Statistical state dynamics reveals mechanism for organization of coherent structures in turbulence

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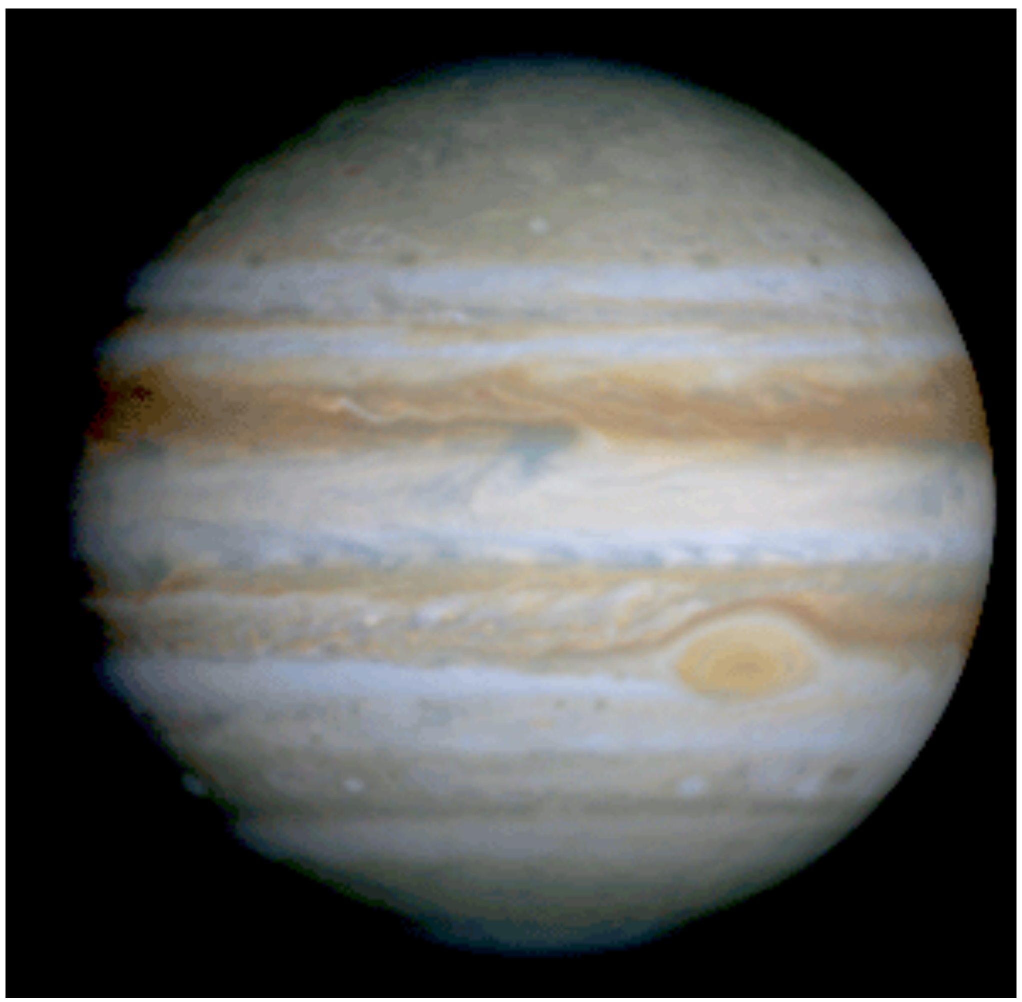
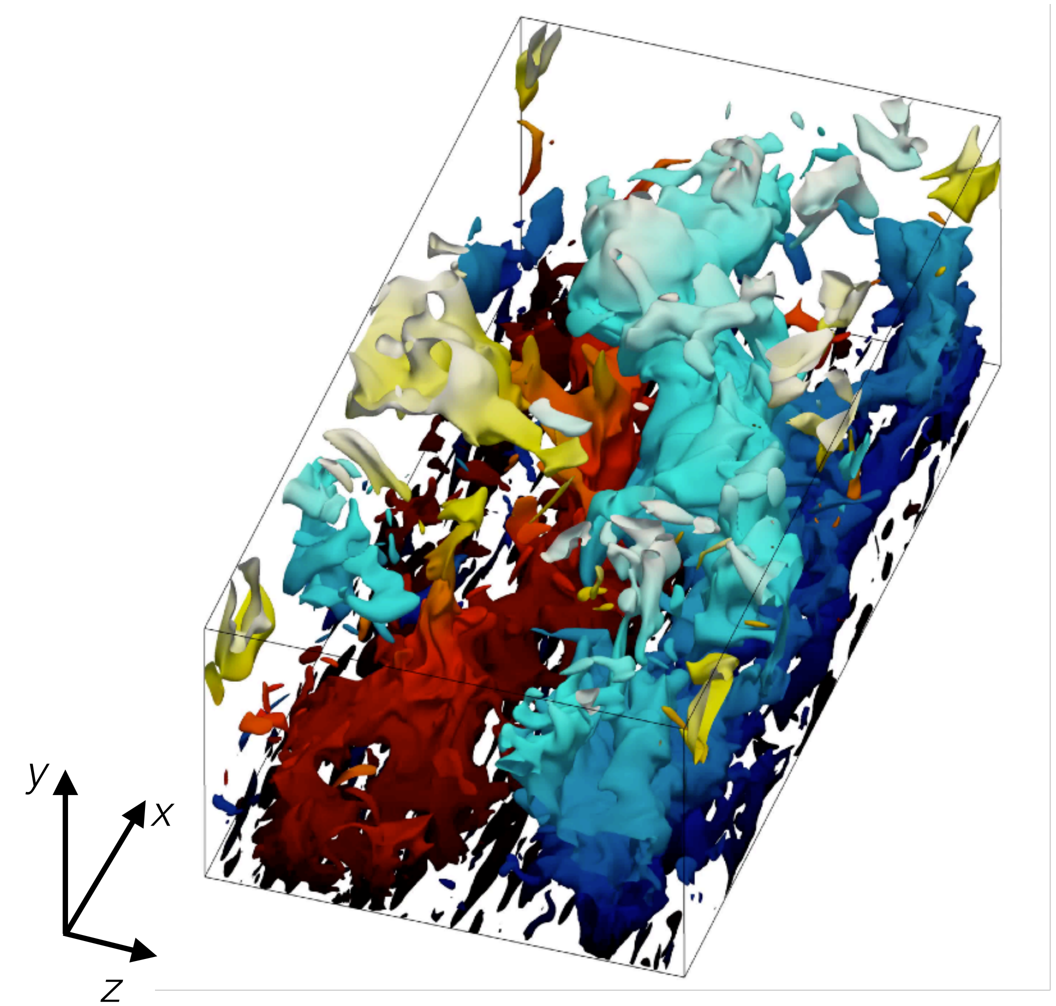




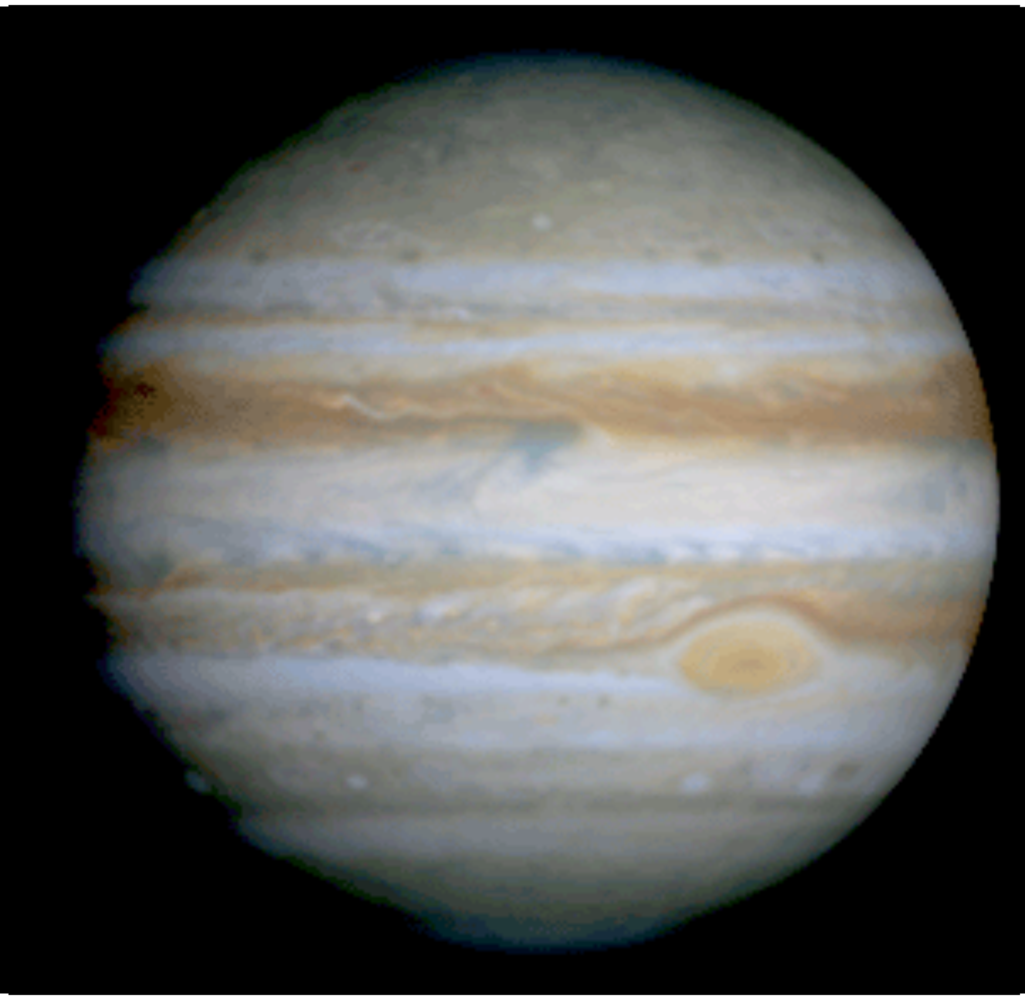
London, August 2018

Acknowledgements go to:
N. Bakas, B. Farrell,
P. Ioannou, M.-A. Nikolaidis

planetary turbulence

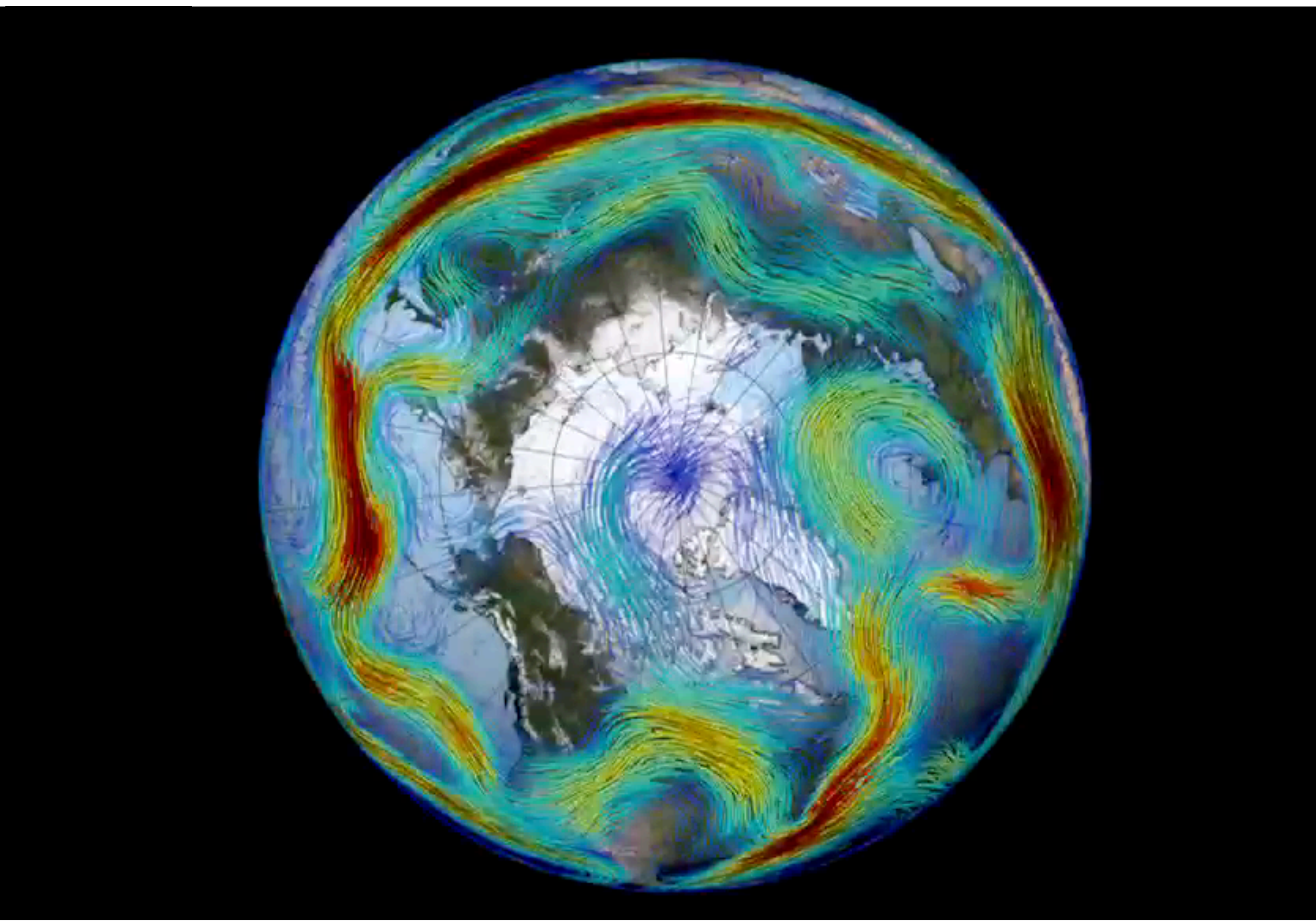
most of the energy of the flow is in large-scale coherent jets and vortices

not at the largest allowed scale (as 2D inverse energy cascade might imply)
arrest of the cascade by jets



banded Jovian jets

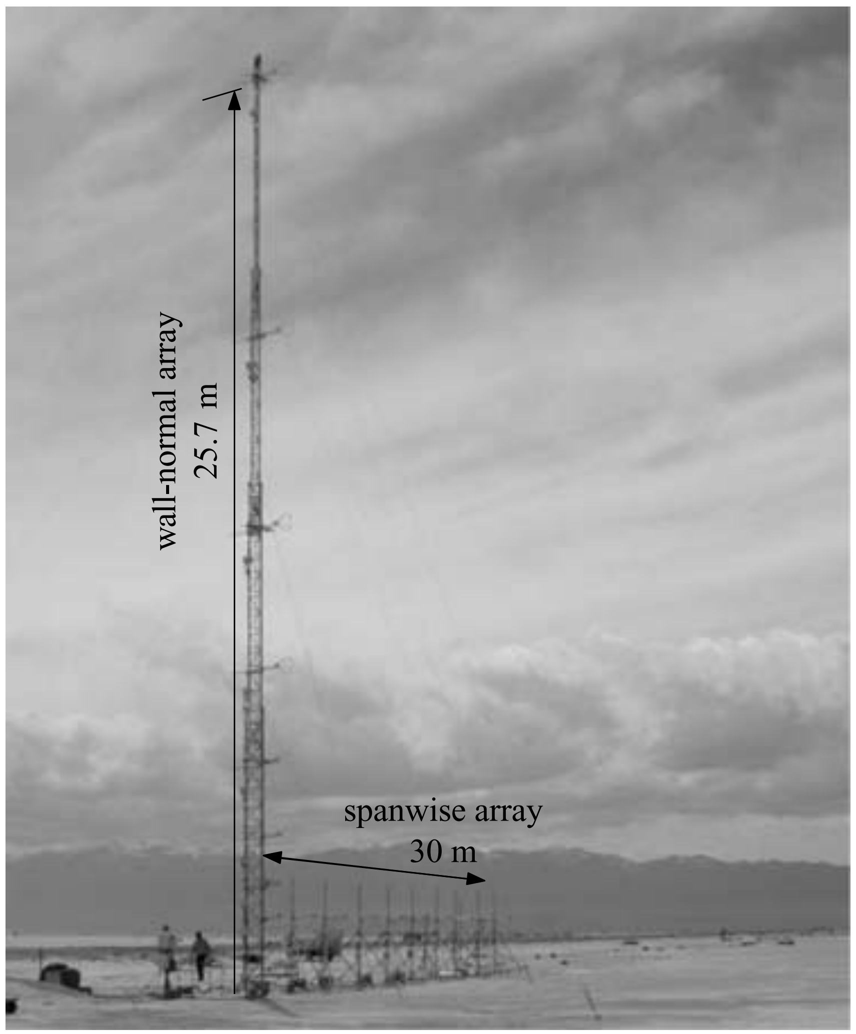
NASA/Cassini Jupiter Images



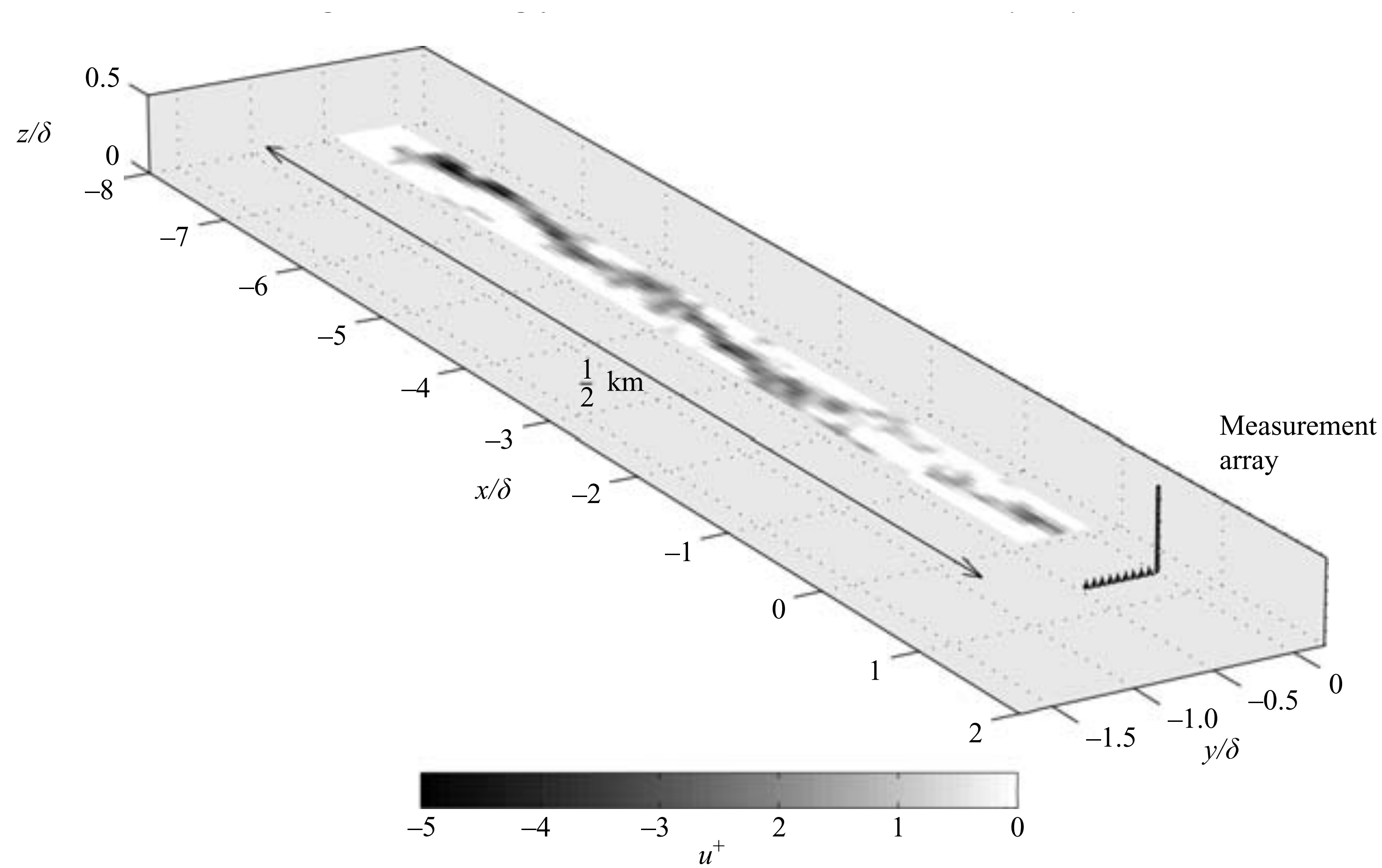
polar front jet

NASA/Goddard Space Flight Center

boundary layer turbulence



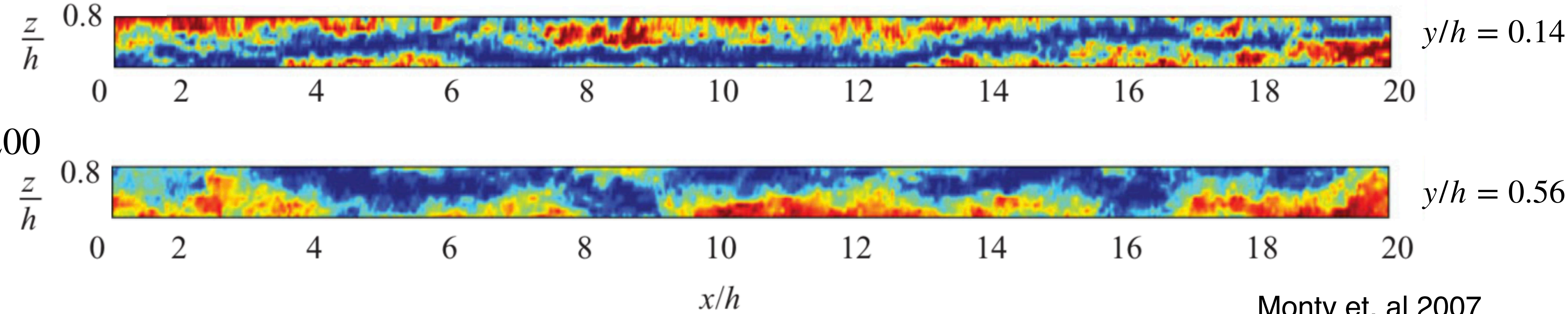
observing boundary layer in
Utah salt lake



