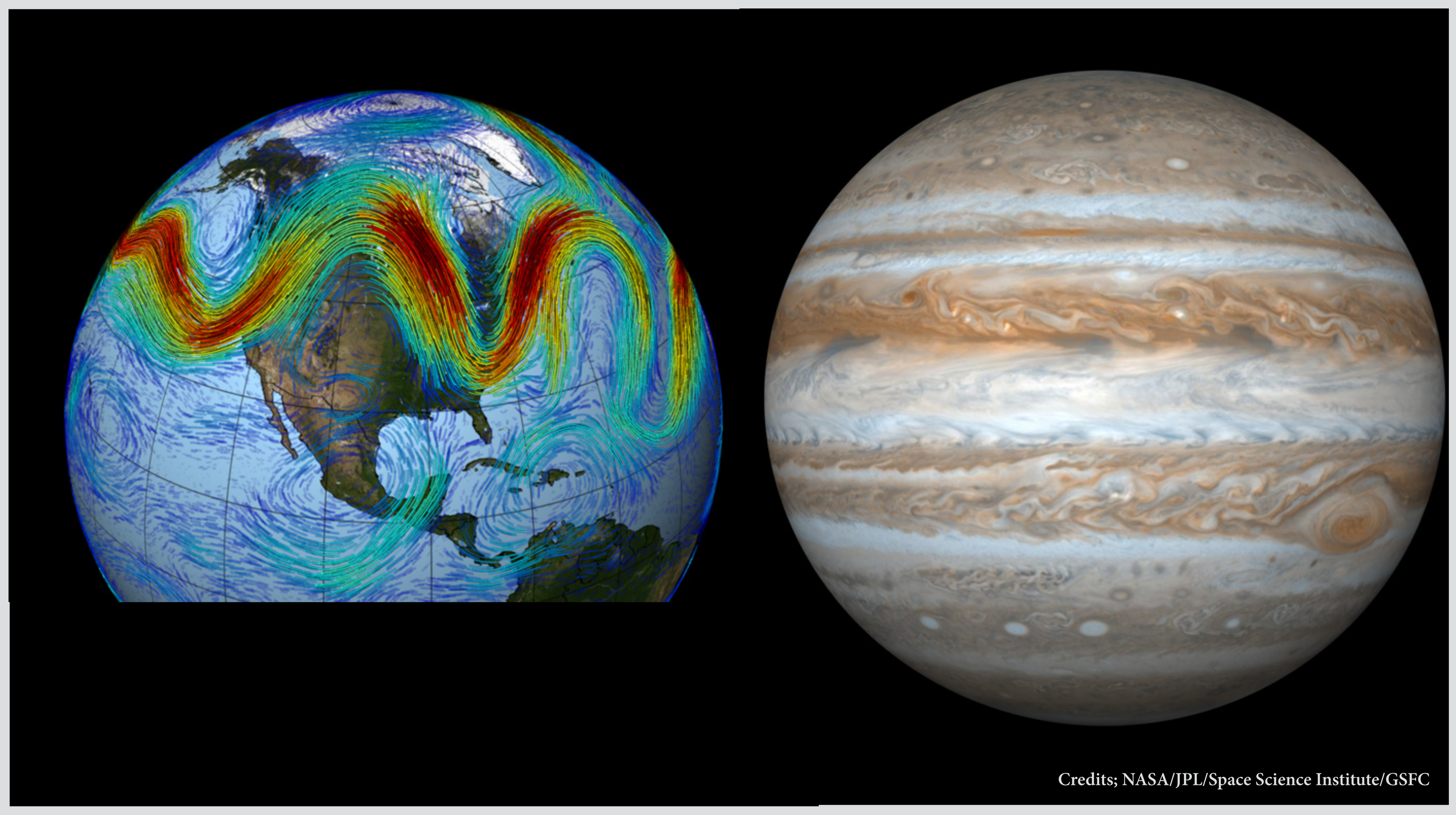
University of Athens

# Emergence and equilibration of zonal winds in turbulent atmospheres

**Navid C. Constantinou and Petros J. Ioannou**

### Atmospheric turbulence self-organizes into zonal jets (midlatitude polar jet)

Theories for jet formation are based on:
- arrest at the jet scale of the turbulent cascade that starts at the small scale of the turbulence excitation
- absolute vorticity mixing arising from wave breaking
- emergence of jets as the statistically most probable state under vorticity mixing




