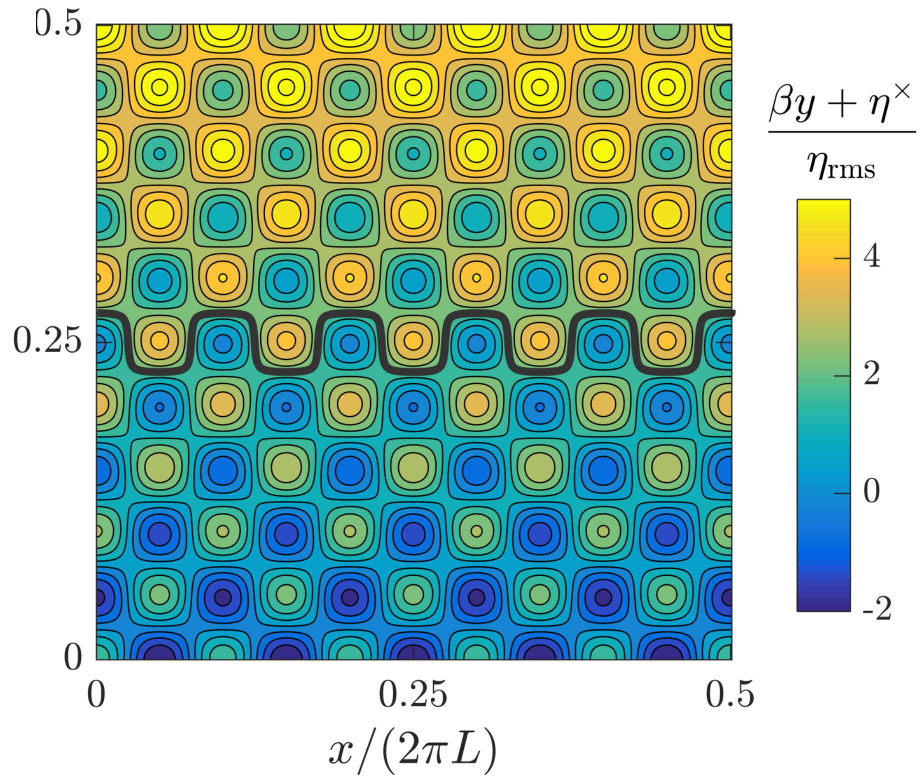
geostrophic contours

$$\beta y + \eta(x, y)$$

this is small-Rossby number expansion of $f/(H+h)$

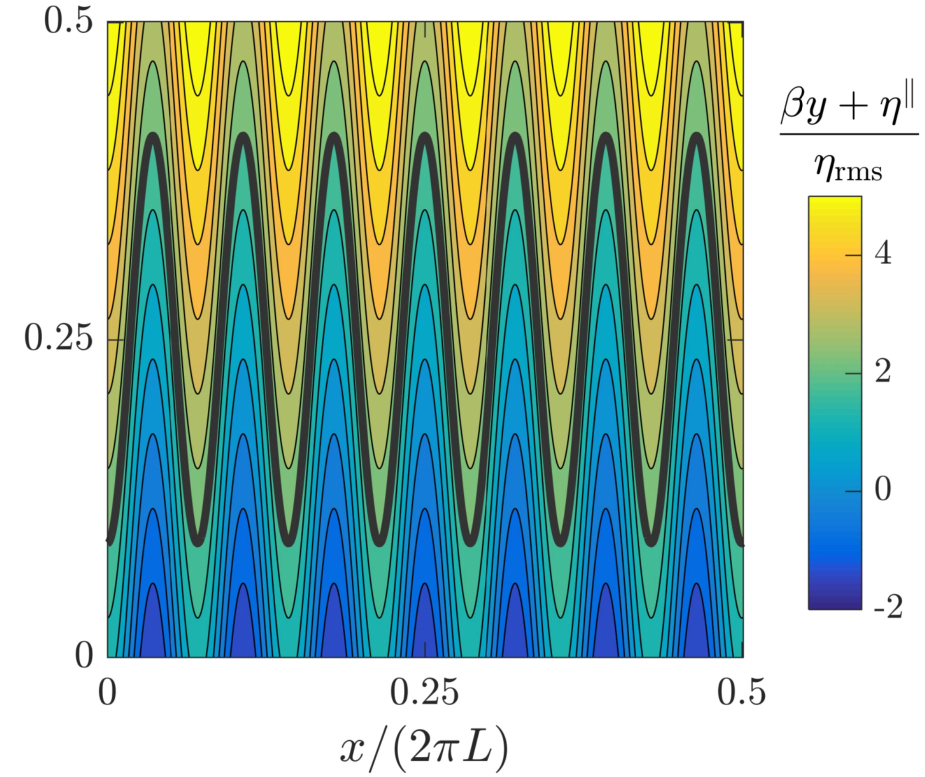
The *main control* parameter for whether eddy saturation occurs is the structure of the geostrophic contours.

