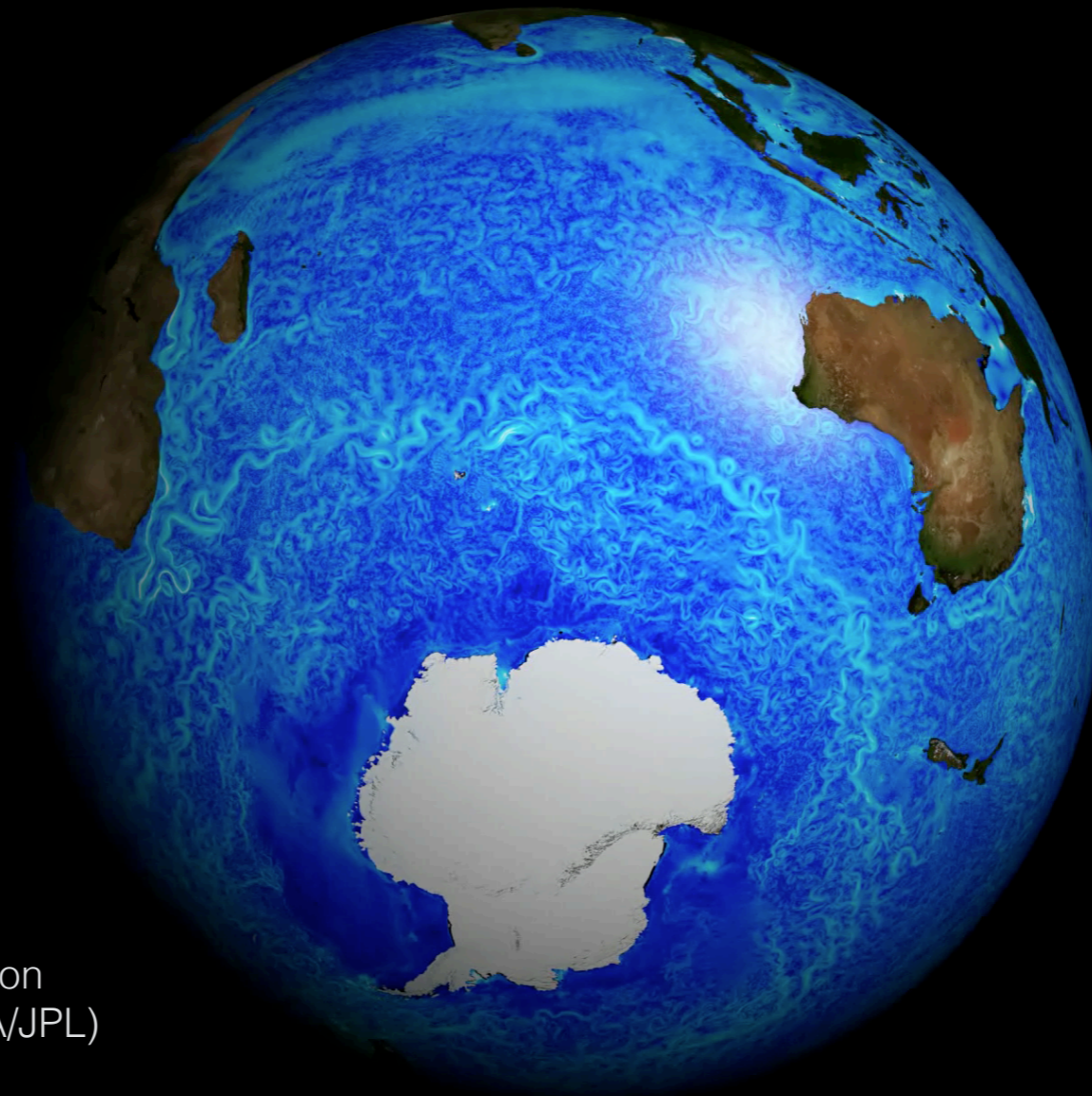


Baroclinic Vs Barotropic eddy saturation



Navid Constantinou
Australian National University
ARC Centre of Excellence for Climate Extremes