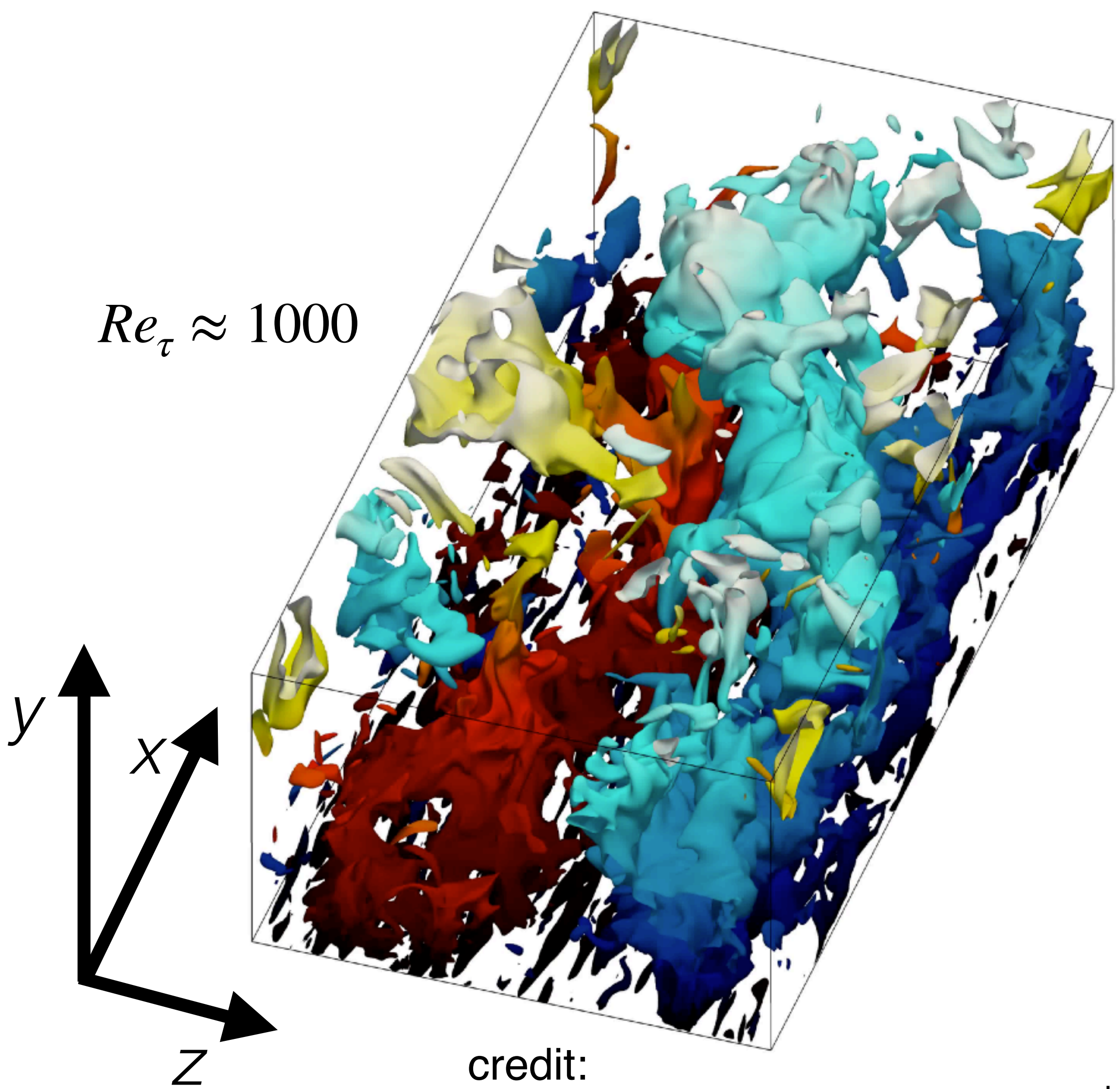
Hutchins & Marusic 2007

large-scale motions in wall-bounded turbulence

$Re_\tau \approx 3200$

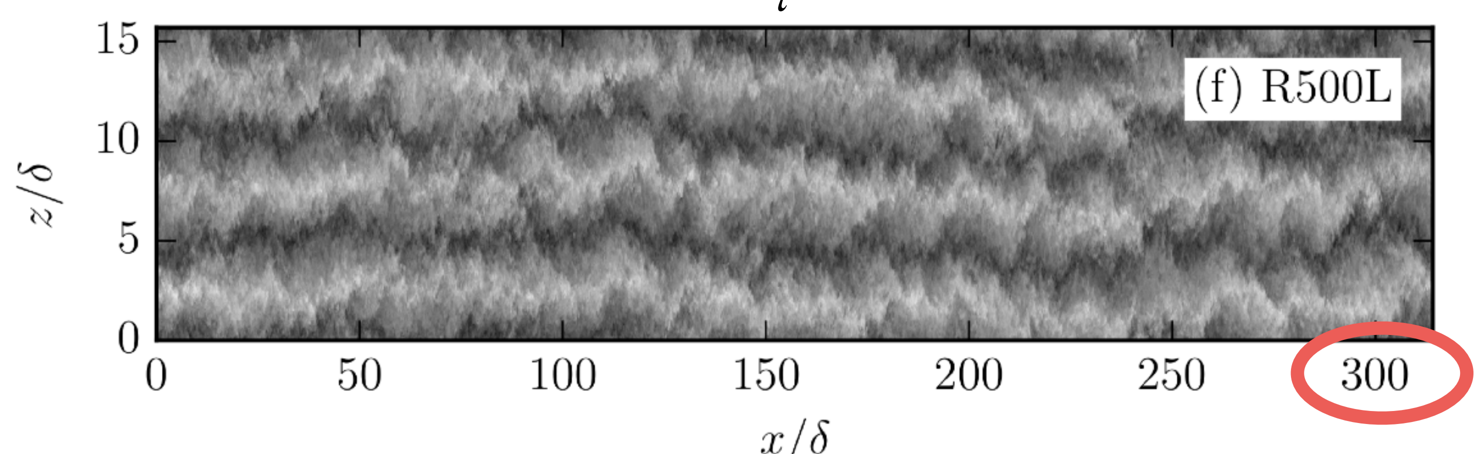


Monty et. al 2007



credit:
A Lozano-Durán

$Re_\tau = 500$



Lee & Moser 2018

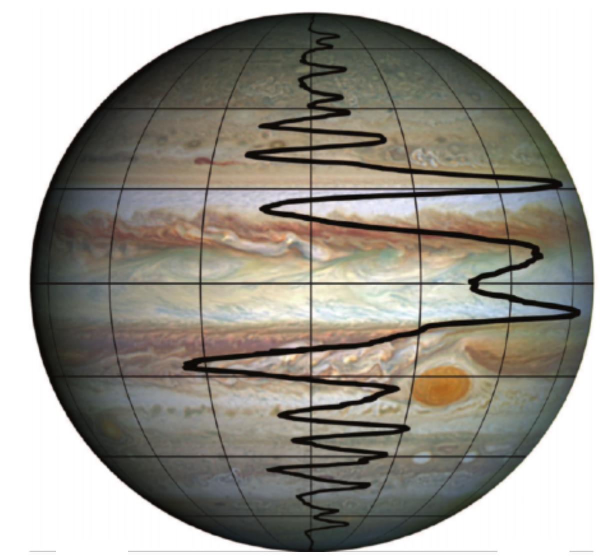
streamwise velocity
(everything *but* the time-mean of the $U(k_x = k_z = 0)$)

The problem to be addressed:

Understand how these *specific* structures arise
and how are they maintained

outline

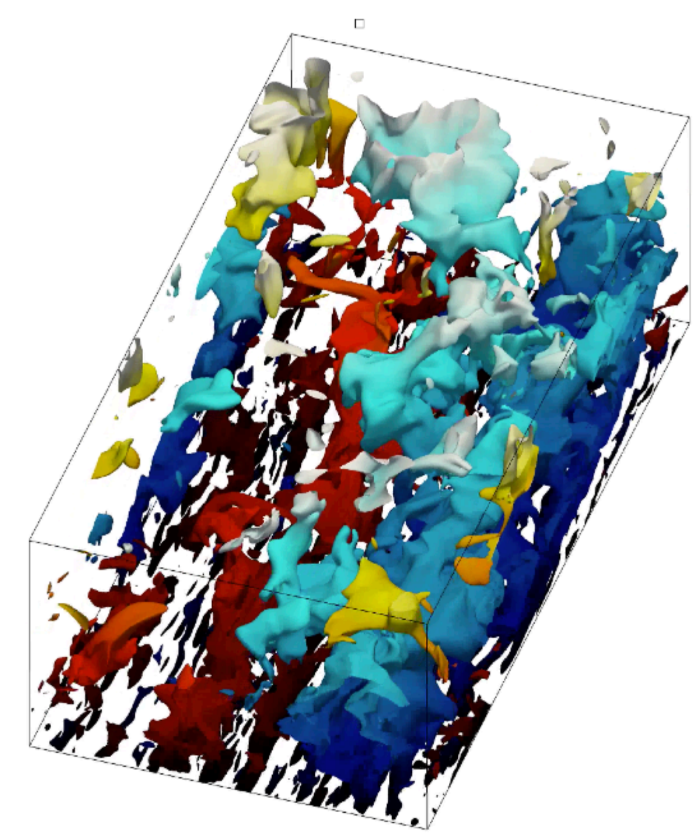
- new framework for studying turbulent flows
  (statistical state dynamics)



begin with

- zonal jet formation in planetary atmospheres

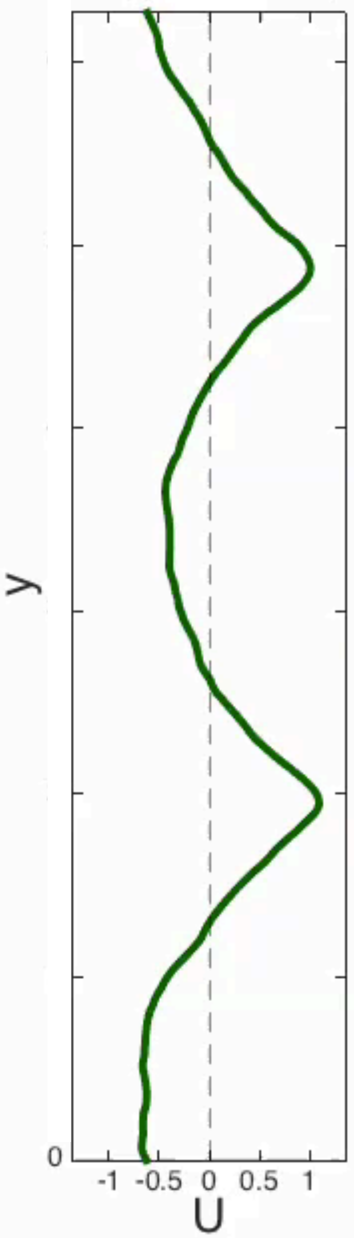
    (familiarise with the basic concepts/ideas)

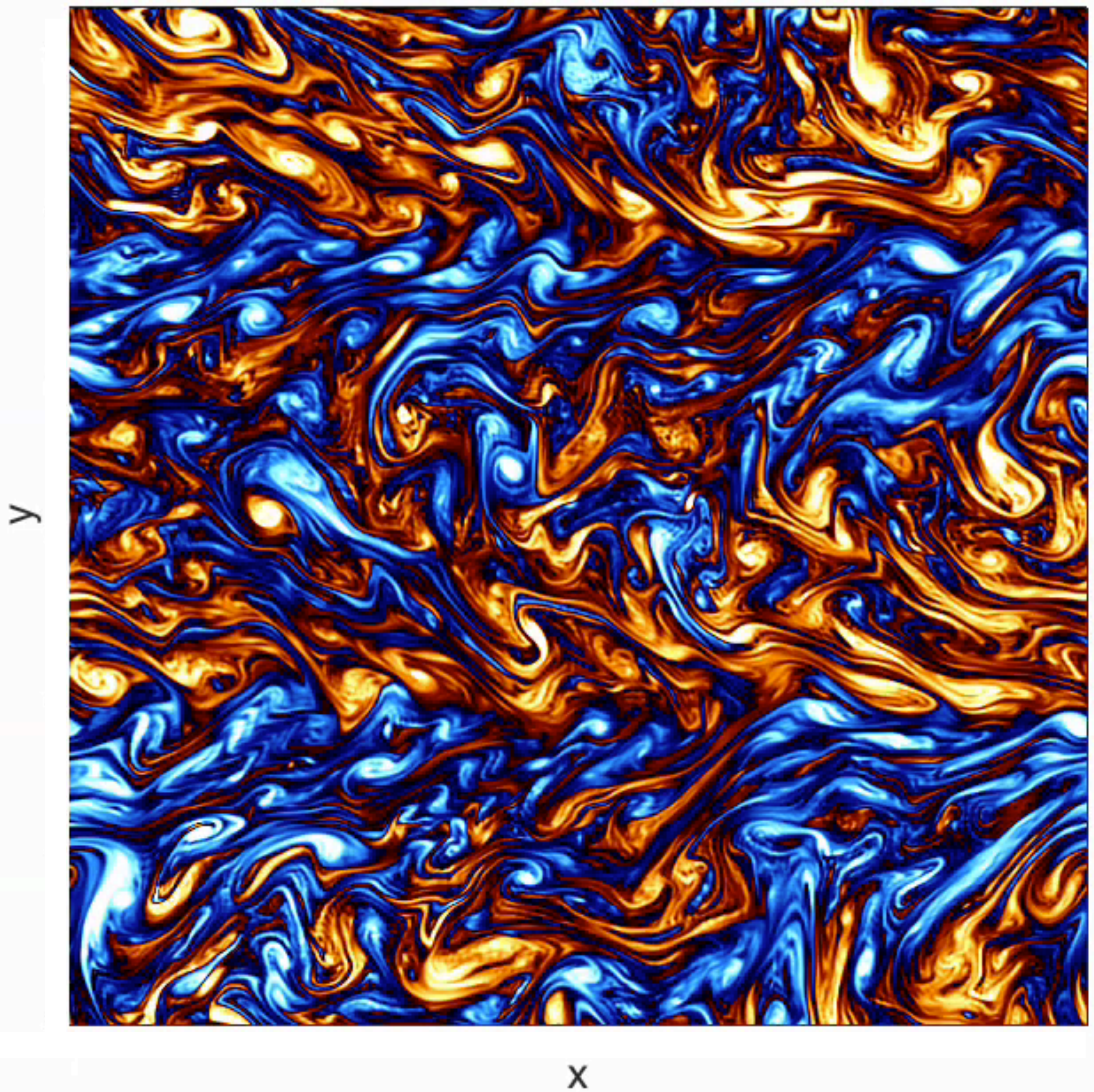


then discuss how SSD illuminates

- streak-roll formation & SSP
  in wall-bounded turbulence

zonal jet formation in forced-dissipative barotropic β plane





statistically homogeneous small-scale forcing

(forcing **does not** impose any inhomogeneity)

random flow inhomogeneities organize the turbulence so that they are reinforced

we observe:

- jet emerge
- jets appear to change *much slower* compared to the eddies
- jets may merge

non-dimensional parameters

$\varepsilon k_f^2 / \mu^3 = 10^6$ (≈"amplitude of forcing")

$\beta / (k_f \mu) = 67$ (≈"rotation of the planet")

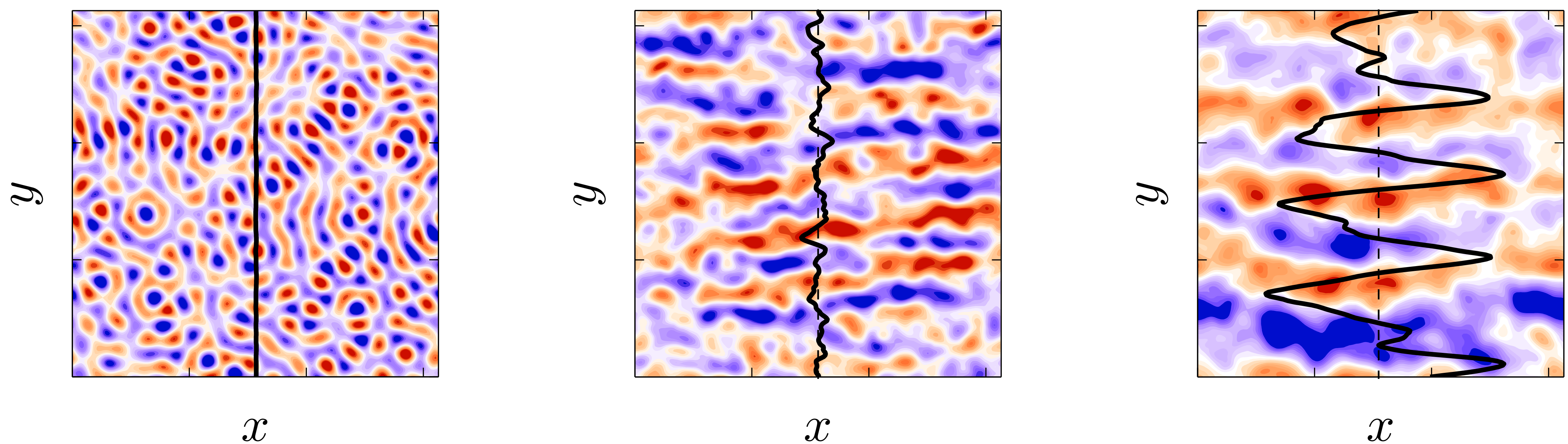
β gradient of Coriolis parameter, μ linear drag, ε energy injection rate by the forcing; k_f characteristic wavenumber of forcing

various β-plane flow regimes flows at statistically steady state:

homogeneous — traveling waves — zonal jets

$$\beta/(k_f r) = 67$$

$\varepsilon k_f^2/\mu^3 = \quad 10^2 \qquad\qquad 5\times 10^3 \qquad\qquad 5\times 10^4$



this suggests that there is some kind of transition as ε is increased

[snapshots of the streamfunction together with instantaneous zonal mean flow $U(y,t)$]

claims

I. The underlying dynamics of structure formation lies in the interaction of turbulent eddies with mean flows

(what part of the flow is the "mean flow"?)