$\eta^{\times}$ closed geostrophic contours



most geostrophic contours are closed

$\eta^{\parallel}$ open geostrophic contours



all geostrophic contours are open

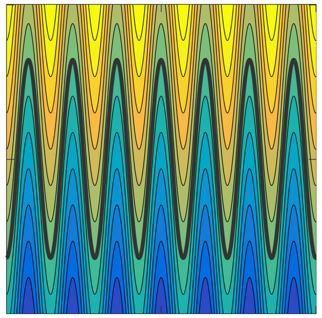
this topography exhibits eddy saturation

geostrophic contours $\beta y + \eta(x, y)$

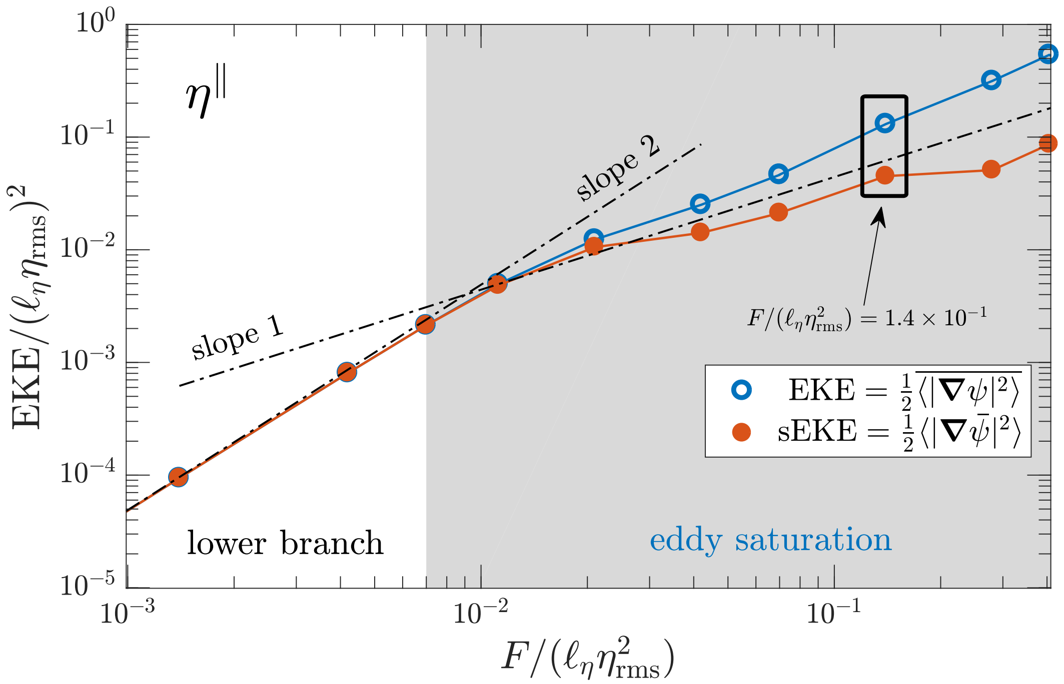
this is small-Rossby number expansion of $f/(H+h)$

Eddy saturation occurs when
the geostrophic contours are "open",
that is, when the geostrophic contours
span the domain in the zonal direction.

this is a general result
we've seen it in various cases
whatever the topography
(random, monoscale, multiscale, etc.)