the Antarctic
Circumpolar
Current

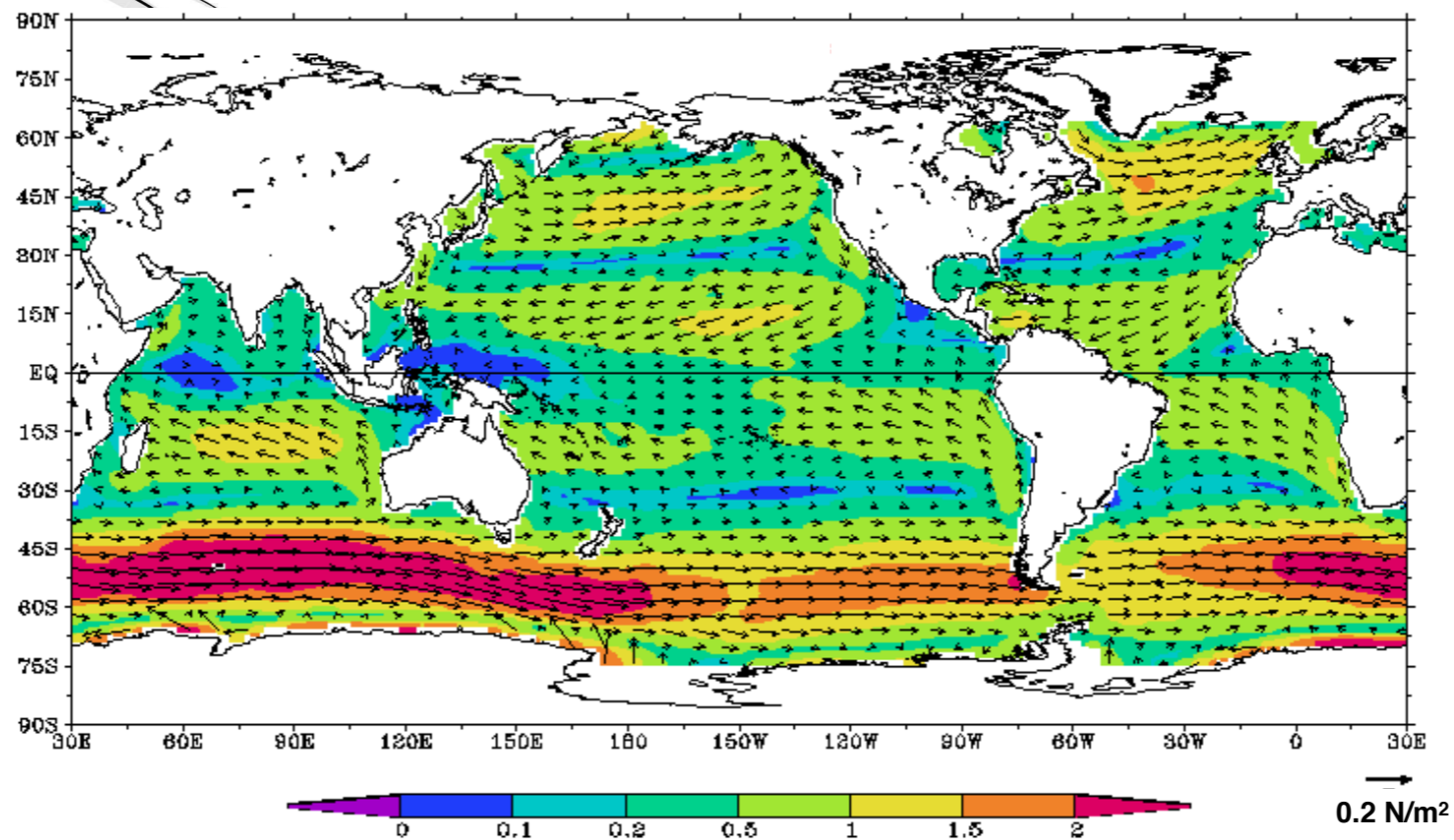
LLC4320 sea surface speed animation
by C. Henze and D. Menemenlis (NASA/JPL)
1/48th degree, 90 vertical levels
MITgcm spun up from ECCO v4 state estimate

what drives the Antarctic Circumpolar Current?