II. Often, structure formation has analytic expression *only* in the Statistical State Dynamics (SSD)

(the dynamics that govern the statistics of the flow rather than the dynamics governing single flow realizations)

III. Because of (**I**) a second-order closure of the SSD is adequate

(given that we decompose our fields into mean+eddies adequately)

Statistical State Dynamics (SSD)

1. split the flow variables into: $\overline{\text{mean}}$ + eddy $'$

$$\boldsymbol{u}(\boldsymbol{x},t) = \overline{\boldsymbol{u}(\boldsymbol{x},t)} + \boldsymbol{u}'(\boldsymbol{x},t)$$

[mean is **not** a time-mean!]

2. form the hierarchy of same-time statistical moments/cumulants

$$\underbrace{\overline{\boldsymbol{u}(\boldsymbol{x}_a,t)}}_{=C_a^{(1)}}\ ,\quad \underbrace{\overline{\boldsymbol{u}'(\boldsymbol{x}_a,t)\boldsymbol{u}'(\boldsymbol{x}_b,t)}}_{=C_{ab}^{(2)}}\ ,\quad \underbrace{\overline{\boldsymbol{u}'(\boldsymbol{x}_a,t)\boldsymbol{u}'(\boldsymbol{x}_b,t)\boldsymbol{u}'(\boldsymbol{x}_c,t)}}_{=C_{abc}^{(3)}}\ ,\quad \dots$$

3. use equations of motion to find how each one of the cumulants evolve

$$\partial_t C_a^{(1)} = \mathcal{F}_1\left(C_a^{(1)}\ ,\ C_{ab}^{(2)}\right)$$

$$\partial_t C_{ab}^{(2)} = \mathcal{F}_2\left(C_a^{(1)}\ ,\ C_{ab}^{(2)}\ ,\ C_{abc}^{(3)}\right)$$

$$\partial_t C_{abc}^{(3)} = \mathcal{F}_3\left(C_a^{(1)}\ ,\ C_{ab}^{(2)}\ ,\ C_{abc}^{(3)}\ ,\ C_{abcd}^{(4)}\right)\quad ,\ \text{etc}\ \dots$$

Hopf 1952
Frisch 1995

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Hopf 1952
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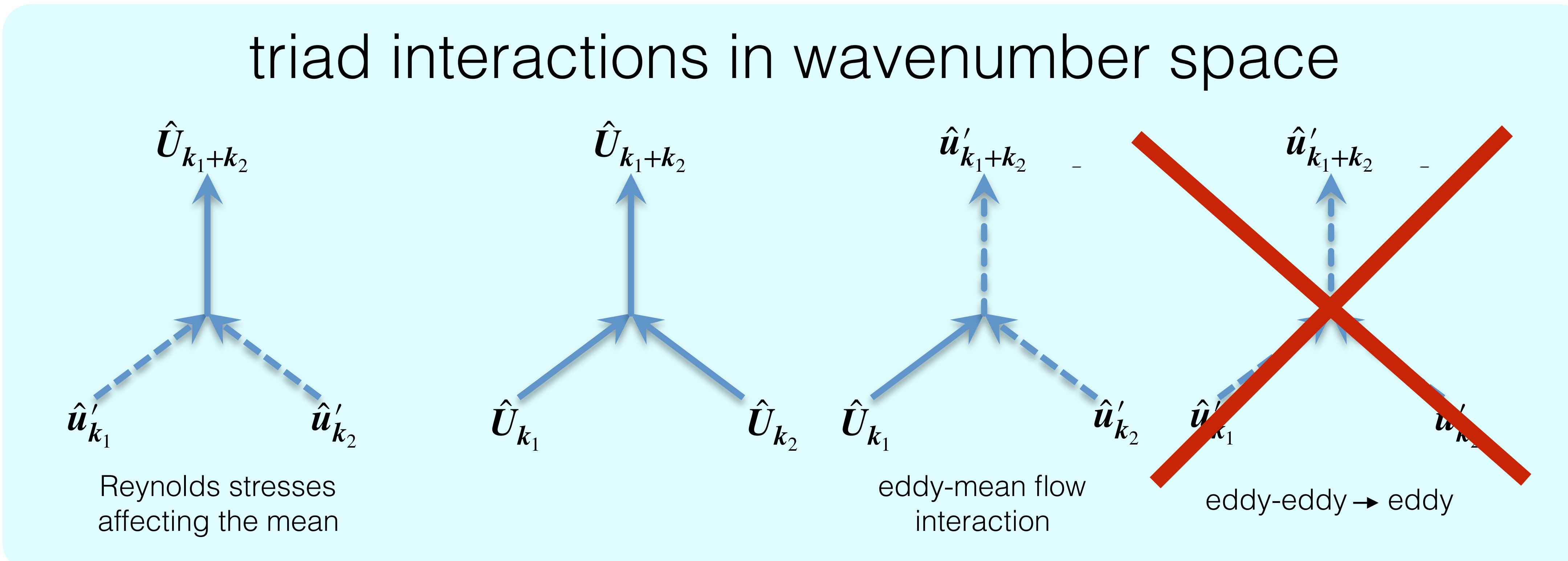
4. SSD closure at second order (called S3T or CE2)

Farrell & Ioannou 2003
Marston et al. 2008

Navier-Stokes-type quadratic nonlinearities



an SSD closure at second order
is ***the same*** as dropping the
eddy-eddy⟶eddy nonlinearity

does it matter what we identify with the $\overline{\text{mean}}$?

$\overline{\text{mean}}$ = $x,\ z,\ t$ -average

$$\overline{\boldsymbol{u}(x,y,z,t)} = U(y)$$

[Reynolds decomposition]
no mean flow dynamics

$\overline{\text{mean}}$ = $x,\ z$ -average

$$\overline{\boldsymbol{u}(x,y,z,t)} = U(y,t)$$

[see Jimenez & Pinelli (1999) experiments:
filtering out the streaks is equivalent with taking
here the streaks as part of the incoherent flow]

$\overline{\text{mean}}$ = x-average

$$\overline{\boldsymbol{u}(x,y,z,t)} = U(z,y,t)$$

[streamwise mean]

$\overline{\text{mean}}$ = small-k_x spatial average

[e.g., NCC, Farrell & Ioannou 2016,
Marston, Tobias, & Chini 2016; Child et al. 2016;
Tobias & Marston 2017]

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remarks on SSD — what is novel here?

$$
\begin{aligned}
\partial_t C_a^{(1)} &= \mathcal{F}_1\left(C_a^{(1)}\ ,\ C_{ab}^{(2)}\right) \\
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\end{aligned}
$$

remarks on SSD — what is novel here?

Usually (motivated by homogeneous isotropic turbulence) people took $\overline{\boldsymbol{u}(\boldsymbol{x},t)} = 0$

$$\partial_t C^{(1)}_{a} = \mathcal{F}_1\left(C^{(1)}_{a}\ ,\ C^{(2)}_{ab}\right)$$

but this is fundamental for structure formation (claim ($\mathbf{I}$))

$$\partial_t C^{(2)}_{ab} = \mathcal{F}_2\left(C^{(1)}_{a}\ ,\ C^{(2)}_{ab}\ ,\ C^{(3)}_{abc}\right)$$

$$\partial_t C^{(3)}_{abc} = \mathcal{F}_3\left(C^{(1)}_{a}\ ,\ C^{(2)}_{ab}\ ,\ C^{(3)}_{abc}\ ,\ C^{(4)}_{abcd}\right) \quad \text{, etc ...}$$

remarks on SSD — what is novel here?

Usually (motivated by homogeneous isotropic turbulence) people took $\overline{\boldsymbol{u}(\boldsymbol{x},t)} = 0$

Main focus/effort was to obtain the equilibrium statistics: $\partial_t = 0$

$$\partial_t C^{(1)}_a = \mathcal{F}_1\left(C^{(1)}_a \ , \ C^{(2)}_{ab}\right)$$

but this is fundamental for structure formation (claim ($\mathbf{I}$))

$$0 \ = \mathcal{F}_2\left(C^{(1)}_a \ , \ C^{(2)}_{ab} \ , \ C^{(3)}_{abc}\right)$$

$$0 \ = \mathcal{F}_3\left(C^{(1)}_a \ , \ C^{(2)}_{ab} \ , \ C^{(3)}_{abc} \ , \ C^{(4)}_{abcd}\right) \quad , \text{etc} \ldots$$

remarks on SSD — what is novel here?

By studying the *dynamics* of the statistics
novel explanations for phenomena become available.

$$
\begin{aligned}
\partial_t C^{(1)}_{a} &= \mathcal{F}_1\left(C^{(1)}_{a}\ ,\ C^{(2)}_{ab}\right)\\
\partial_t C^{(2)}_{ab} &= \mathcal{F}_2\left(C^{(1)}_{a}\ ,\ C^{(2)}_{ab}\ ,\ C^{(3)}_{abc}\right)\\
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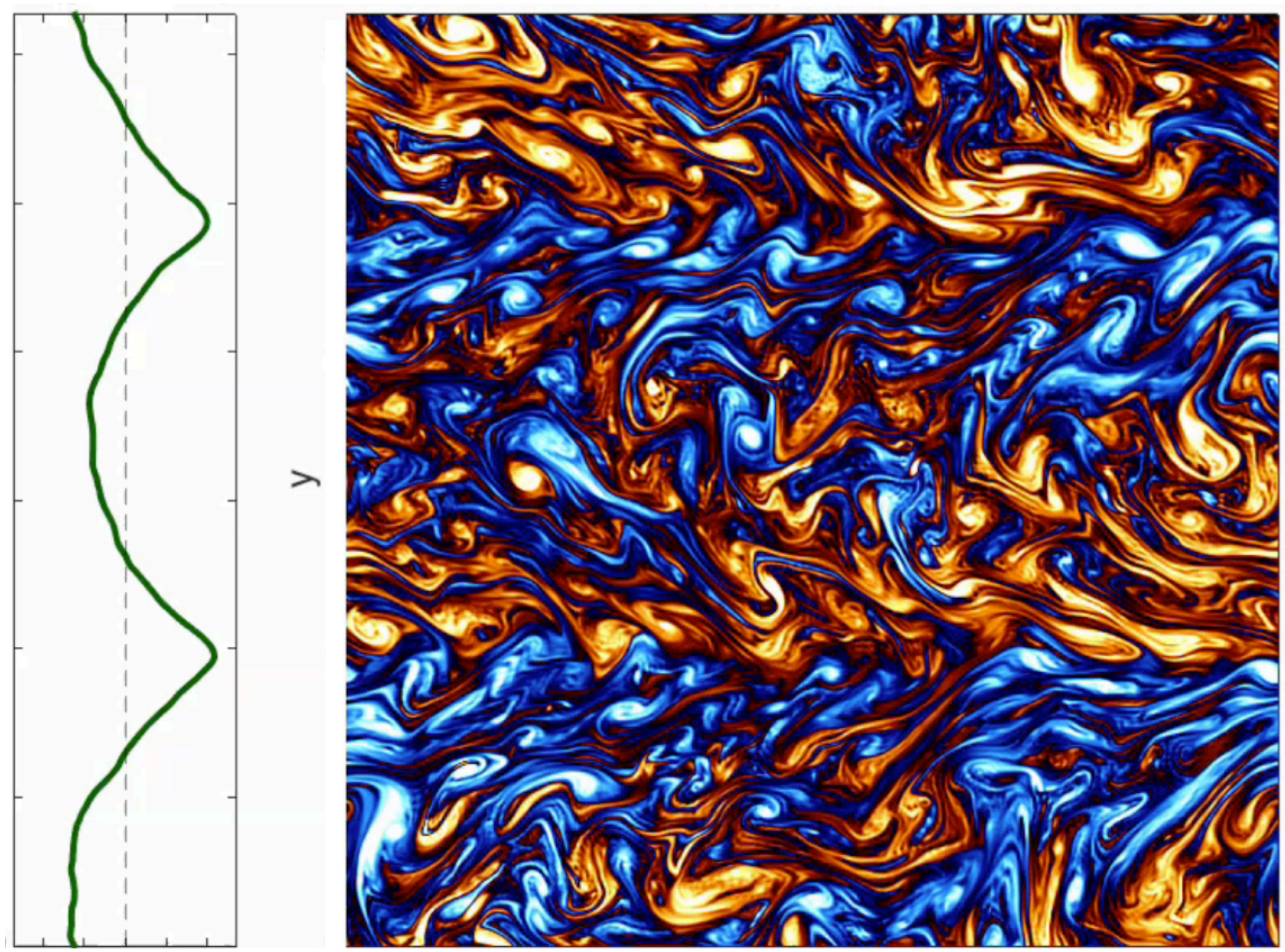
\end{aligned}
$$

While flow realizations exhibit the phenomena,
analytic expression of the phenomena requires the SSD.

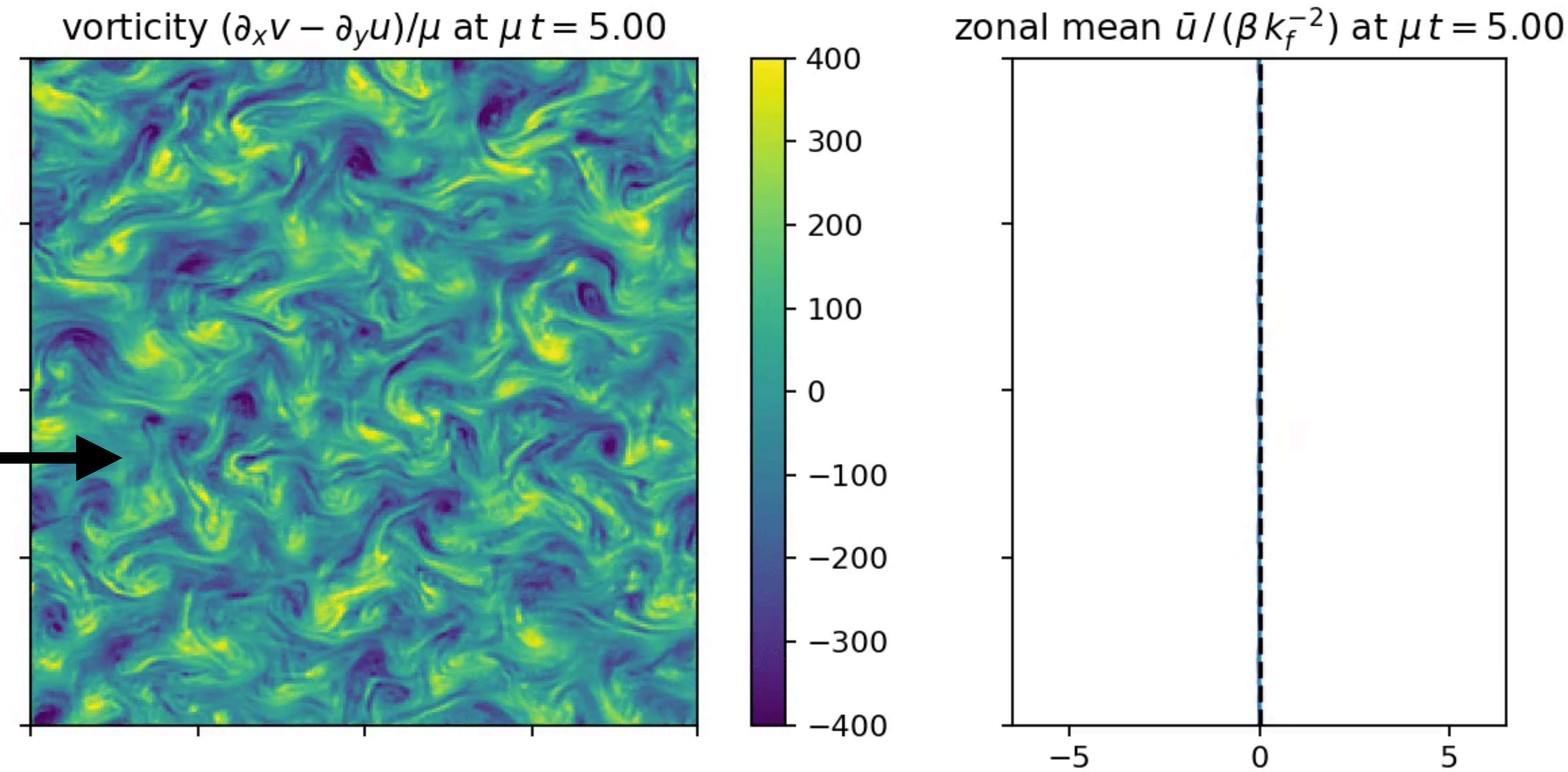
examples?

understanding zonal jet formation through SSD

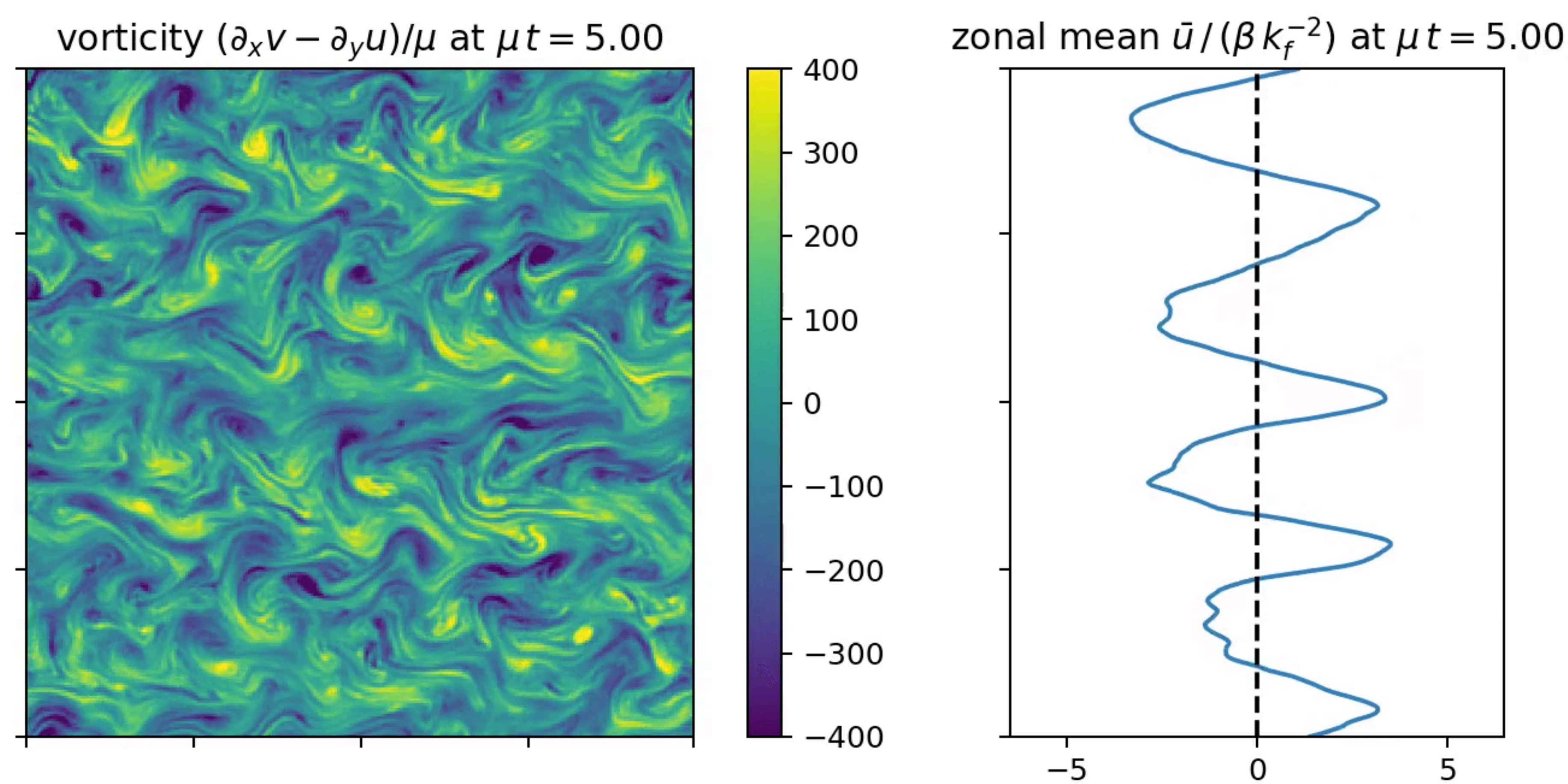


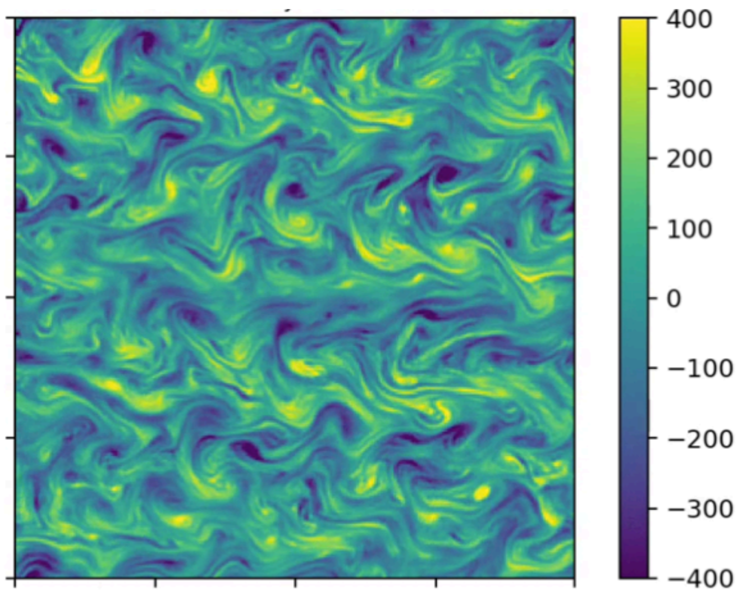
how do we show that a flow like this ...