further “symptoms” of eddy saturation



wind stress

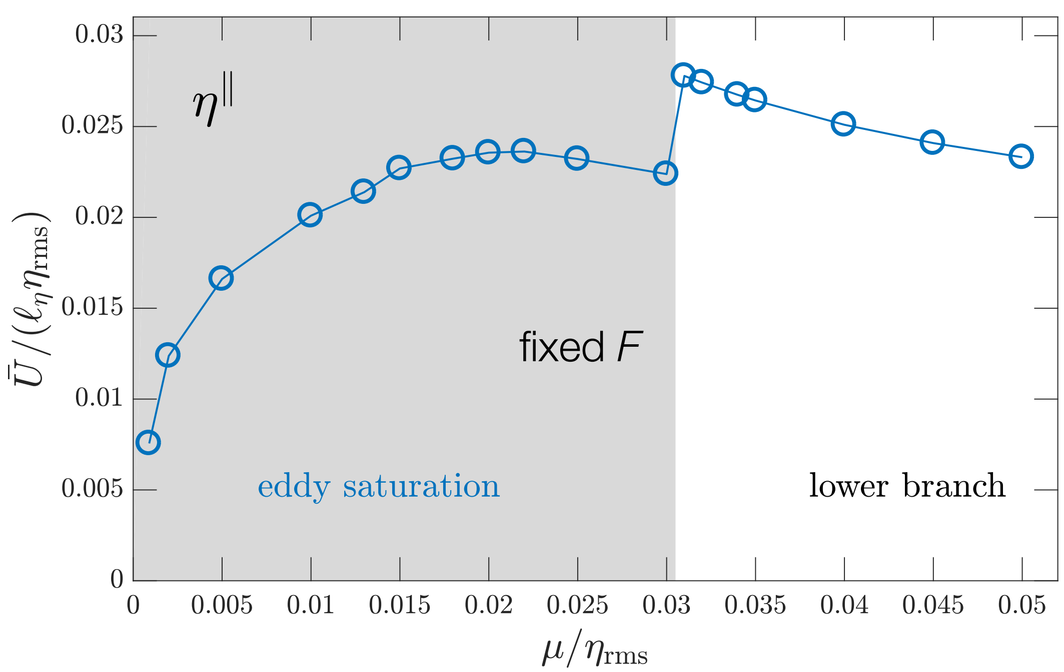
EKE grows roughly linearly with wind stress

large-scale zonal mom. eq.

$$0 = F - \mu\bar{U} - \langle \bar{\psi}\eta_x \rangle$$

energy power integral

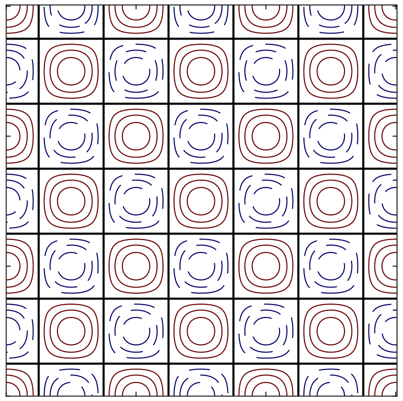
$$\underbrace{\bar{U}\langle\bar{\psi}\eta_x\rangle}_{\sim F} + \underbrace{\overline{U'\langle\psi'\eta_x\rangle}}_{\text{negligible}} = 2\mu\underbrace{\langle \tfrac{1}{2}\overline{|\nabla\psi|^2}\rangle}_{\overset{\text{def}}{=}\text{EKE}} + \text{small hyperviscous dissipation}$$



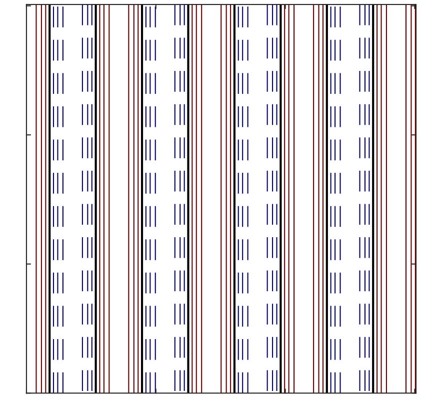
bottom drag

transport grows with increasing bottom drag

Increasing *drag* damps the eddies responsible for form stress.
Thus, *U* increases if the drag is larger.