Climate Prediction Center

GODAS Wind Stress, 1982-2004 Annual



strong westerly winds blow over the Southern Ocean transferring momentum through wind stress at the surface

how is this momentum balanced? bottom drag?

Note on the Dynamics of the Antarctic Circumpolar Current

By W. H. MUNK and E. PALMÉN

1951

Abstract

Unlike all other major ocean currents, the Antarctic Circumpolar Current probably does not have sufficient frictional stress applied at its lateral boundaries to balance the wind stress. The balancing stress is probably applied at the bottom, largely where the major submarine ridges lie in the path of the current. The meridional circulation provides a mechanism for extending the current to a large enough depth to make this possible.



W.H. Munk
(101 bday on Oct 19th, 2018)

Note on the Dynamics of the Antarctic Circumpolar Current

By W. H. MUNK and E. PALMÉN

1951

Abstract

Unlike all other major ocean currents, the Antarctic Circumpolar Current probably does not have sufficient frictional stress applied at its lateral boundaries to balance the wind stress. The balancing stress is probably applied at the bottom, largely where the major submarine ridges lie in the path of the current. The meridional circulation provides a mechanism for extending the current to a large enough depth to make this possible.



W.H. Munk
(101 bday on Oct 19th, 2018)

start with the zonal
angular momentum
equation

$f(y)$ is the Coriolis parameter
 $f = 2\Omega \sin \vartheta$

$$(\partial_t + u\partial_x + v\partial_y + w\partial_z) \underbrace{\left(u - \int^y f(y') dy' \right)}_{\stackrel{\text{def}}{=} a} + p_x = \tau_z$$

angular momentum

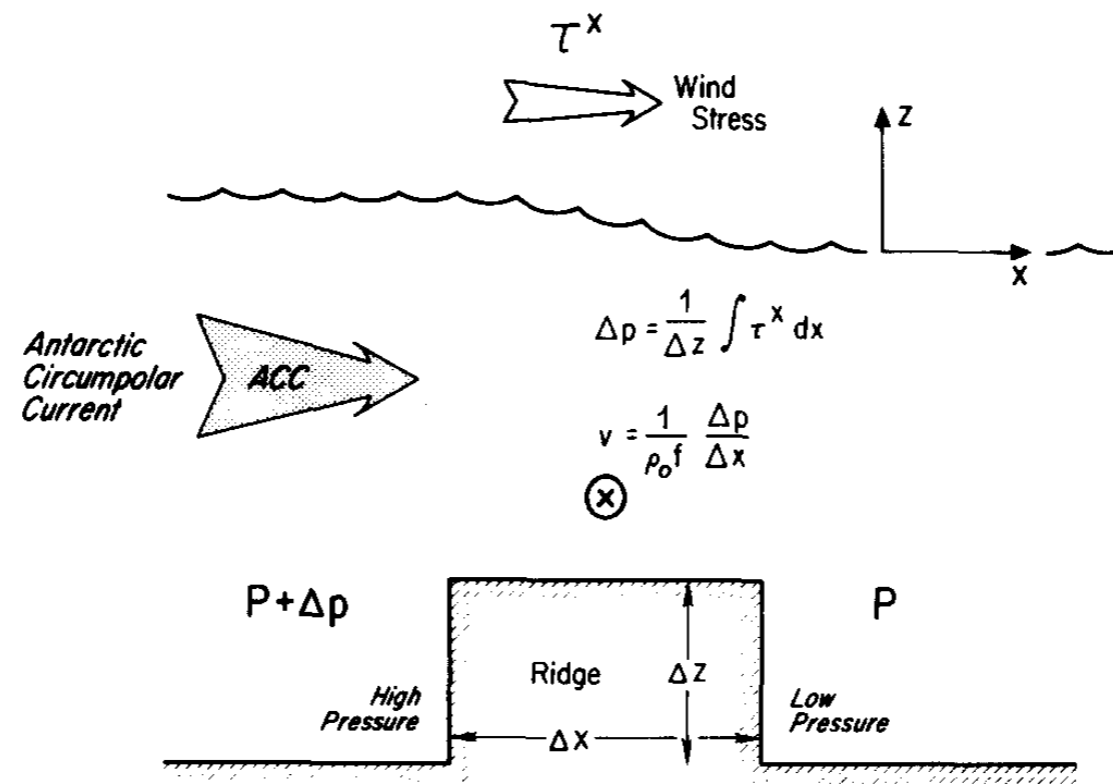
vertically integrate,
top $z=0$ to bottom $z=-h(x,y)$