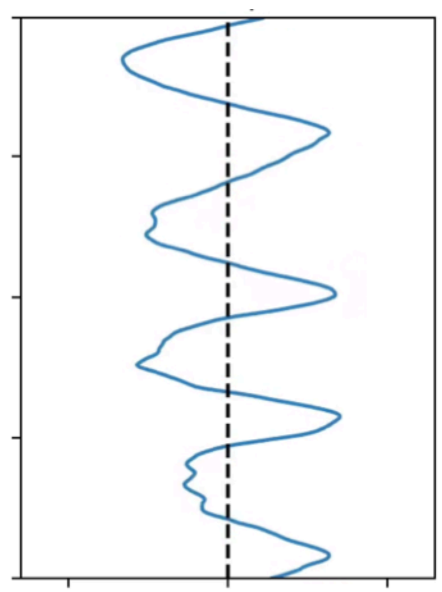
[simulation in which we kill the $k_x = 0$ component at each time step]

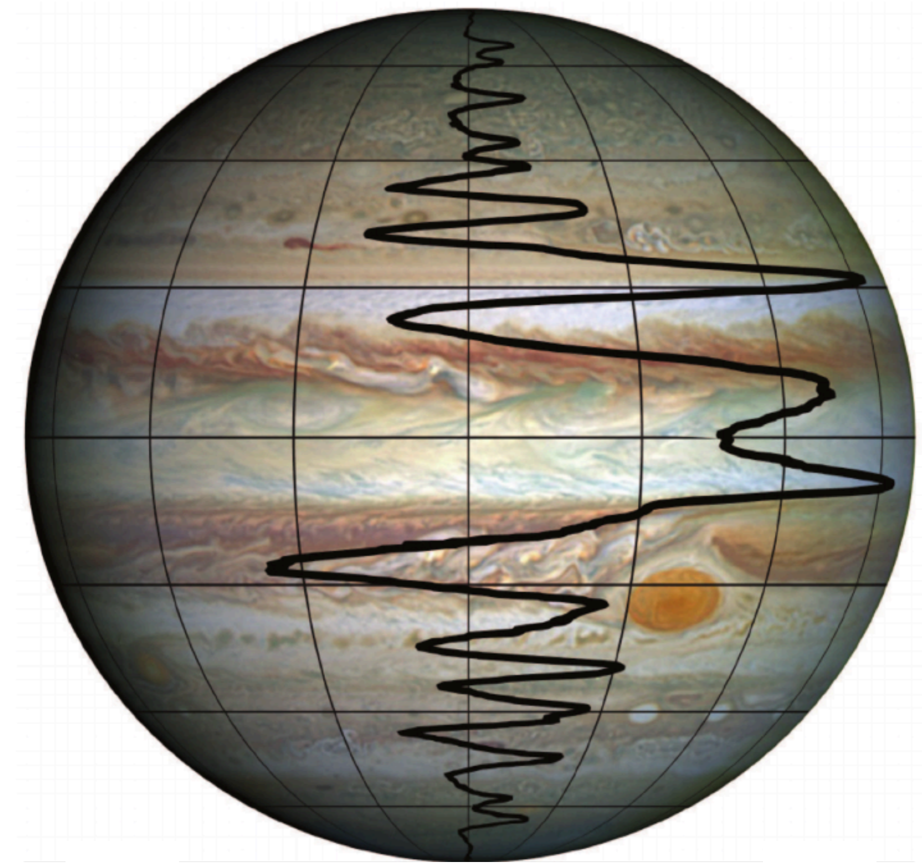


... is ***unstable*** leading to forming four jets?









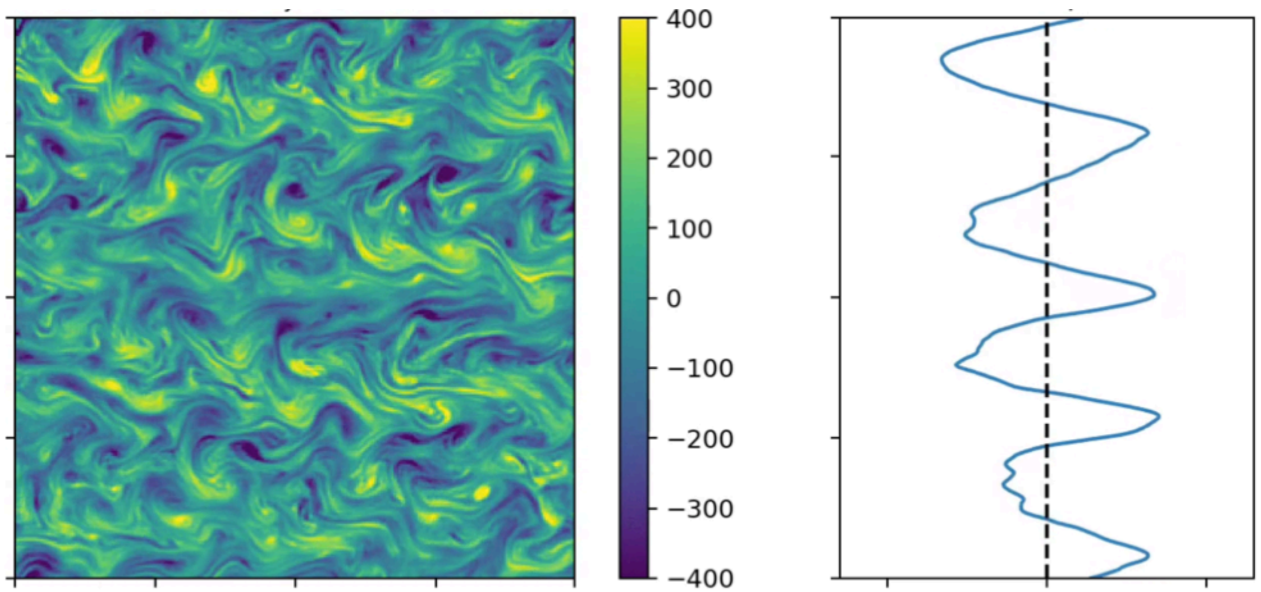
at statistical equilibrium:

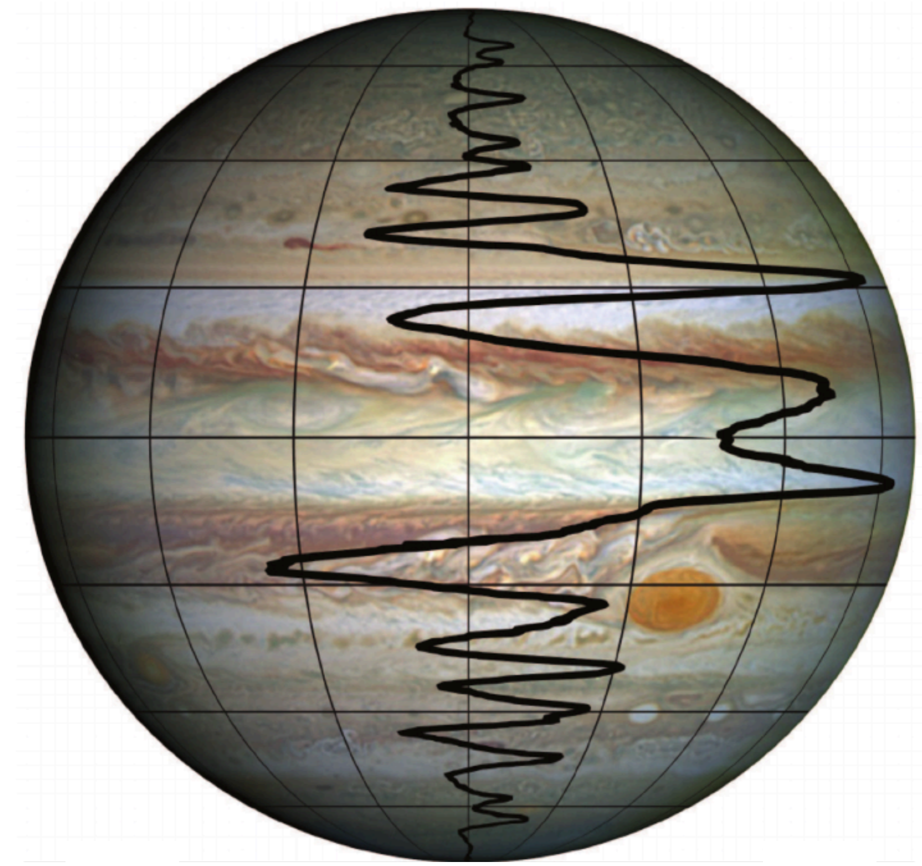
realization dynamics

jets + turbulent eddies

≈ steady

strongly
time-dependent





at statistical equilibrium:

statistical state dynamics

jets **+** second-order eddy **statistics**

≈steady ≈stationary

Farrell & Ioannou 2003, 2007; Srinivasan & Young 2012; Tobias & Marston 2013; NCC, Farrell & Ioannou 2014, 2016; Bakas, NCC & Ioannou 2015, 2018; Bakas & Ioannou 2013, 2014, 2018; Parker & Krommes 2013, 2014; Marston, Tobias, Chini, 2016; Ait-Chaalal, Schneider, Meyer, & Marston; Marston & Tobias 2017, NCC & Parker 2018

S3T second-order closure of SSD

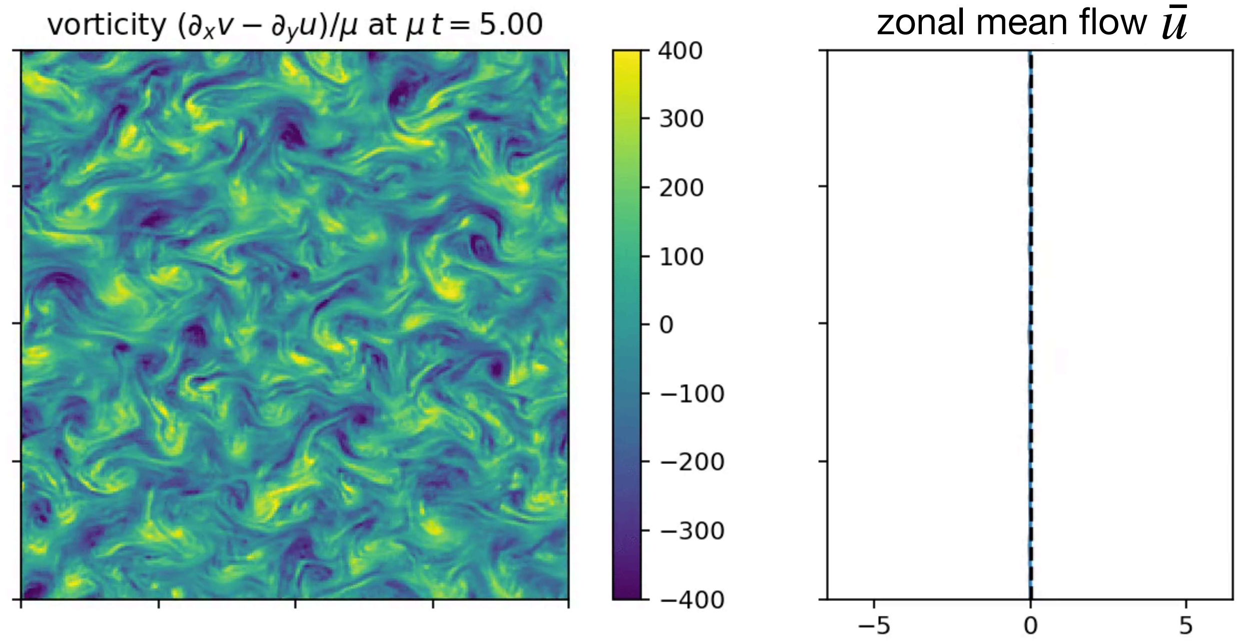
$$\partial_t C_a^{(1)} = \mathscr{F}_1\big(C_a^{(1)}, C_{ab}^{(2)}\big)$$

$$\partial_t C_{ab}^{(2)} = \mathscr{F}_2\big(C_a^{(1)}, C_{ab}^{(2)}, Q_{ab}\big)$$

mean flow
(jets)

second-order
eddy statistics

spatial covariance
of forcing
second-order
(**known**)



homogeneous
stationary
second-order
eddy statistics

+ no mean flow

this is a fixed point of the SSD closure

$$\partial_t C^{(1)}_{a} = \mathscr{F}_1\big(C^{(1)}_{a}, C^{(2)}_{ab}\big)$$

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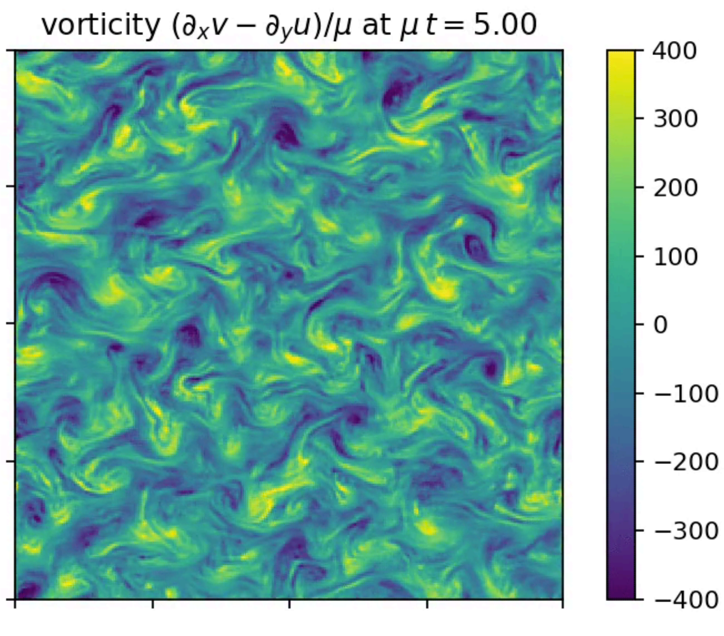
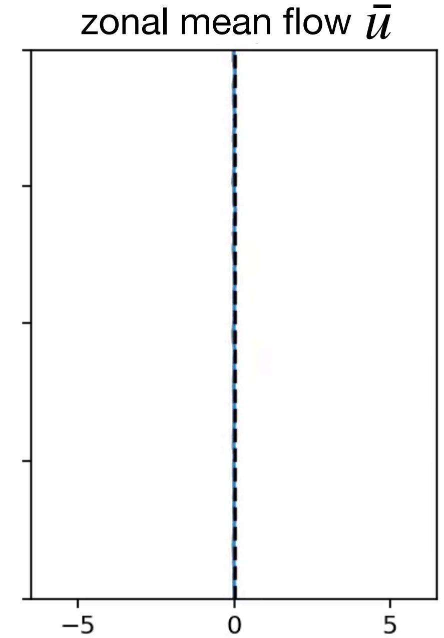




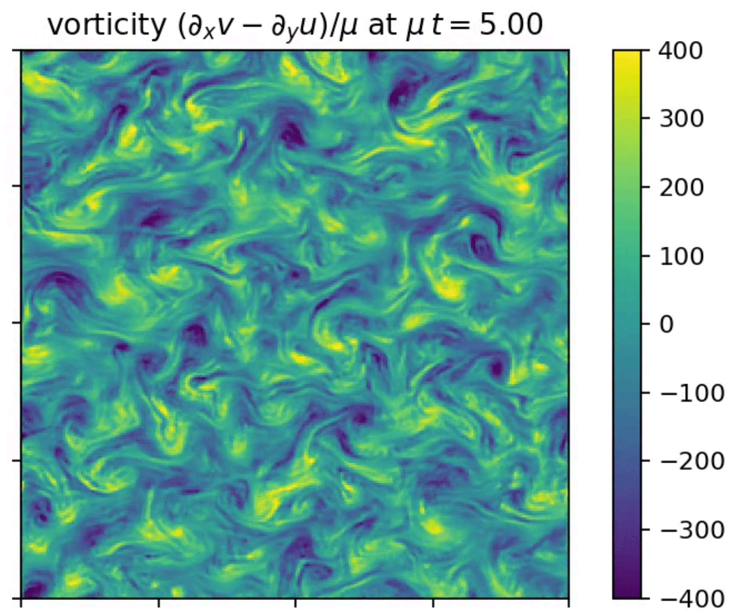
homogeneous
stationary
second-order
eddy statistics

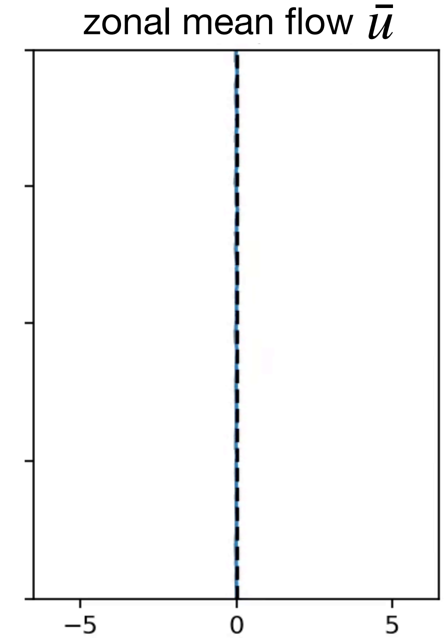
+ no mean flow

this is a fixed point of the SSD closure

let's perturb it and study its stability...
(doable, but we have to solve an eigenvalue problem of dimension $\mathcal{O}(n^4 \times n^4)$)

note: we've linearized about a turbulent state!





homogeneous
stationary
second-order
eddy statistics

+ no mean flow

as we cross a threshold value of $\varepsilon k_f^2/\mu^3$
the homogeneous turbulent state ***becomes unstable***
to infinitesimal zonal jet mean flow perturbations

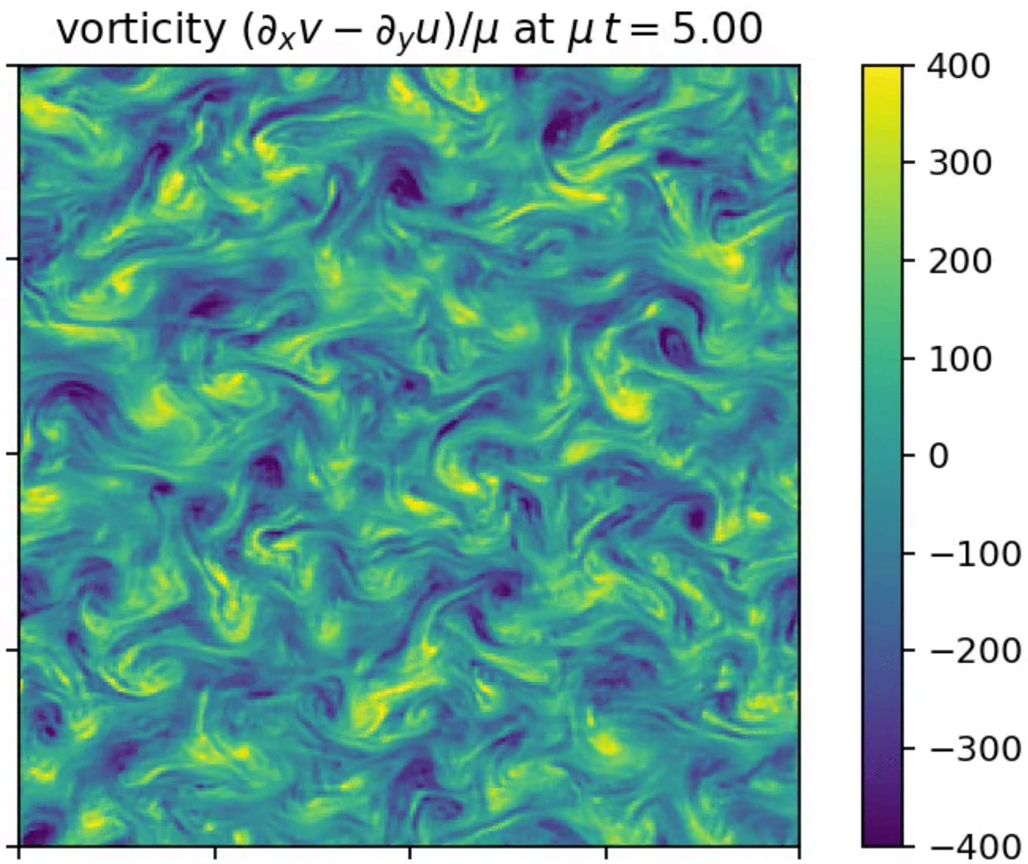
how does this flow-forming instability manifest?