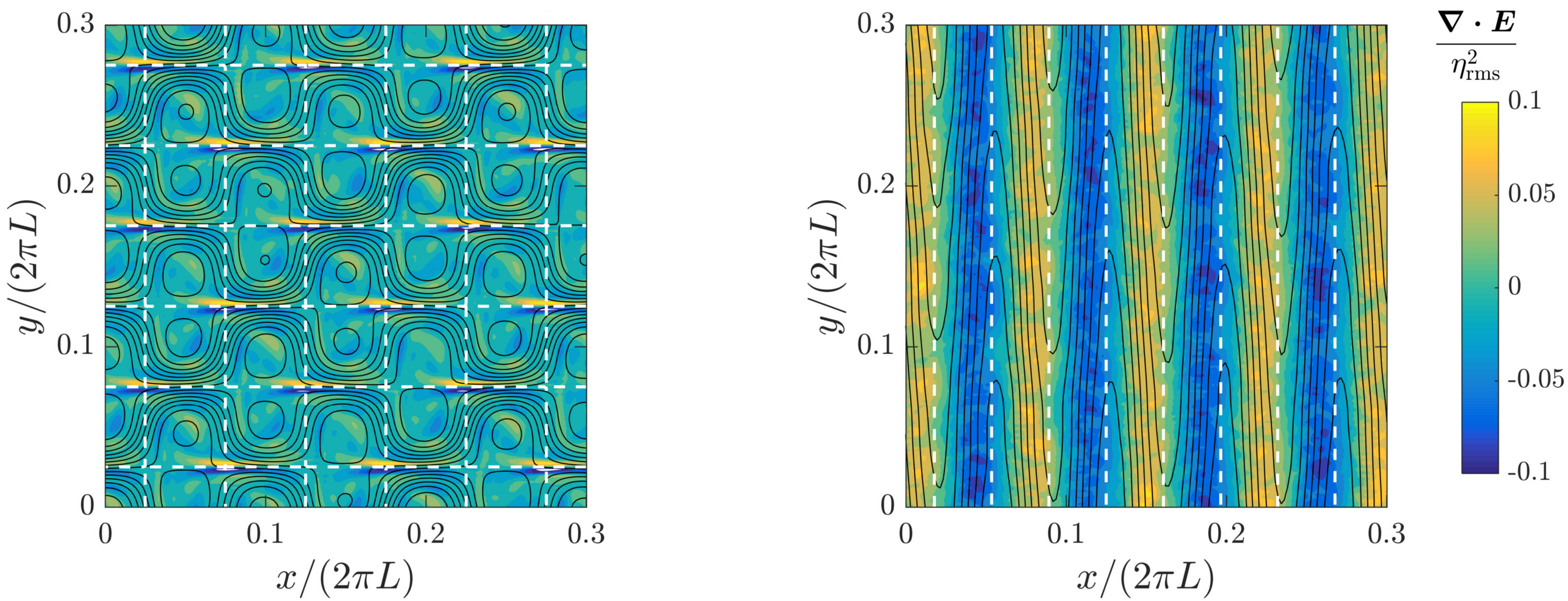


the role of transient eddies



eddy PV fluxes: $\boldsymbol{E} = \left(\overline{(U' + u')\nabla^2\psi'}, \overline{v'\nabla^2\psi'}\right)$

$\eta^{\times}$



$\eta^{||}$

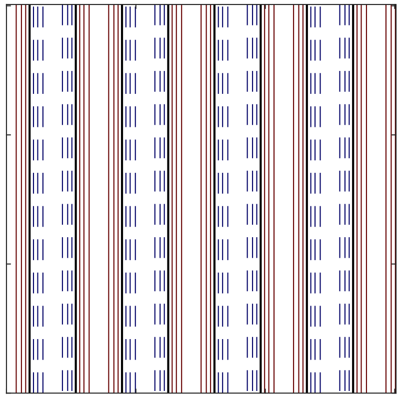
colors: $\nabla \cdot \boldsymbol{E}$

black contours: time-mean streamlines $\bar{\psi} - \bar{U}y$

white contours: $\eta = 0$

We want to investigate the role of the transient eddies
in producing eddy-saturated states.