$$\begin{aligned} \partial_t \int_{-h}^0 a dz + \partial_x \left[\int_{-h}^0 ua + p dz \right] + \partial_y \int_{-h}^0 va dz = \\ = \underbrace{\tau(0)}_{\text{wind stress}} - \underbrace{\tau(-h)}_{\text{bottom drag}} + \underbrace{h_x p(-h)}_{\text{form stress}} \end{aligned}$$

we've used
integration by parts:

$$\int_{-h}^0 p_x dz = \partial_x \int_{-h}^0 p dz - h_x p(-h)$$

topographic form stress



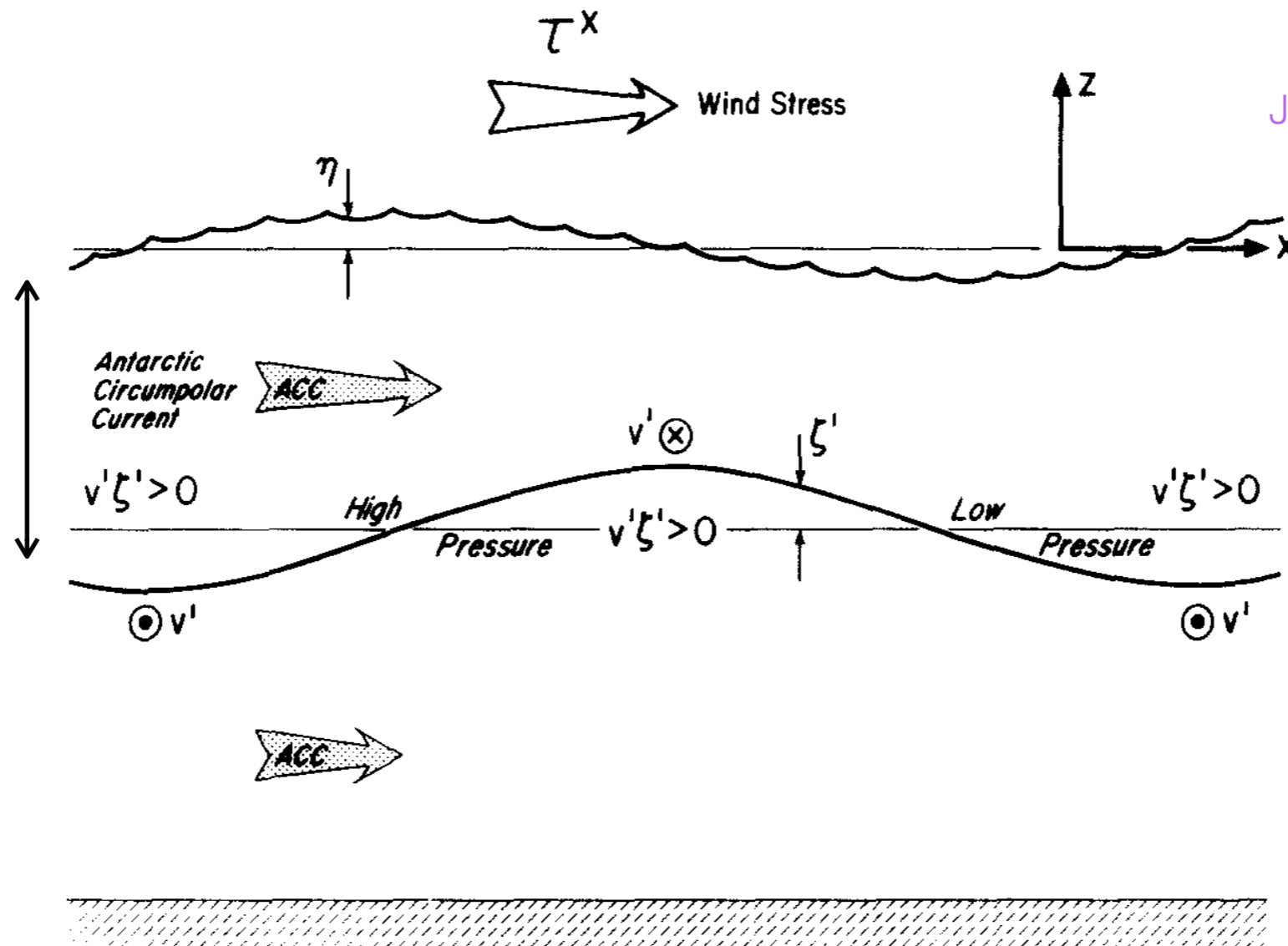
Schematic presentation of bottom form drag or mountain drag. Wind stress imparted eastward momentum in the water column is removed by the pressure difference across the ridge.