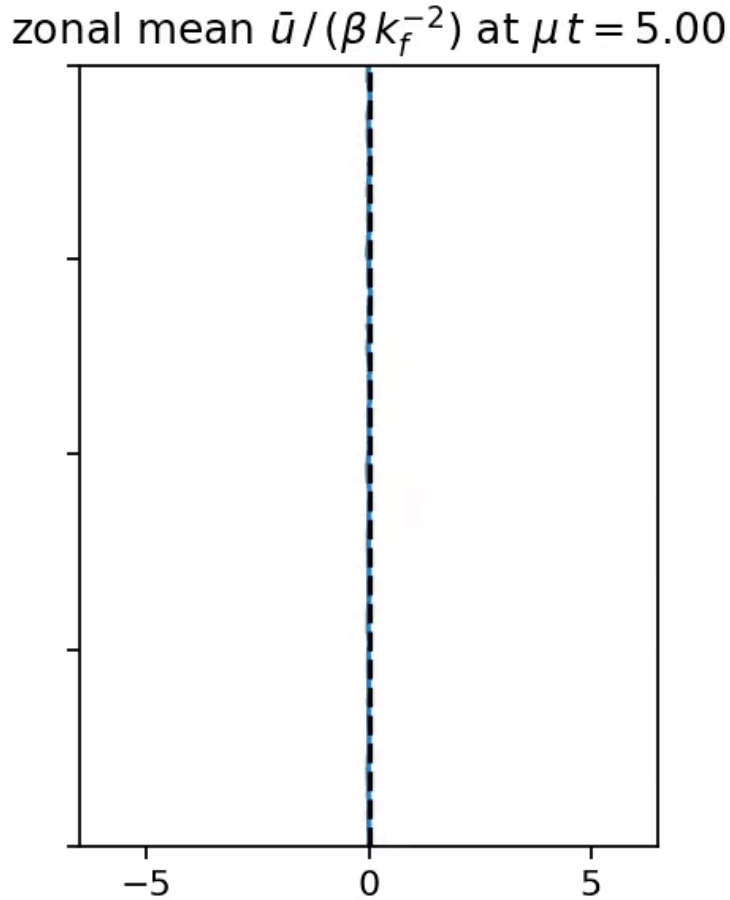
the (infinitesimal) jet organizes the turbulent field
so that it produces Reynolds stresses
that reinforce *the very jet itself* !

this process is obscured in realization dynamics...
but it becomes evident in the statistical state dynamics

SSD reveals what is obscured in single flow realizations

$$\partial_t U = -\mu U - \partial_y \overline{u'v'}$$





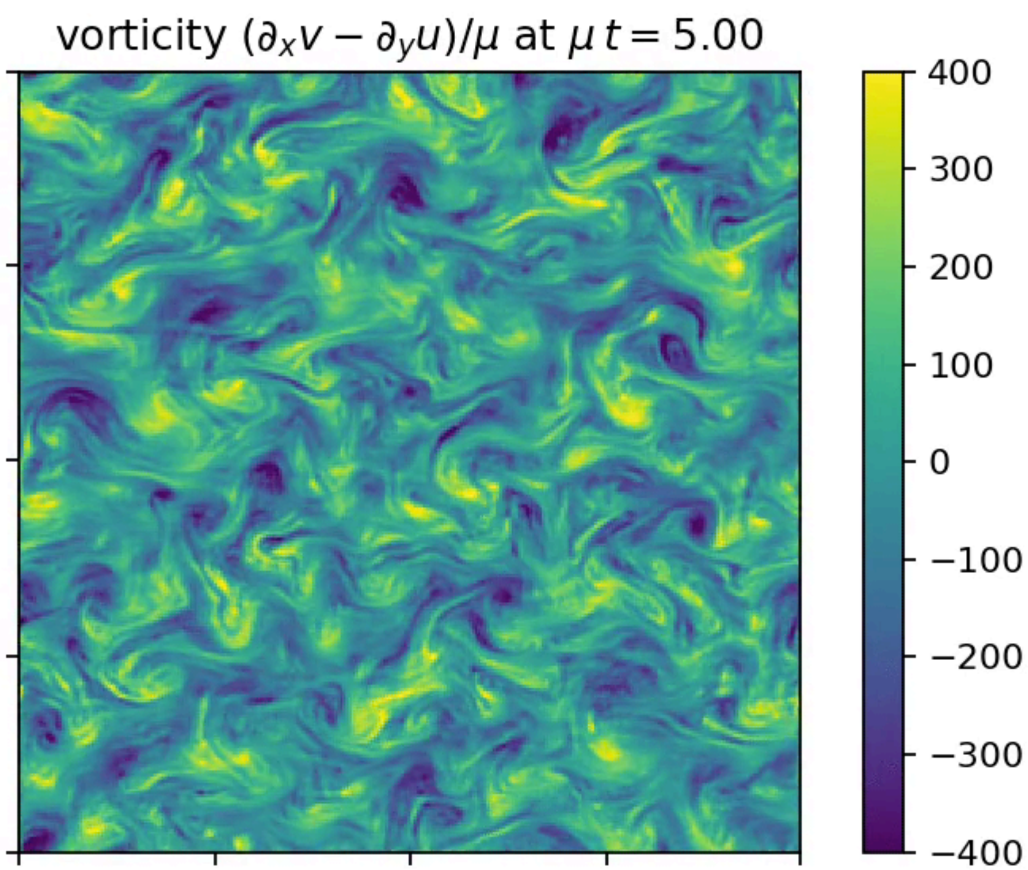
homogeneous turbulence with stationary second-order eddy statistics

no mean flow

SSD reveals what is obscured in single flow realizations

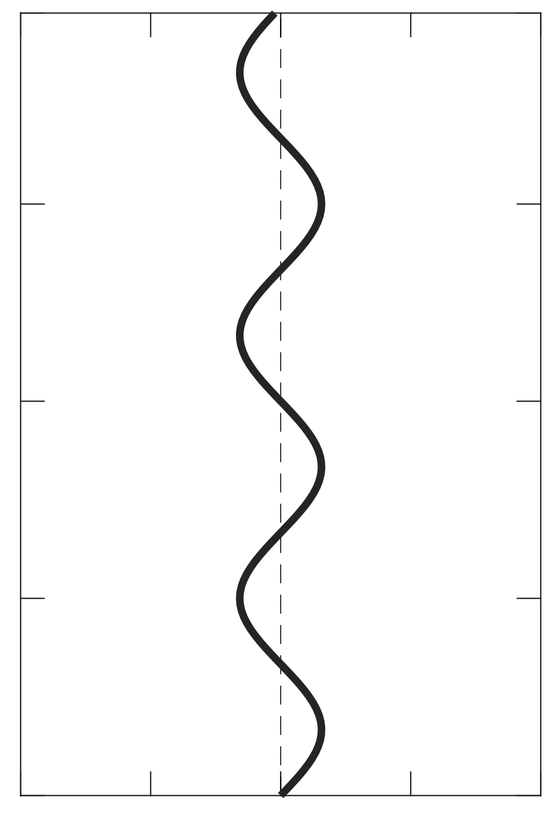
20 independent perturbation realizations

$$\partial_t U = -\mu U - \partial_y \overline{u'v'}$$



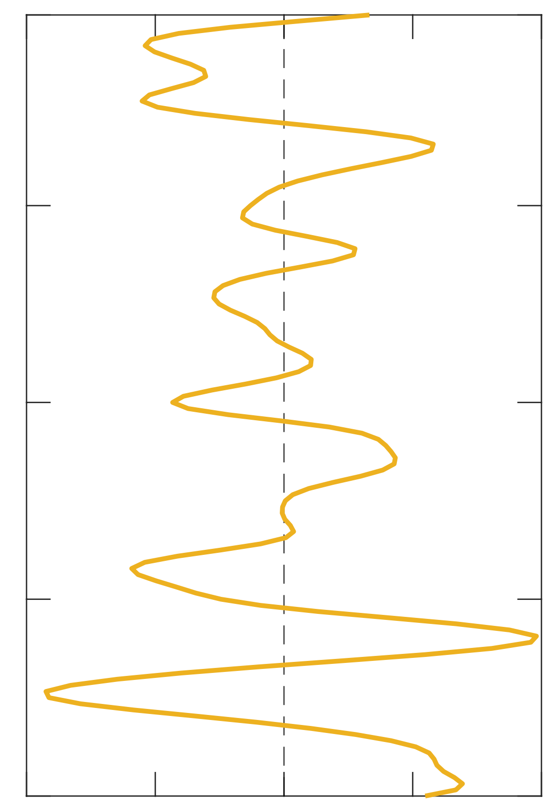
homogeneous turbulence with stationary second-order eddy statistics

+



δU

infinitesimal zonal jet perturbation



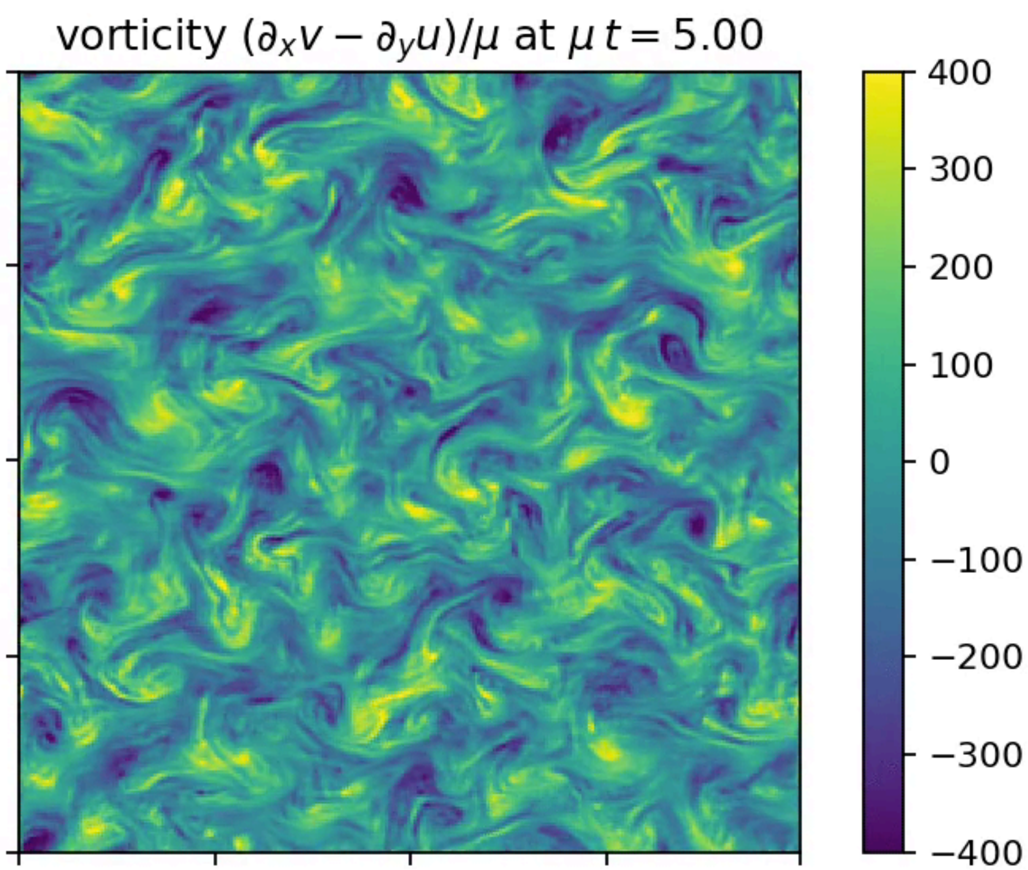
$-\partial_y \overline{u'v'}$

resulting average infinitesimal Reynolds stress divergence

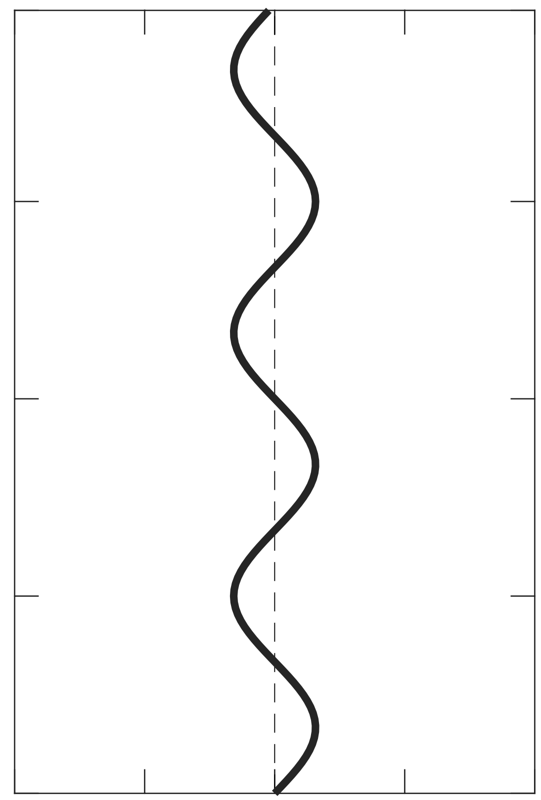
SSD reveals what is obscured in single flow realizations

200 independent perturbation realizations

$$\partial_t U = -\mu U - \partial_y \overline{u'v'}$$

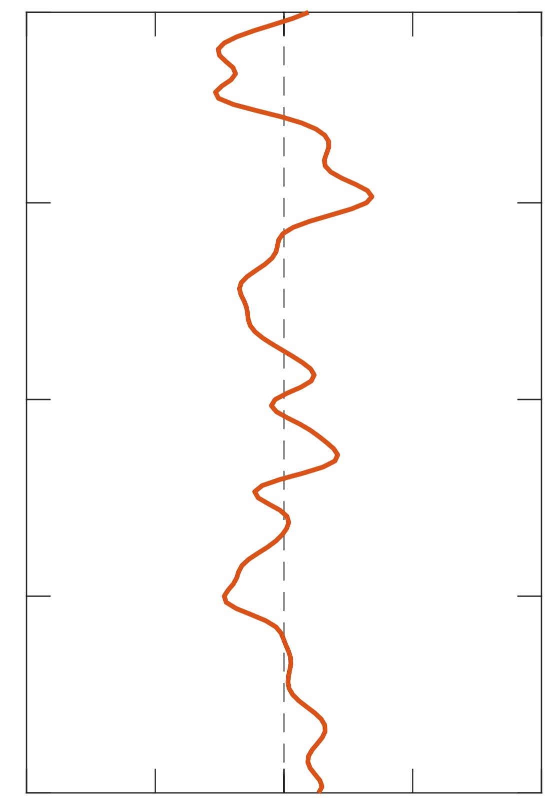


homogeneous turbulence with stationary second-order eddy statistics



$$\delta U$$

infinitesimal zonal jet perturbation



$$-\partial_y \overline{u'v'}$$

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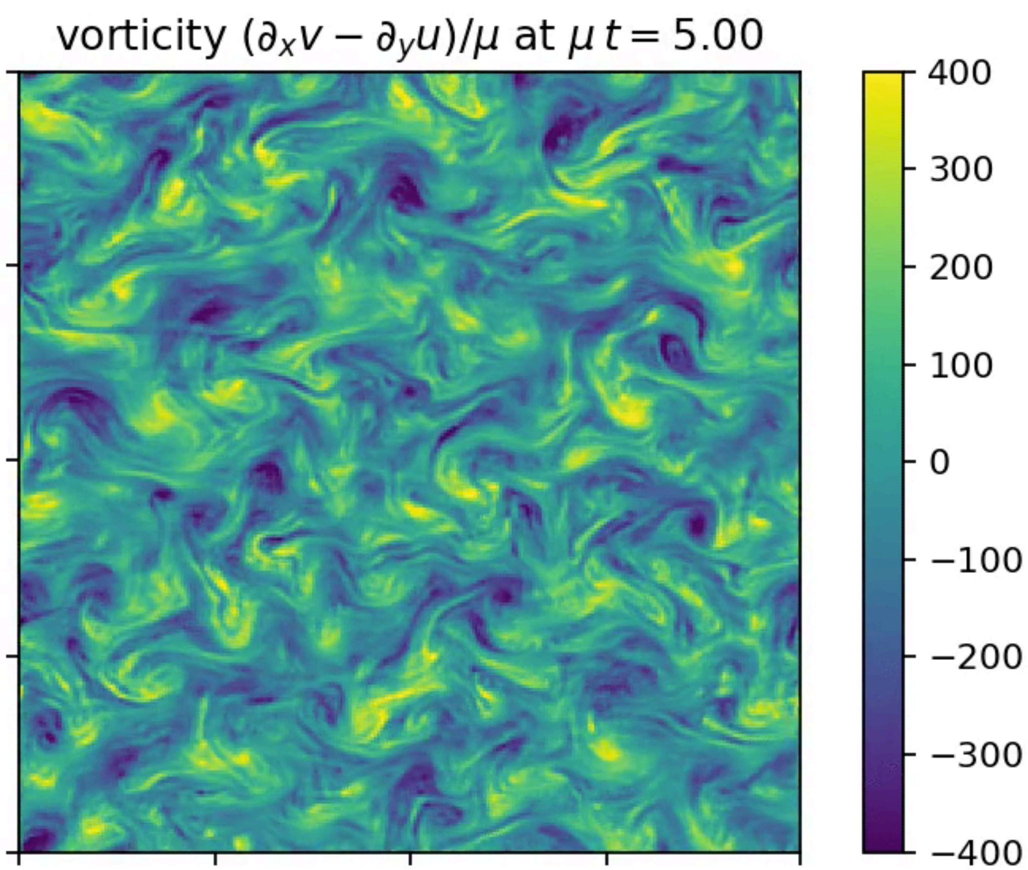

homogeneous turbulence
with stationary
second-order
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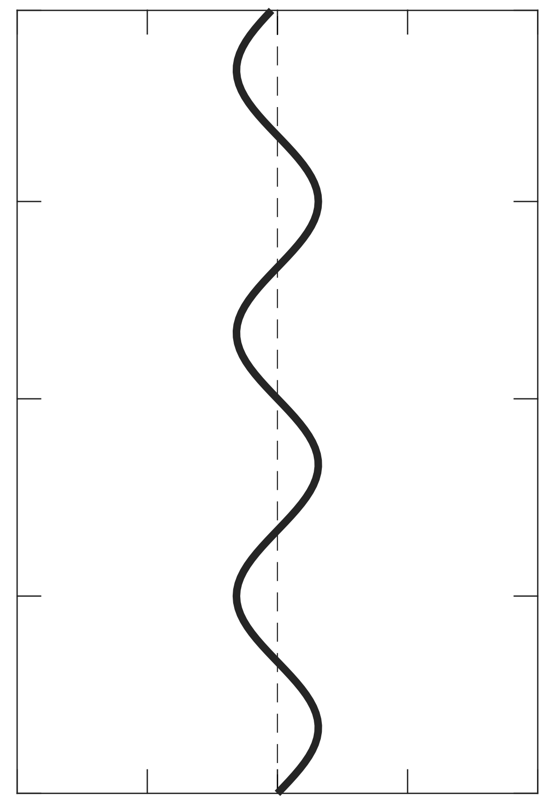
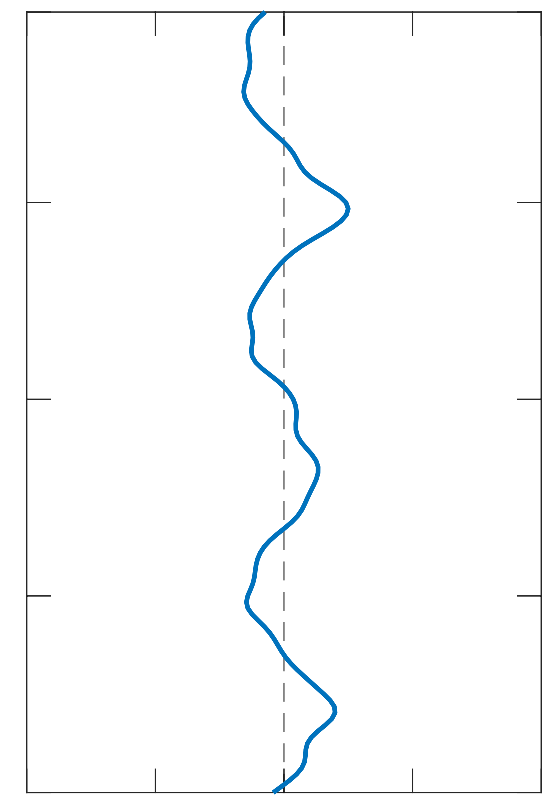