stability analysis for $\eta^{||}=\eta_0 \cos(mx)$



For $\eta = \eta_0 \cos(mx)$ there exist a low-dimensional manifold

$$\psi = \left[S(t)\sin{(mx)} + C(t)\cos{(mx)}\right]/m \qquad \& \qquad U(t)$$

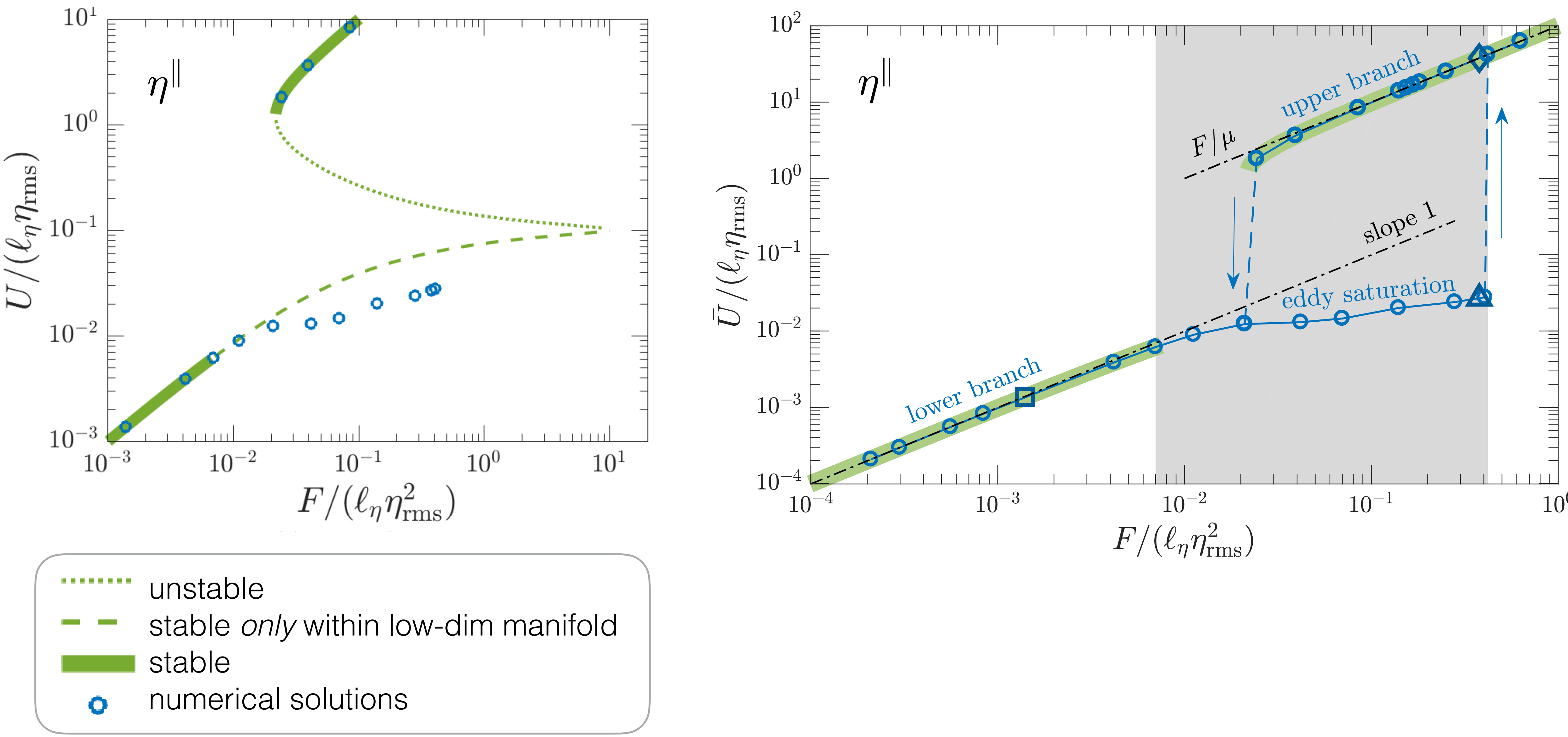
Lower and upper branch solutions are steady solutions *within* this manifold.

Stability of within the low-dim manifold; done by Hart (1979).