$$\begin{aligned}
 \partial_t \int_{-h}^0 a \, dz + \partial_x \left[\int_{-h}^0 ua + p \, dz \right] + \partial_y \int_{-h}^0 va \, dz = \\
 = \underbrace{\tau(0)}_{\text{wind stress}} - \underbrace{\tau(-h)}_{\text{bottom drag}} + \underbrace{h_x p(-h)}_{\text{form stress}}
 \end{aligned}$$

Topographic form stress is a purely **barotropic** process.

interfacial form stress

Johnson & Bryden 1989



vertically integrate from the sea-surface down to a moving buoyancy surface

(i.e., integrate within a layer of constant density)

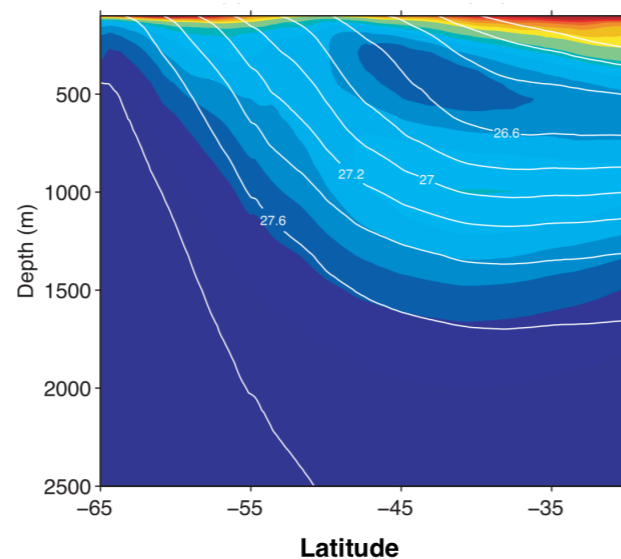
Schematic presentation of interfacial form drag. Correlations of perturbations in the interface height, ζ' , and the meridional velocity, V' (\odot indicating poleward flow and \otimes indicating equatorward flow), which are related to pressure perturbations by geostrophy, allow the upper layer to exert an eastward force on the lower layer and the lower layer to exert a westward force on the upper layer; thus effecting a downward flux of zonal momentum.

Interfacial form stress requires **baroclinicity**.

the most popular scenario for the momentum balance

- momentum is imparted at the surface by wind,
- isopycnals slope \longrightarrow **baroclinic** instability,
- momentum is transferred downwards by **interfacial eddy form stress**
- momentum reaches the bottom where it is transferred to the solid Earth by **topographic form stress**.