δU

infinitesimal
zonal jet perturbation



$-\partial_y \overline{u'v'}$

resulting average
infinitesimal
Reynolds stress
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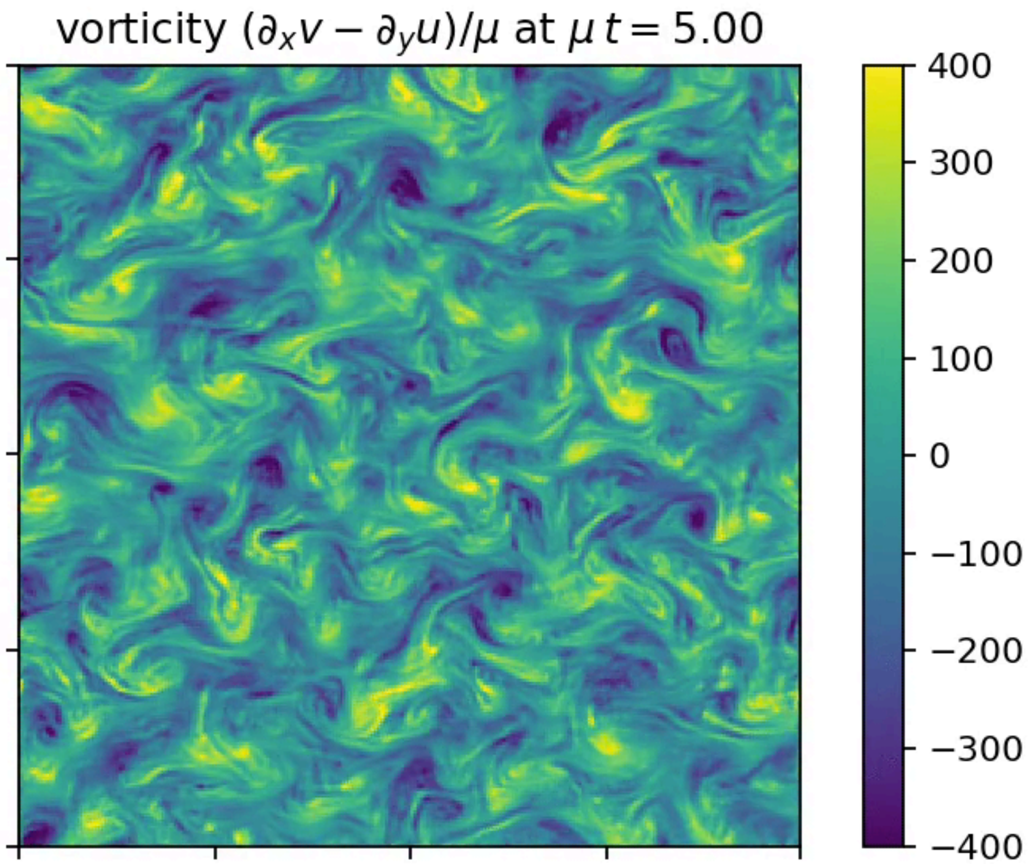
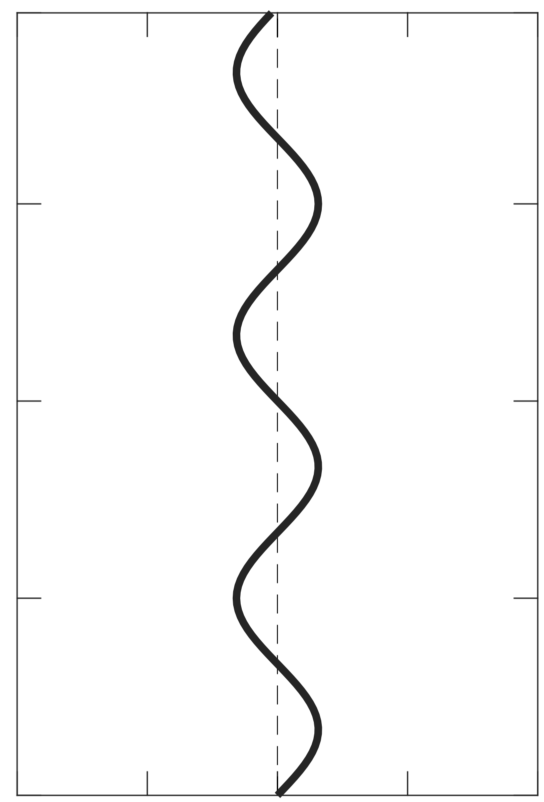
$$\partial_t U = -\mu U - \partial_y \overline{u'v'}$$

2000 independent perturbation realizations

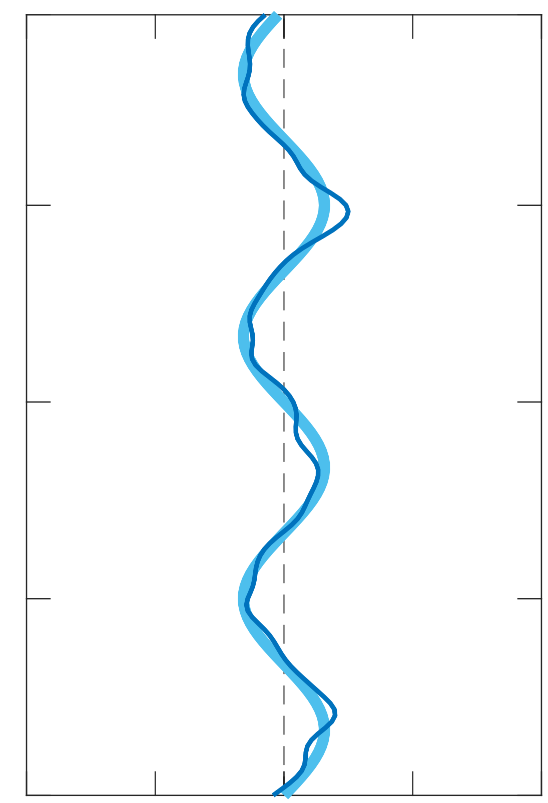
& SSD closure



+



δU



$-\partial_y \overline{u'v'}$

homogeneous turbulence with stationary second-order eddy statistics

infinitesimal zonal jet perturbation

resulting average infinitesimal Reynolds stress divergence

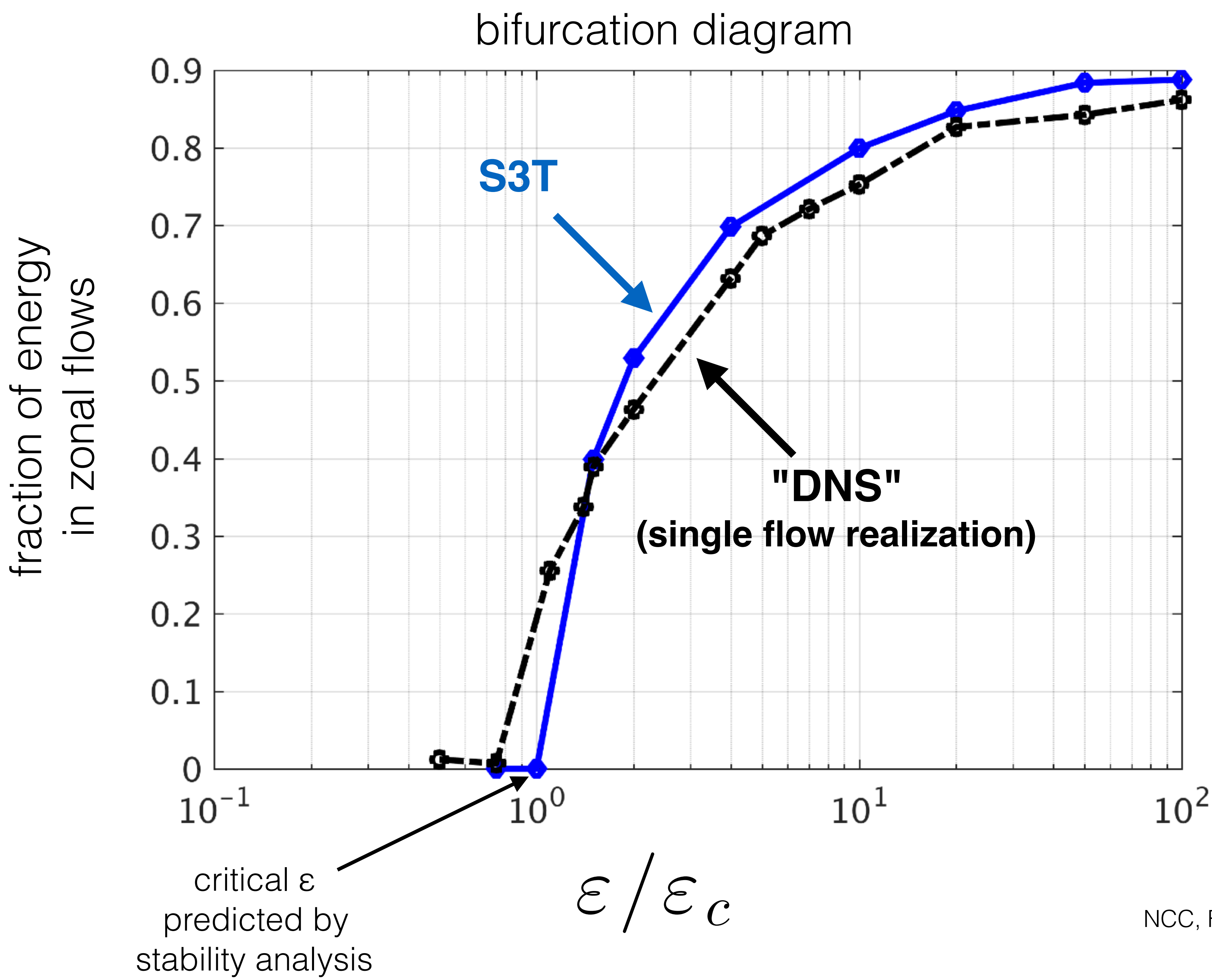
how does this flow-forming instability manifest?

the jet organizes the turbulent field
so that it produces Reynolds stresses
that reinforce *the very jet itself* !

this is a very robust process; not only in jet formation

verification of S3T predictions for the jet formation bifurcation

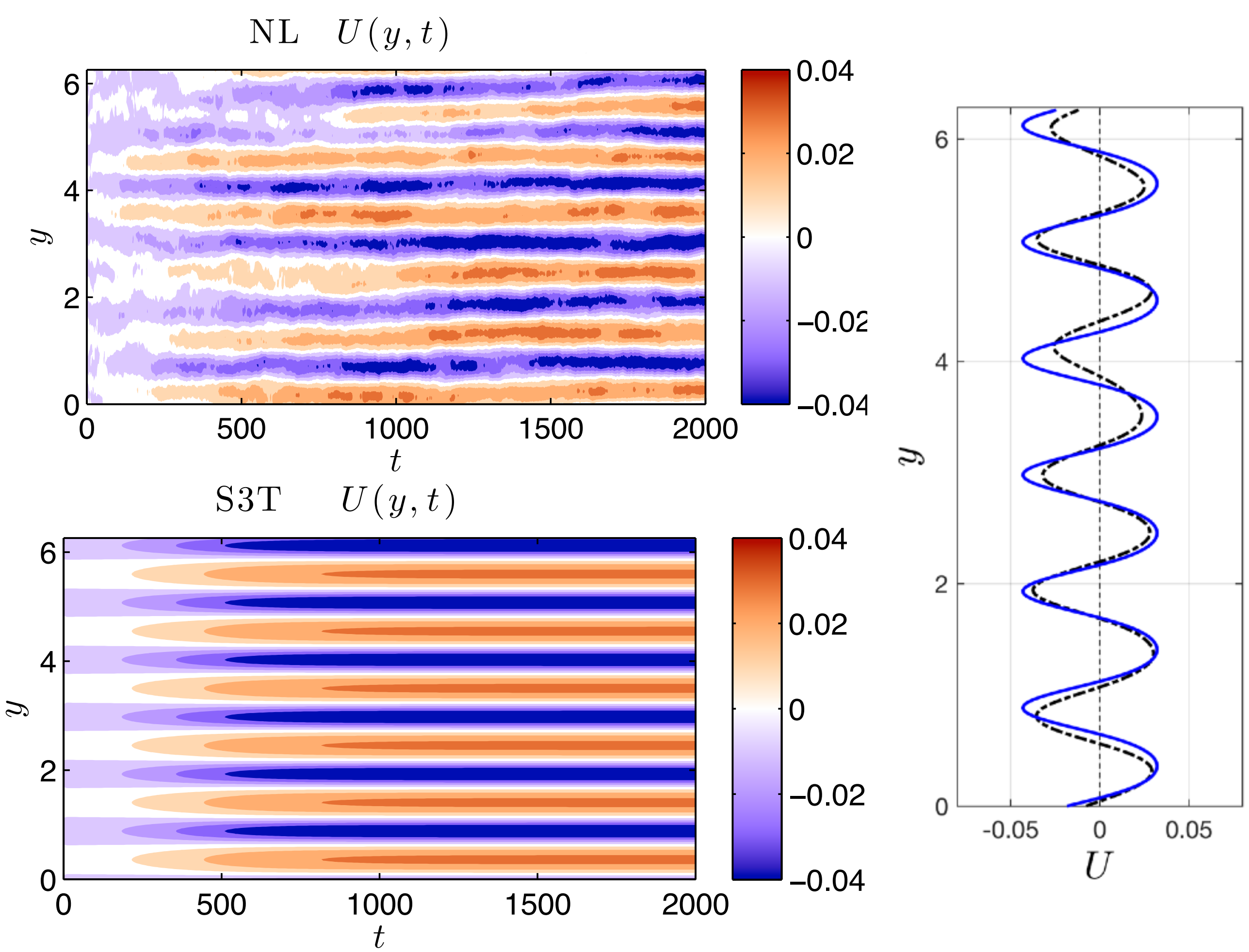
(best case)



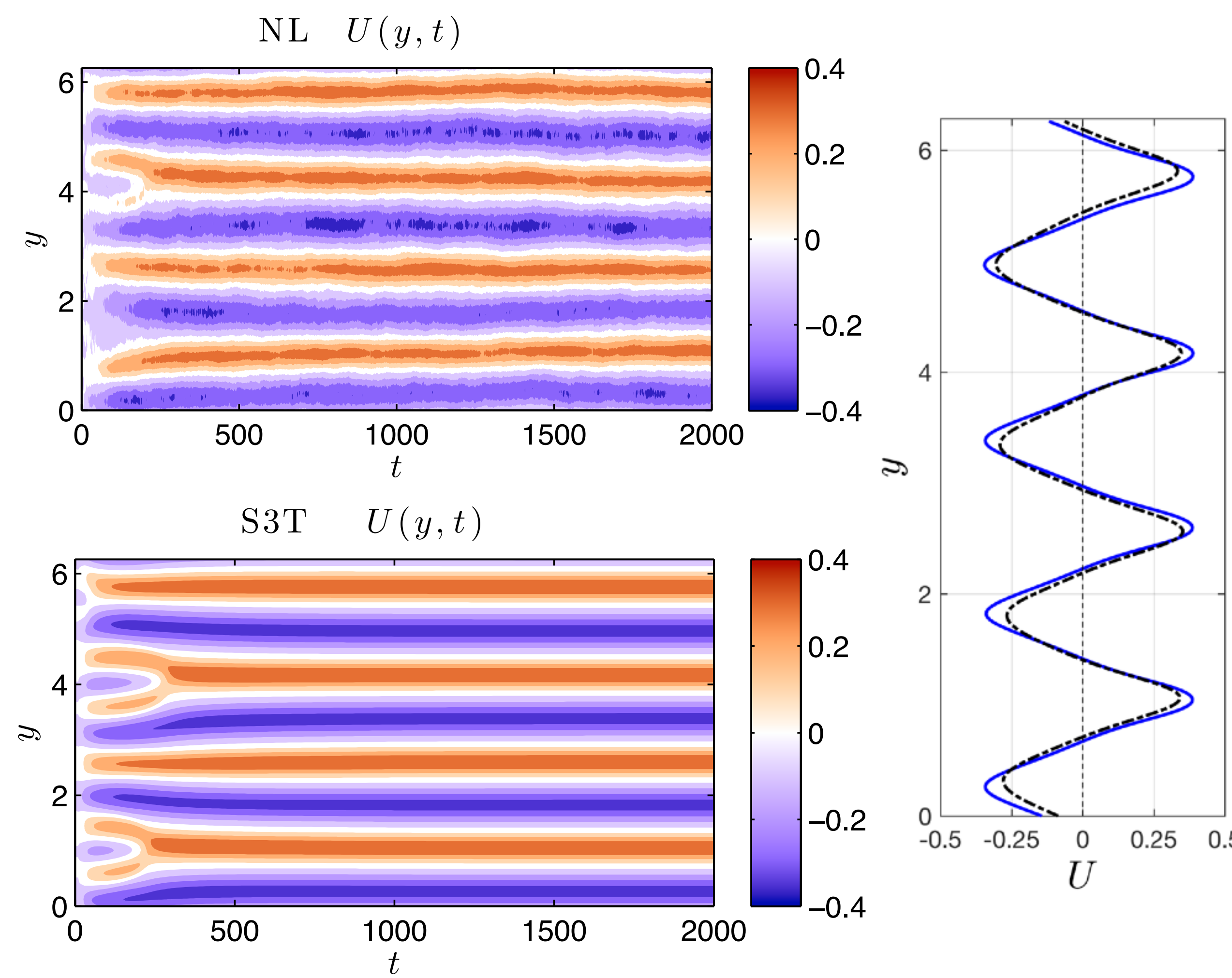
NCC, Farrell & Ioannou 2014

verification of the S3T predictions for the structure of the finite amplitude jet equilibria