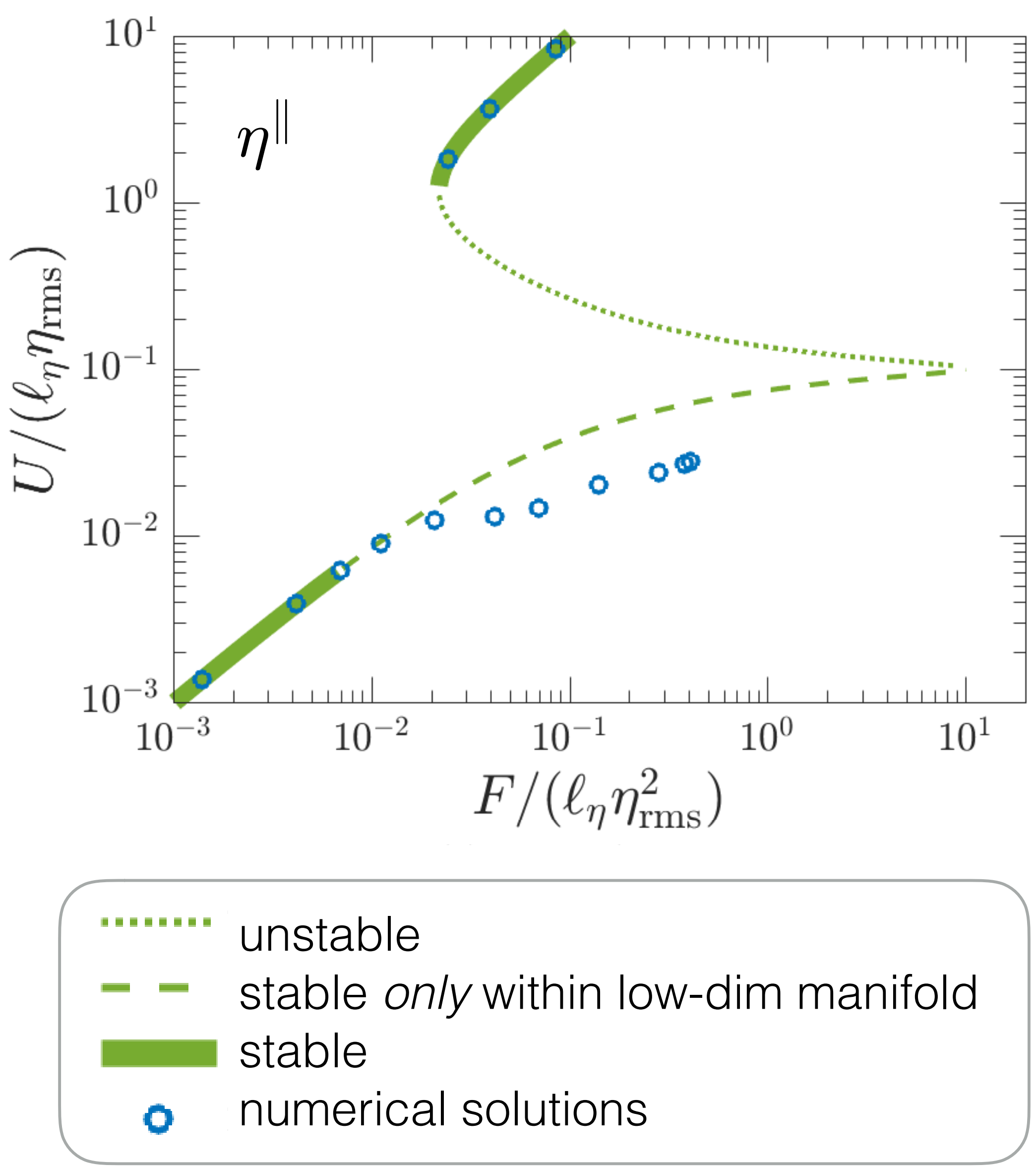
Stability of steady solutions with respect to general perturbations *outside* the low-dim manifold shed light on the role of transient eddies in eddy saturation.

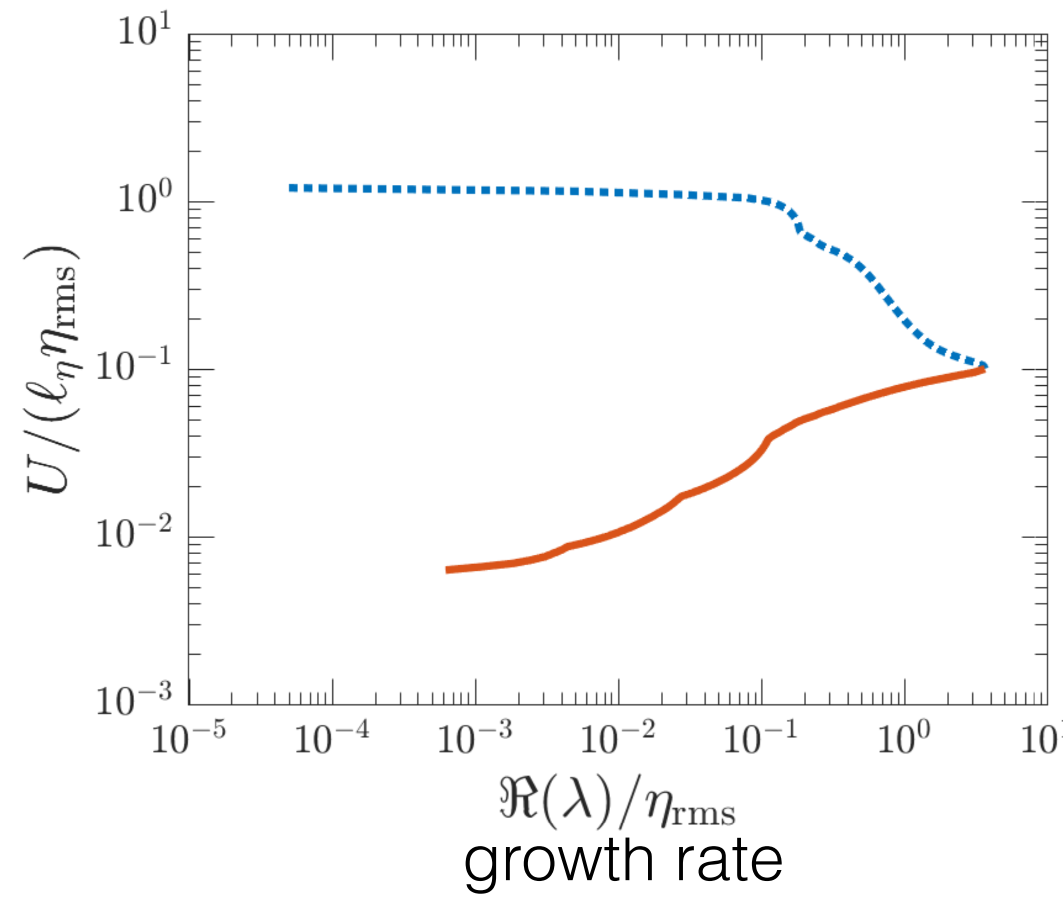
(similar stability analysis was done by Charney & Flierl 1980 but treating *U* as an external parameter)