We study the organization of zonal jets in turbulent atmospheres using a new theory: S3T

We show that the jet formation process is unaffected by imposing a quasi-linear restriction on the nonlinear equations which eliminates:
- the turbulent cascade
- wave breaking
- nonlinear vorticity mixing

The mechanism of jet formation is identified as organization of turbulent Reynolds stress by the jet. This results in a cooperative wave/mean-flow interaction instability the eigenmodes of which are the emergent jets. Observed planetary scale jets are identified with the nonlinear equilibria associated with these instabilities.

## THEORY

Simplest setting that exhibits jet formation:
nonlinear barotropic vorticity equation on a beta-plane ($x$: zonal, $y$: meridional)

$$\partial_t \zeta + u\partial_x \zeta + v\partial_y \zeta + \beta v = -r\zeta + \sqrt{\varepsilon}\, f$$

$f$ is external forcing that parameterizes processes absent in the dynamics (i.e. cascade of energy from baroclinic to barotropic eddies or convection) and it is modeled as homogeneous random stirring delta-correlated in time

$$\langle f(\mathbf{x}_a,t) f(\mathbf{x}_b,t')\rangle = \Xi(\mathbf{x}_a-\mathbf{x}_b)\,\delta(t-t')$$

Decomposing flow fields into zonal mean - eddies:

$$\zeta = Z + \zeta' \,,\qquad u = U + u' \,,\qquad \dots \qquad \left(\text{i.e. } Z = \frac{1}{L_x}\int_0^{L_x} \zeta \,\mathrm{d}x\right)$$

**Nonlinear (NL) system**

$$\partial_t U = \overline{v'\zeta'} - r_m U$$
$$\partial_t \zeta' = \mathcal{A}(U)\,\zeta' + \left[\partial_y\left(\overline{v'q'}\right) - \partial_y(v'q') - \partial_x(u'q')\right] + \sqrt{\varepsilon} f$$
$$\mathcal{A}(U) = -U\partial_x - [\beta - (\partial_{yy}U)]\partial_x \Delta^{-1} - r$$

Neglect eddy-eddy interactions responsible for turbulent cascade, wave breaking and nonlinear vorticity mixing.

**Quasilinear (QL) system**

$$\partial_t U = \overline{v'\zeta'} - r_m U$$
$$\partial_t \zeta' = \mathcal{A}(U)\,\zeta' + \cancel{\left[\partial_y\left(\overline{v'q'}\right) - \partial_y(v'q') - \partial_x(u'q')\right]} + \sqrt{\varepsilon} f$$

Construct the equations for the evolution of the eddy statistics of QL

**S3T system**

$$\partial_t U = \frac{1}{2}\left[(\partial_{x_a}\Delta_a^{-1} + \partial_{x_b}\Delta_b^{-1})C_{ab}\right]_{\mathbf{x}_a=\mathbf{x}_b} - r_m U$$
$$\partial_t C_{ab} = \left[\mathcal{A}_a(U) + \mathcal{A}_b(U)\right] C_{ab} + \varepsilon \Xi_{ab}$$
$$C_{ab}(t) = \langle \zeta'(\mathbf{x}_a,t)\zeta'(\mathbf{x}_b,t)\rangle : \text{ eddy vorticity covariance}$$

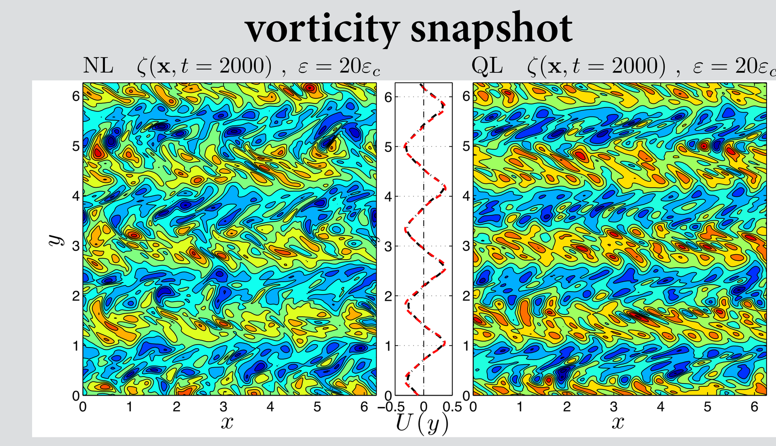
S3T system is a closed, autonomous, fluctuation-free deterministic dynamical system for the evolution of the mean flow $U$ and its associated second order eddy statistics $C$.