$\varepsilon/\varepsilon_c = 1.5$



$\varepsilon/\varepsilon_c = 20$

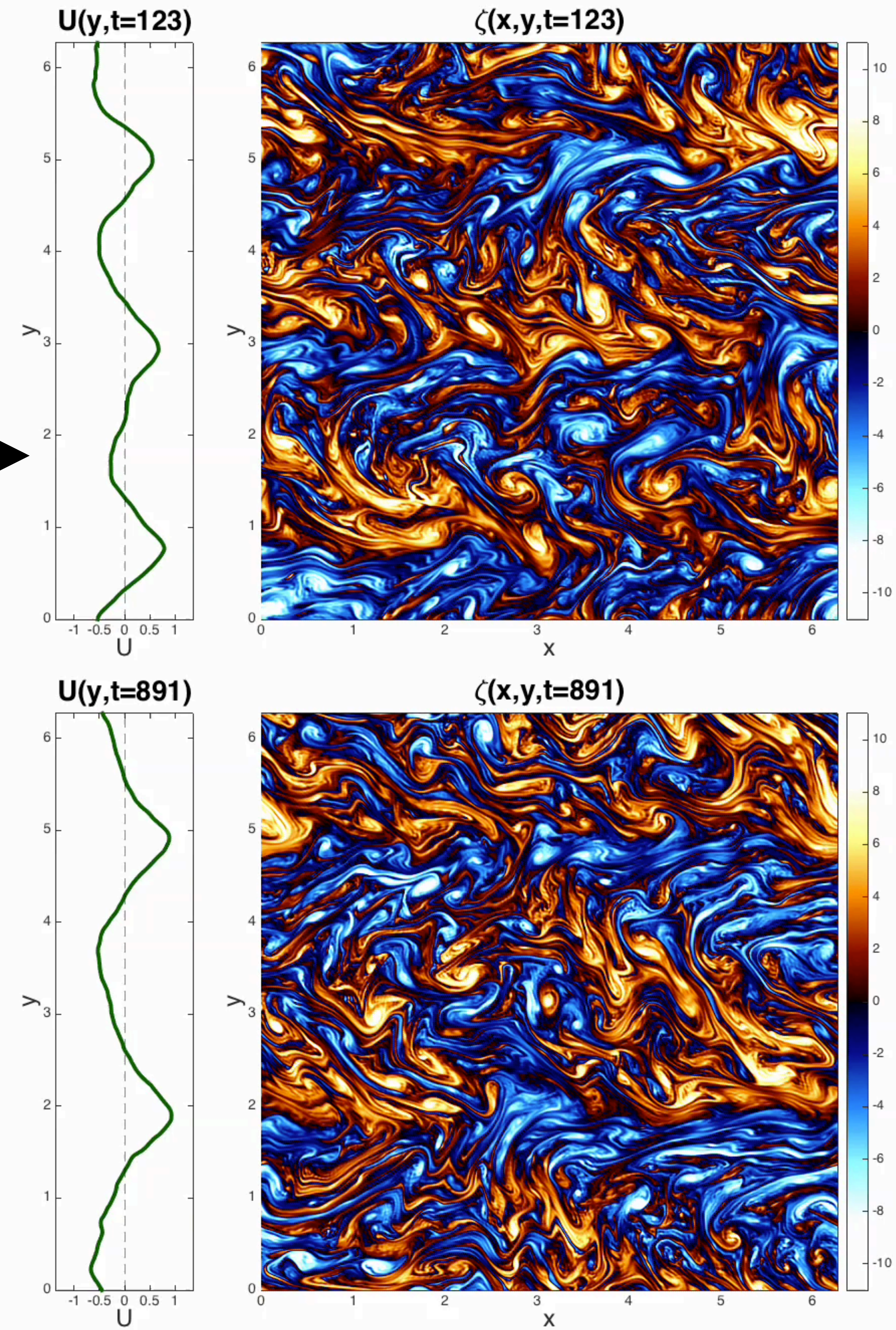


S3T instabilities grow and reach finite amplitude to produce new inhomogeneous S3T equilibria

NCC, Farrell & Ioannou 2014

what more can we do?

how do we predict that
a flow with
three turbulent jets
like this ...



... is ***unstable*** leading
to forming two jets?

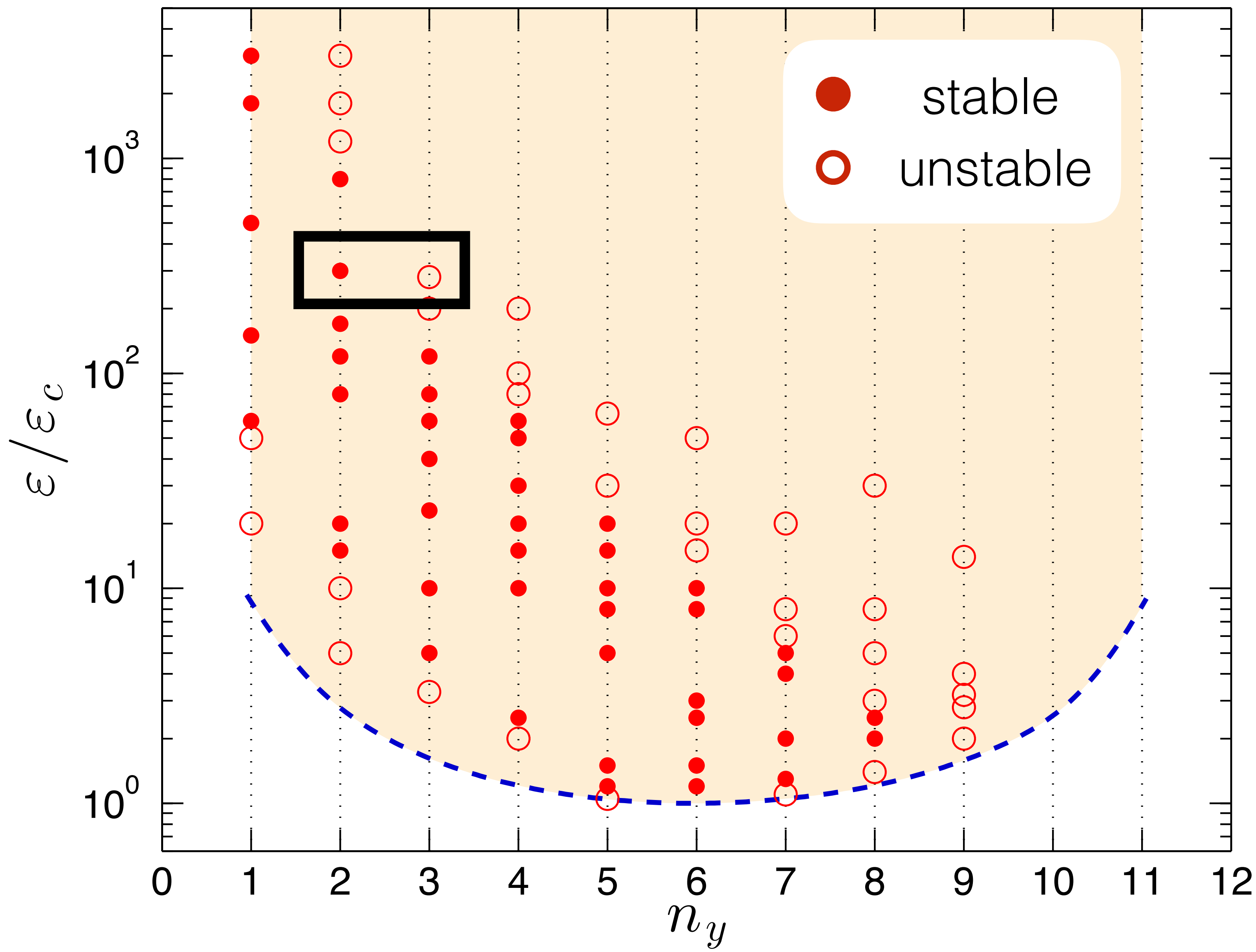
$\varepsilon k_f^2/\mu^3 = 10^6$

$\beta/(k_f\mu) = 67$

stability of zonal jet S3T equilibria to zonal jet perturbations

Stability analysis of inhomogeneous turbulent states with zonal jets predicts:

- existence of multiple equilibria and their domain of attraction
- merging of jets as ε increases



(= number of jets for a $2\pi \times 2\pi$ domain)

stability of zonal jet S3T equilibria to zonal jet perturbations

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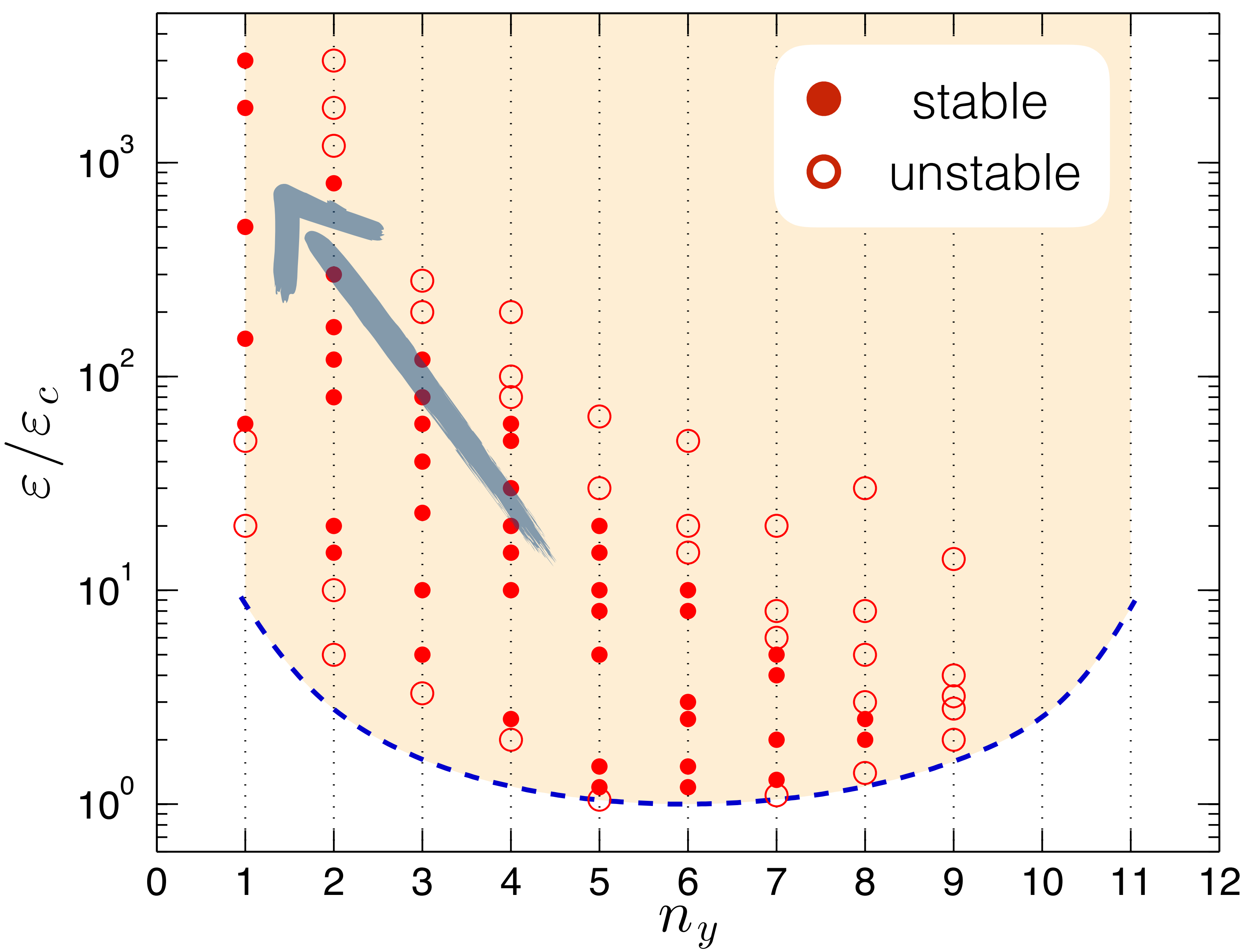


(= number of jets for a $2\pi \times 2\pi$ domain)

For higher energy input rates equilibria become S3T unstable and move towards the left of the diagram

streak-roll formation & SSP
in wall-bounded turbulence

I'll touch briefly on these topics:

I. Single realizations of the 2nd-order SSD closure captures the essence of DNS turbulence
[**RNL models** (see Dennice's talk), i.e., DNS with eddy-eddy→ eddy nonlinearity suppressed]

Farrell et al. 2012; NCC et al. 2014b; Thomas et al. 2014, 2015; Bretheim et al. 2015;
Farrell et al 2016; Farrell, Gayme, & Ioannou 2017; Bretheim, Meneveau, & Gayme 2018

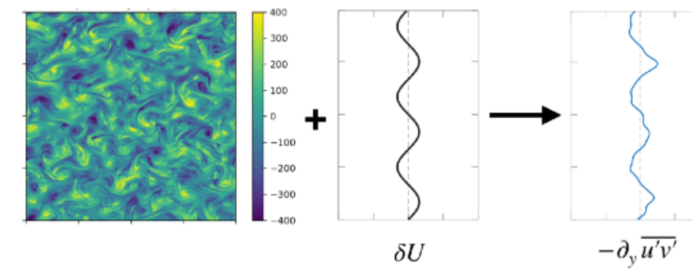
II. Identified the roll-streak formation in pre-transitional free-stream Couette turbulence

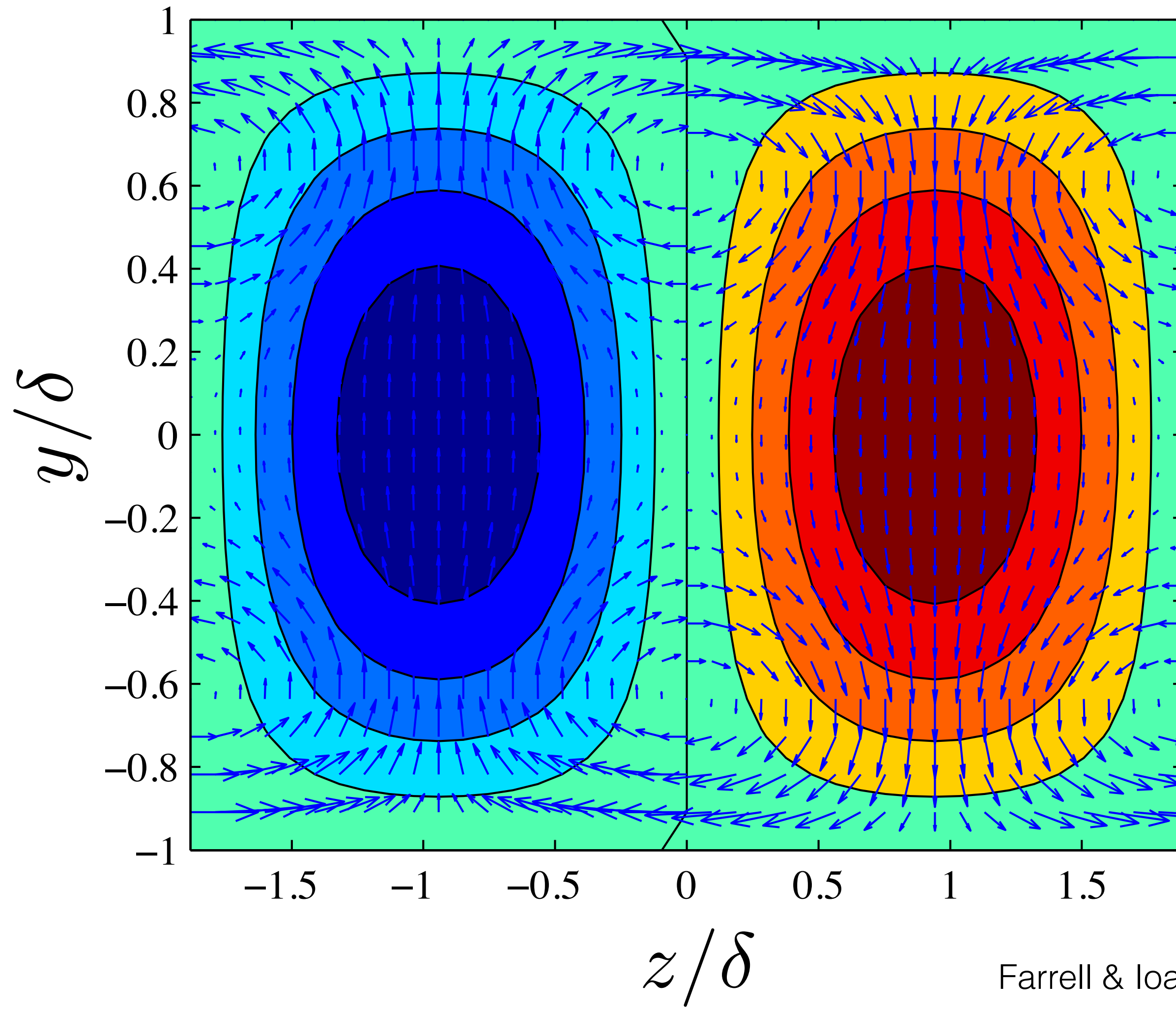
Farrell, Ioannou, & Nikolaidis 2017

III. Identified the structures responsible for closing the loop in the SSP
[active Lyapunov vectors identified by RNL]

Farrell, Ioannou, & Nikolaidis 2018 CTR

II. streaks organize the turbulent stresses in such manner to reinforce themselves...



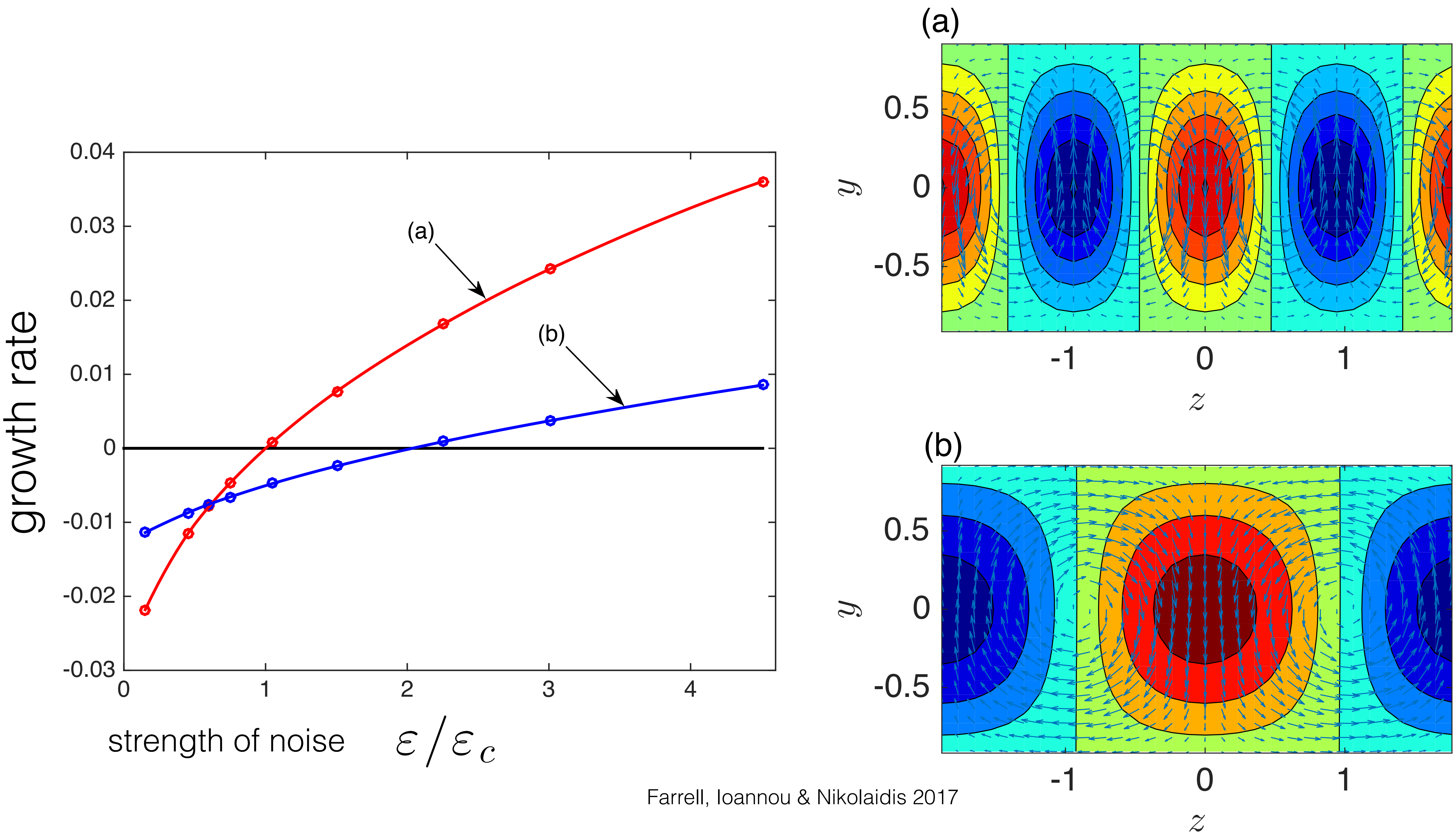


Farrell & Ioannou 2012

Couette
turbulence
minimal channel
Re=400

mean profile $U(y)$ (Couette/Poiseuille) & homogeneous eddy field + infinitesimal streak → Reynolds stresses organize and force a roll → Roll acts to reinforce the streak