stability analysis for $\eta^{\|}=\eta_0 \cos(mx)$



stability analysis for $\eta^{\|}=\eta_0 \cos(mx)$





Max instability growth rate increases ~10^4 times with a 10-fold increase in U!

Minor changes in U ⟶ large transient energy production.

Transient eddies balance most of the momentum imparted by F ⟶ eddy saturation.

(Similarly as in the baroclinic scenario.)

conclusion and discussion

The barotropic scenario for the momentum balance is viable.

This barotropic QG model shows eddy saturation
when geostrophic contours are open.
This is surprising! All previous arguments were based on baroclinicity.

The barotropic—topographic instability is able to produce transient eddies in this model in a similar manner as baroclinic instability.

We need new process models of baroclinic turbulence
in which the mean flow is wind-driven and topography exerts form stress.

(work in progress with Bill Young)

thank you

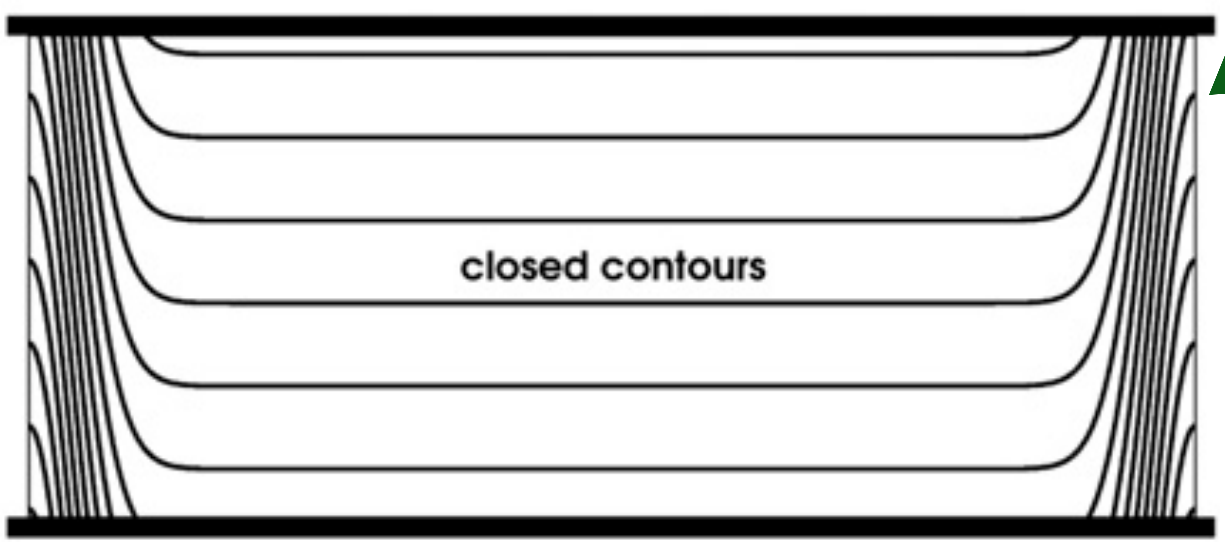
Constantinou (2017). A barotropic model of eddy saturation. *J. Phys. Oceanogr.* (under review, arXiv:1703.06594).
Constantinou and Young (2017). Beta-plane turbulence above monoscale topography. *J. Fluid Mech.*, **827**, 415-447.