S3T system has equilibrium solutions $(U^e, C^e)$ that define stationary statistical states of the turbulent flow (climate state of the system).

S3T can study the stability of equilibria and predict abrupt transition to a different climate state as a function of climate parameters.

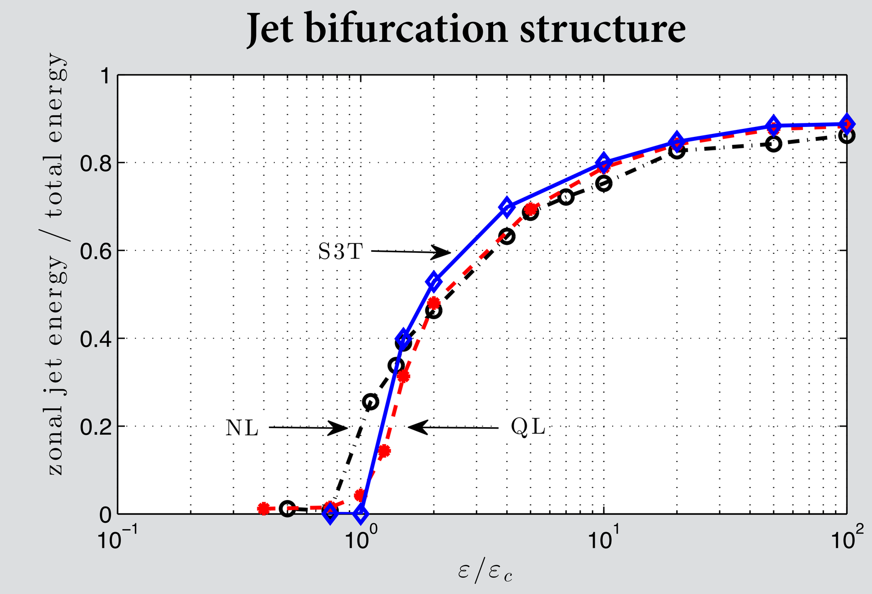
## RESULTS

vorticity snapshot




The reduced quasilinear (QL) system accurately captures the dynamics of the nonlinear (NL) system.