II. eigenvalues/eigenmodes of the least stable S3T roll/streak modes

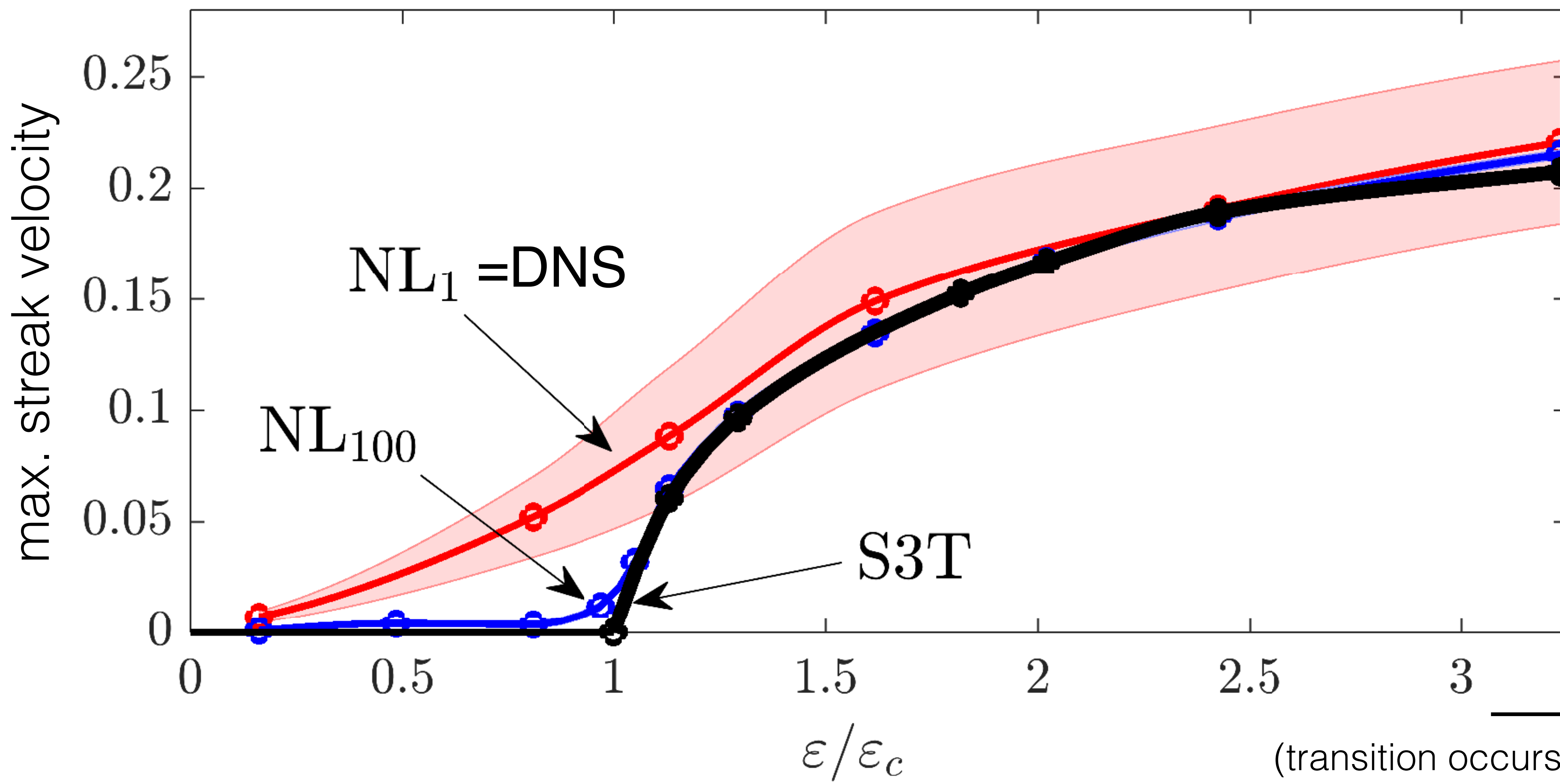


Farrell, Ioannou & Nikolaidis 2017

minimal channel: $L_x = 1.75\pi$, $L_z = 1.2\pi$, $Re = 400$, stochastic excitation at $k_x = 2\pi/L_x$

ε_C sustains turbulence with energy 0.14% of the Couette flow energy.

bifurcation structure

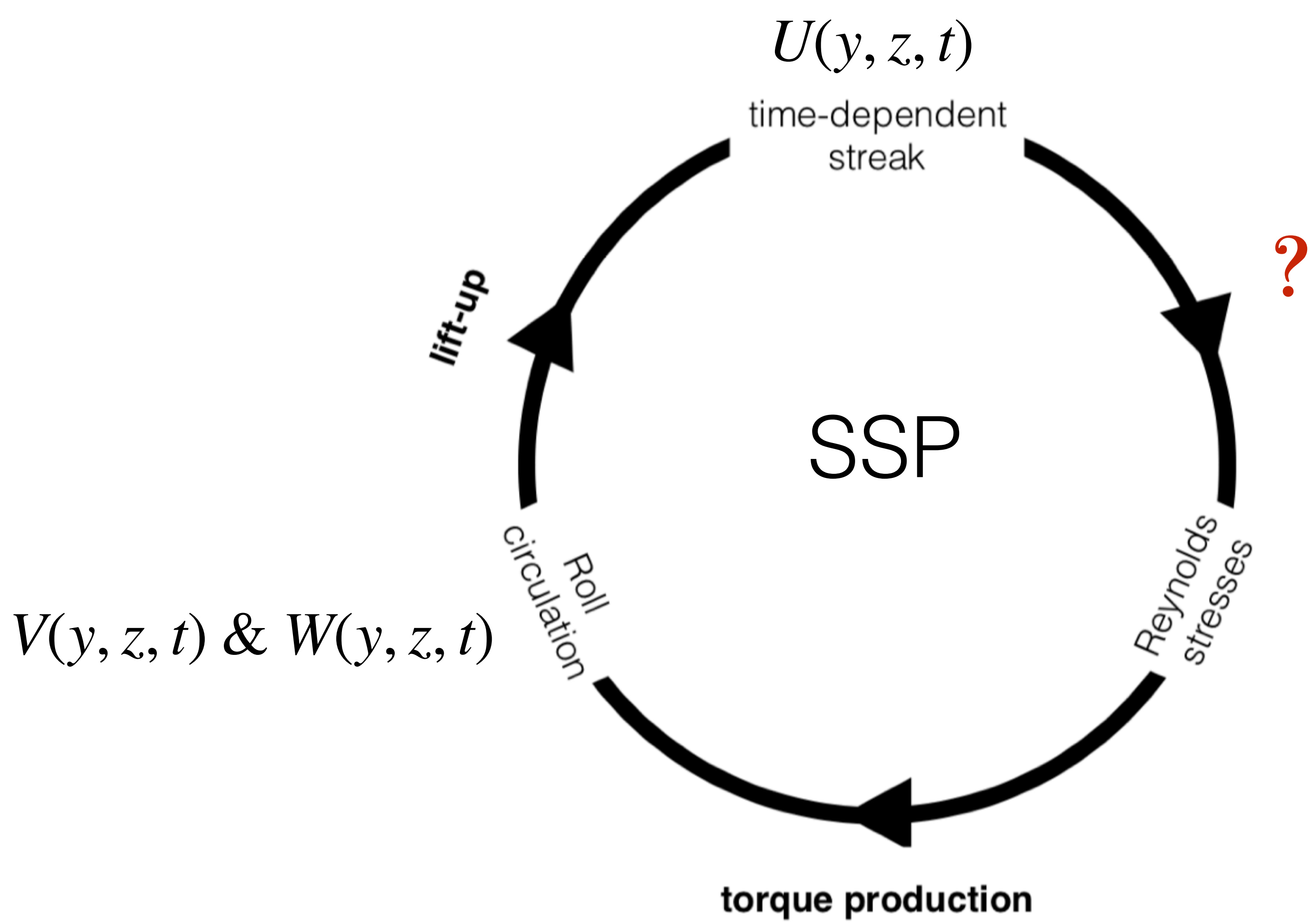


(transition occurs at ~6)

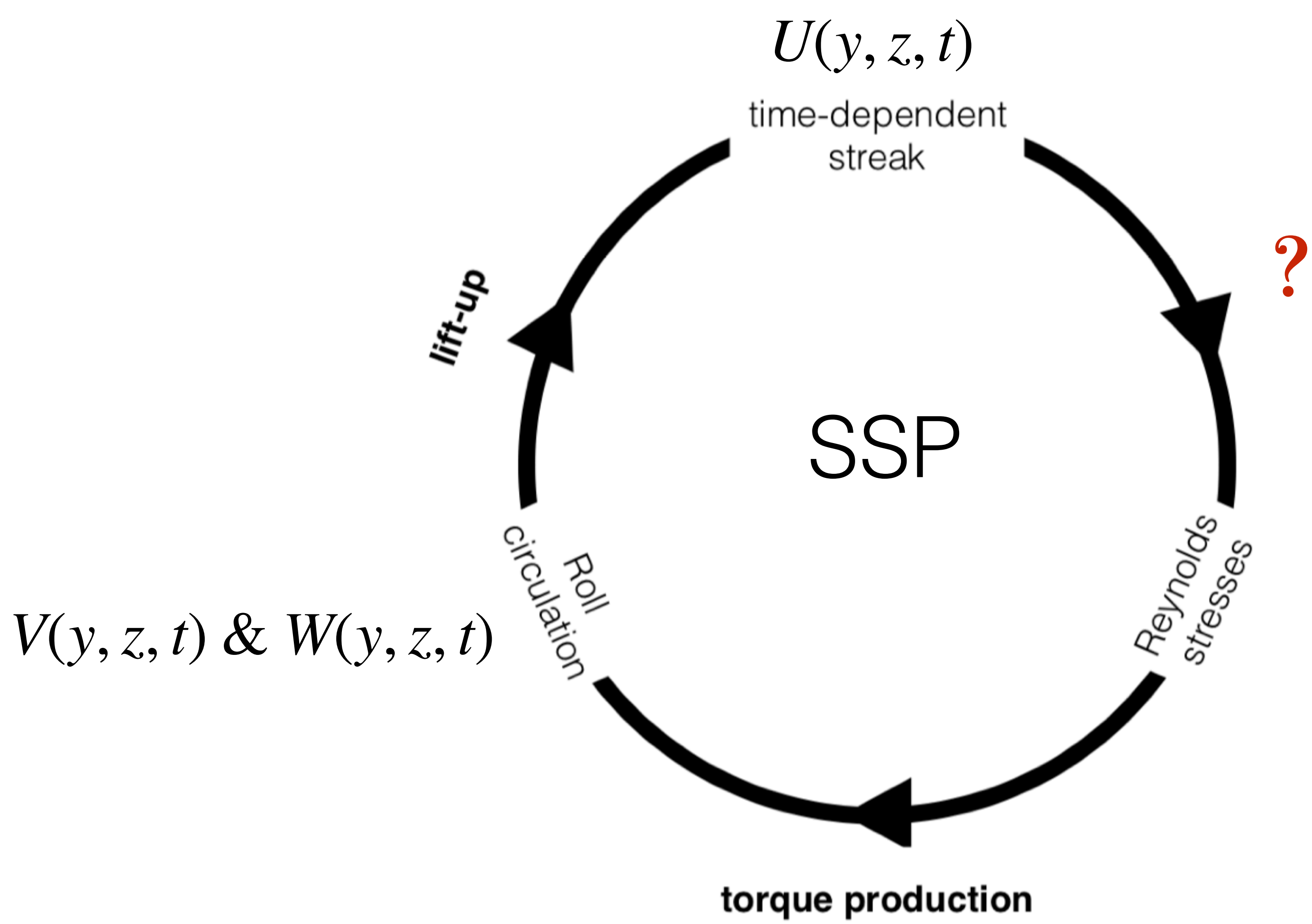
Farrell, Ioannou & Nikolaidis 2017

minimal channel
Re=400

III. closing the loop in the SSP



III. closing the loop in the SSP



In RNL the only way energy can transfer from the mean flow to the perturbations is through the parametric instability of the time-dependent streak

Farrell, & Ioannou (2017) *PRF*, **2 (8)**, 084608

Is this the case in DNS?

III. closing the loop in the SSP

1. Run DNS
2. Take the $U(y, z, t)$ from DNS and compute the Lyapunov vectors
3. Go back to DNS and project-out the active LVs from perturbations (continuously)

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2. Take the $U(y, z, t)$ from DNS and compute the Lyapunov vectors
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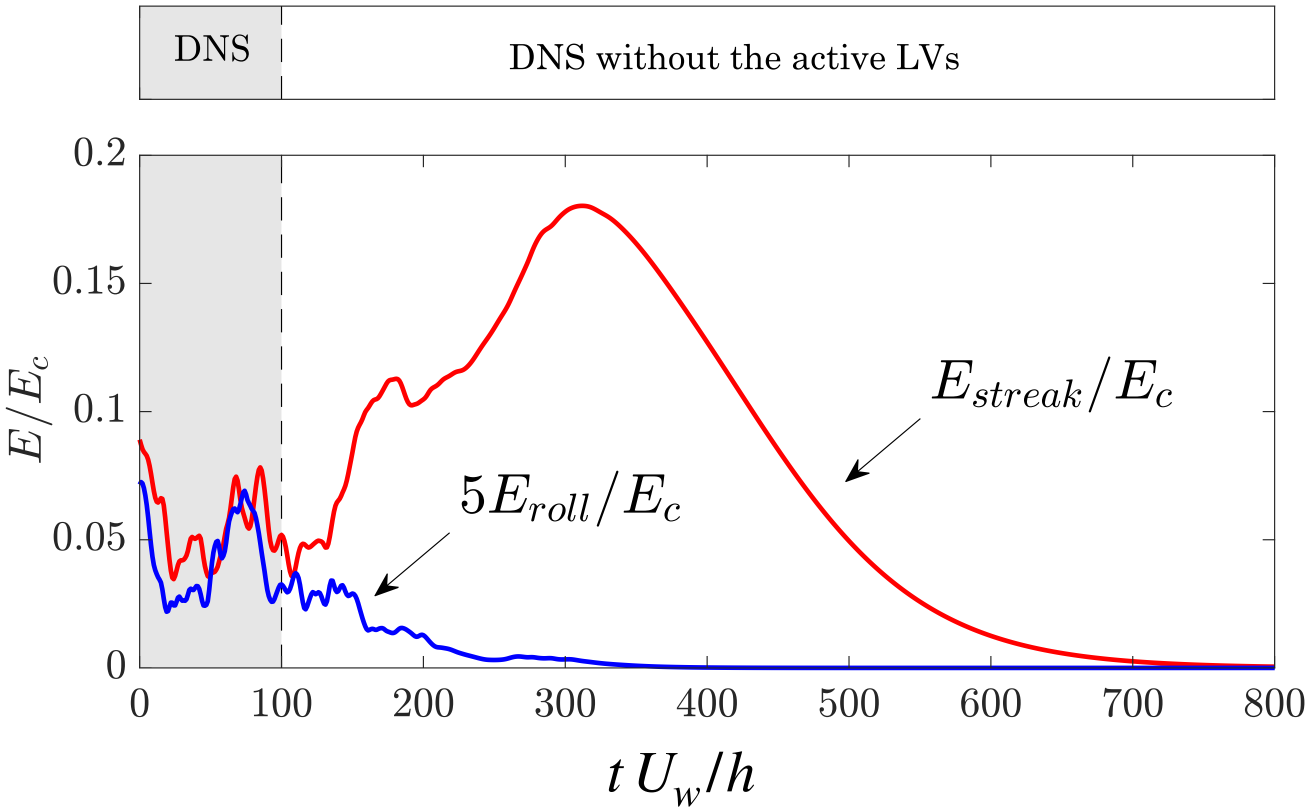

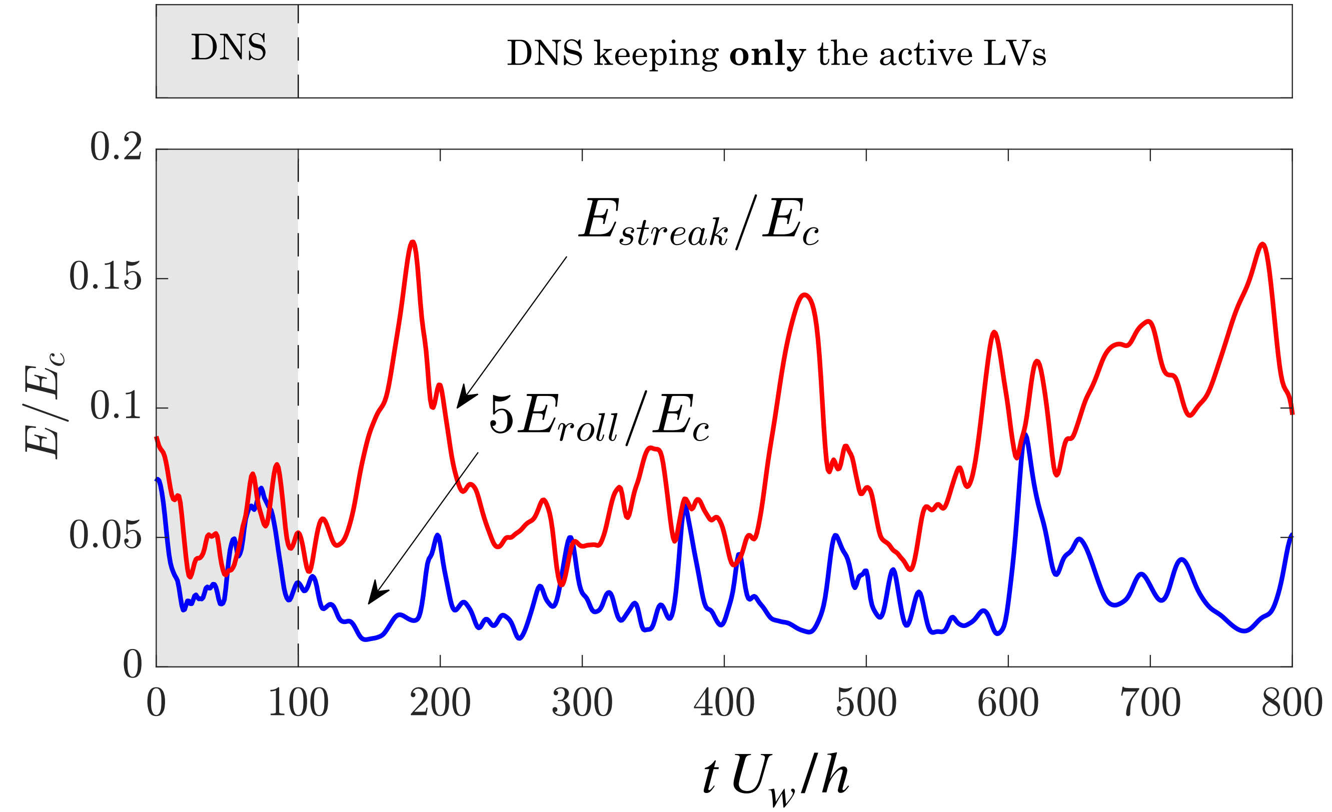

Couette turbulence
at *Re*=600

4 active LVs in this case
containing ~20% of the perturbation energy

Conclusions

- Perturbation S3T generalizes the hydrodynamic stability theory of Rayleigh to study stability of statistical equilibria of turbulent flows.
- Emergence of coherent structures in turbulence is predicted analytically and understood to result from instability of the turbulent state.
- SSD provides analytical methods for studying dynamics and understanding mechanism in turbulent flows.

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