extra slides

characterizing geostrophic contours

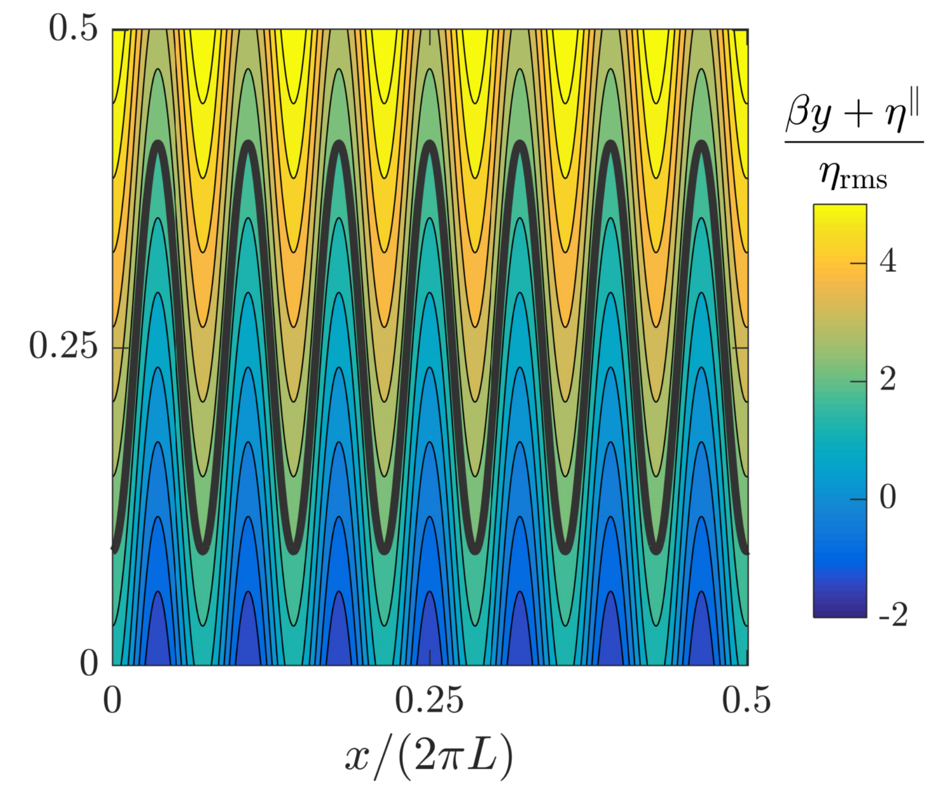
$$\beta y + \eta(x, y)$$

closed/blocked geostrophic contours



Nadeau & Ferrari 2015

open geostrophic contours



Constantinou 2017
Constantinou & Young 2017

without channel walls
both geostrophic contours look alike

decomposing the ACC transport

the time-mean zonal flow:

$$\bar{u}(x,y,z) = \underbrace{\bar{u}(x,y,z) - \bar{u}_{\text{bot}}(x,y)}_{\overset{\text{def}}{=}\bar{u}_{\text{tw}}(x,y,z)} + \bar{u}_{\text{bot}}(x,y)$$

"thermal wind" flow

bottom flow

$$\partial_z \bar{u} = -\partial_y \bar{b}$$

$$\underbrace{\int_{-H}^{0} \mathrm{d}z \int \mathrm{d}y \int \frac{\mathrm{d}x}{L_x}\, \bar{u}}_{\overset{\text{def}}{=}T_{\text{ACC}}} = \underbrace{\int \mathrm{d}y \int \frac{\mathrm{d}x}{L_x}\, \bar{u}_{\text{bot}}}_{\overset{\text{def}}{=}T_{\text{bot}}} + \underbrace{\int_{-H}^{0} \mathrm{d}z \int \mathrm{d}y \int \frac{\mathrm{d}x}{L_x}\, \bar{u}_{\text{tw}}}_{\overset{\text{def}}{=}T_{\text{tw}}}$$

total transport

bottom

"thermal wind"

not included in the barotropic QG model