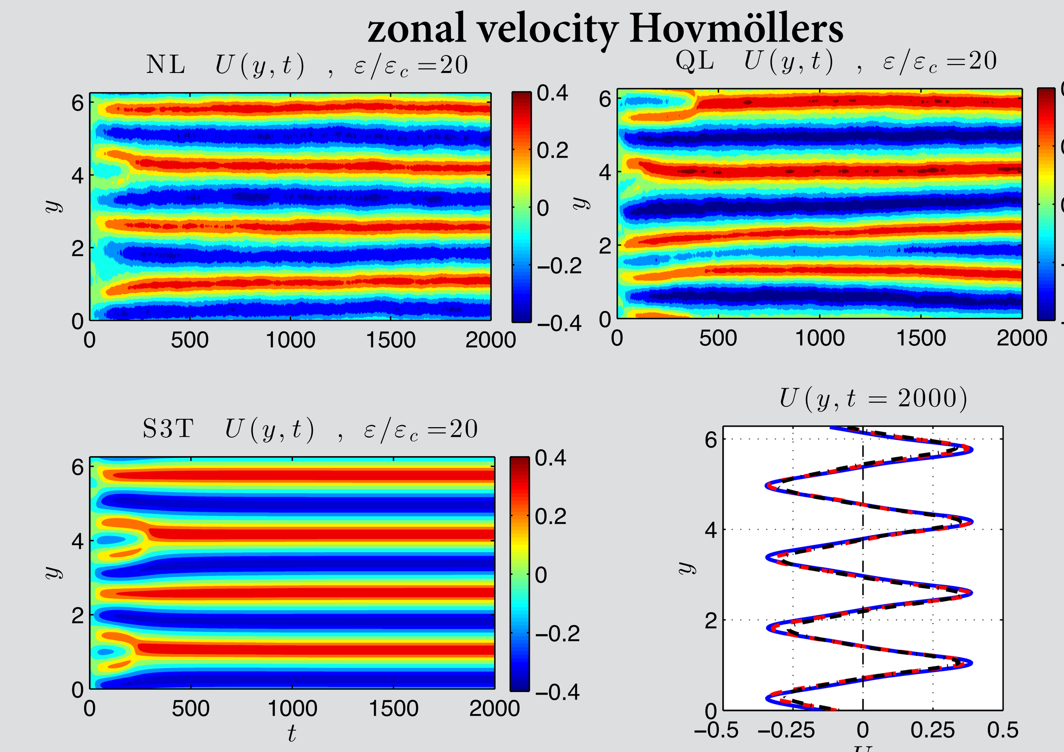
Jet bifurcation structure




Jets form when turbulence forcing exceeds a critical value ($\varepsilon > \varepsilon_c$)

Jets equilibrate at larger amplitude as turbulence forcing increases.

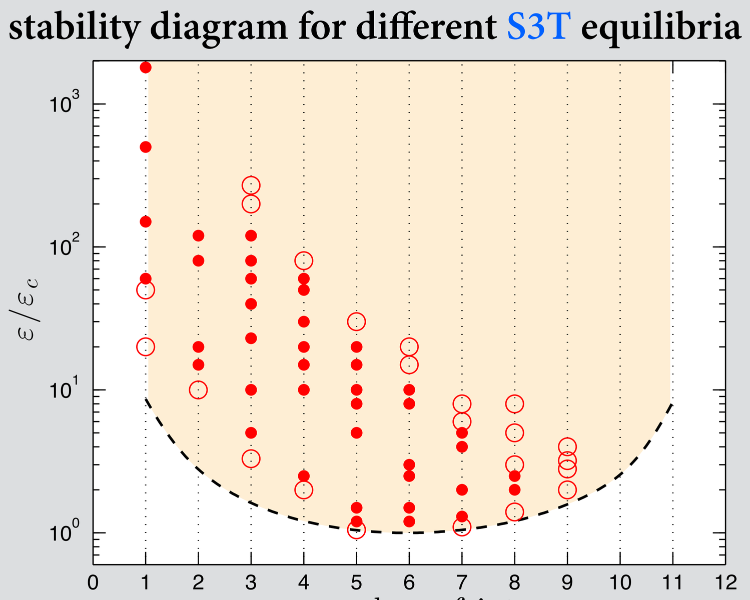
zonal velocity Hovmöllers




S3T stability analysis of the homogeneous turbulent equilibrium with no mean flow $(U^e = 0\,,\ C^e = \varepsilon\Xi/(2r))$ accurately predicts $\varepsilon_c$ and the emergent jet structure.

The S3T system predicts the structure and amplitude of the equilibrated jets

stability diagram for different S3T equilibria




S3T stability analysis of finite amplitude jet equilibria predicts:
- existence of multiple climate states for given parameters
- transition from one climate state to the other as turbulence forcing increases

## CONCLUSIONS

- Jet emergence occurs through a bifurcation in turbulence forcing.
- Snapshots and Hovmöller diagrams of jet emergence and nonlinear equilibration show that QL and S3T predict the growth and equilibration of jets, exactly as simulated by NL, despite the fact that these dynamics do not support a turbulent cascade, wave breaking, or nonlinear turbulent mixing.
- S3T provides the dynamics of the statistics of the flow. Equilibria of the S3T system define the climate state of the flow.
- Stability analysis of S3T equilibria accurately predicts abrupt transition to a different climate state (in this case different structure of zonal jets and eddy statistics).

Constantinou, N. C., B. F. Farrell and P. J. Ioannou, 2014: Emergence and equilibration of jets in beta-plane turbulence: applications of Stochastic Structural Stability Theory, *J. Atmos. Sci.*, **71** (5), 1818-1842.

This work has been supported by Onassis Foundation