Johnson & Bryden 1989



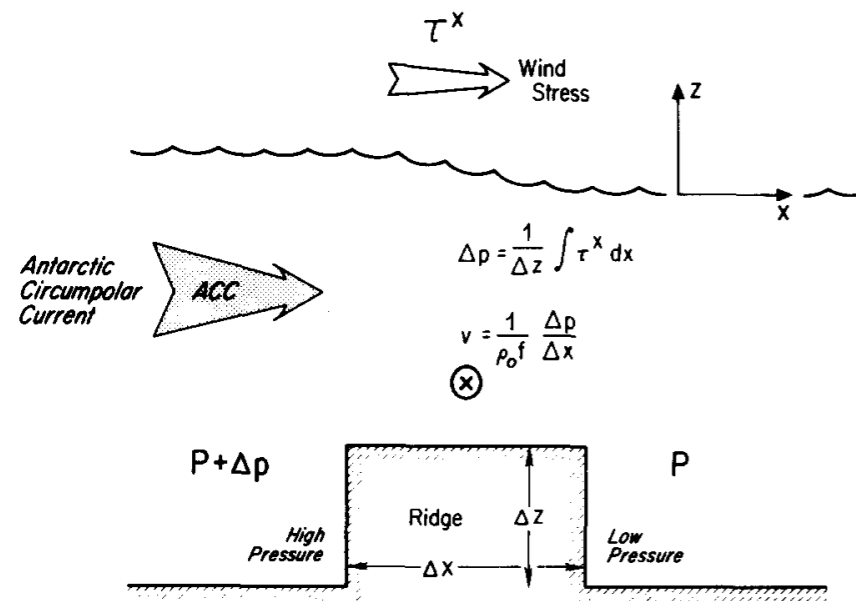
Meredith et al. 2012

$$\text{isopycnal slope} = \left[-\frac{\tau_s}{f \kappa} \right]^{1/2}$$

Marshall & Radko 2003

This **baroclinic** scenario sets up the ACC transport (e.g. the transport through Drake Passage).

but what about **barotropic** dynamics?



The sea surface pressure gradient can be *directly* communicated to the bottom.

And it will be, unless compensated by internal isopycnal gradients.

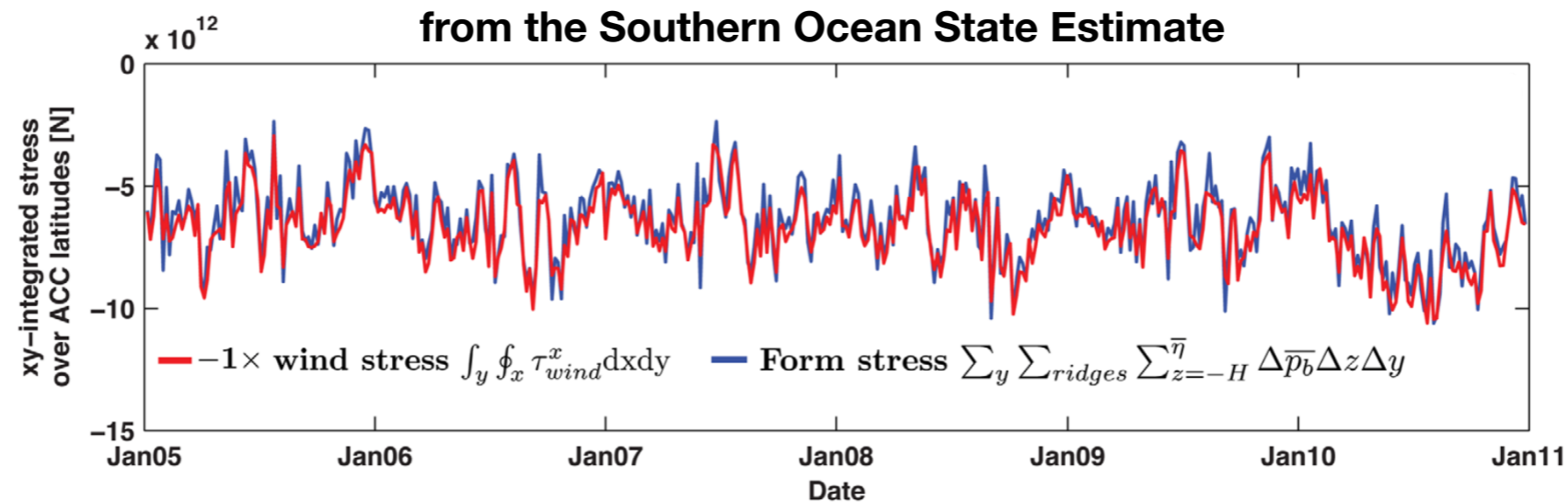
Isn't **barotropic** "communication" much "easier"?

wind stress is *rapidly* communicated to the bottom through **barotropic** processes



Barotropic processes are fast (~days).

Baroclinic processes are much slower (~years).



Masich, Chereskin, and Mazloff 2015

~90% of variance in the topographic form stress signal is explained by the 0-day time lag.

Similar statements also made by:

Straub 1993, Ward & Hogg 2011, Rintoul et al. 2014, Peña Molino et al. 2014, Donohue et al. 2016.

the plan

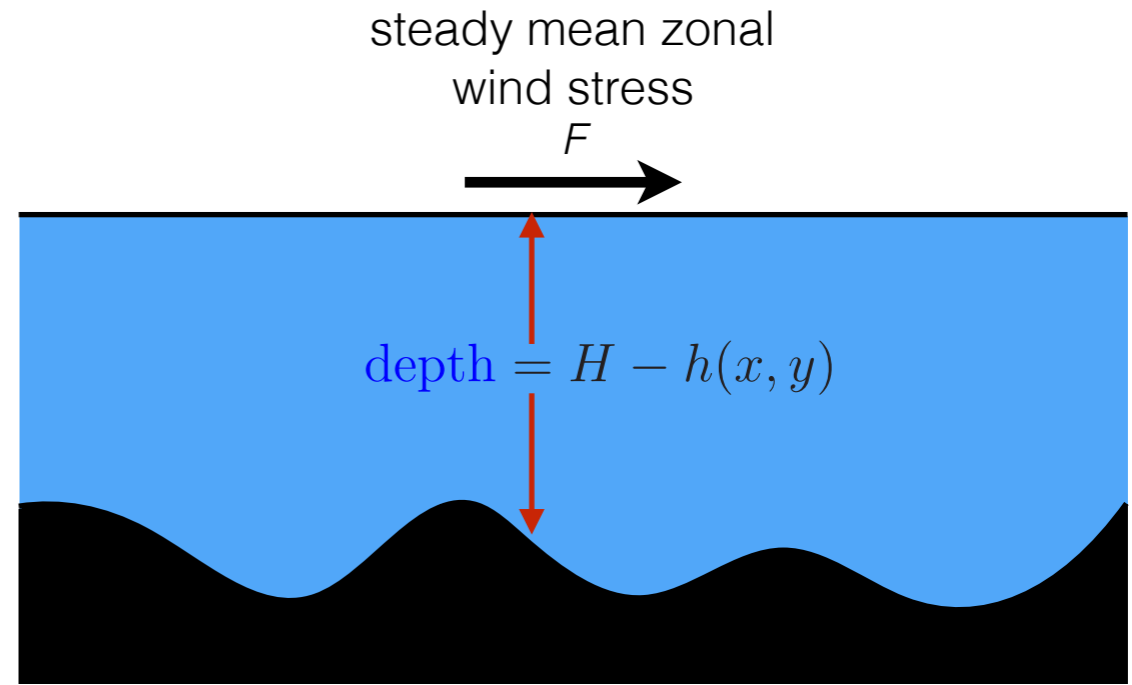
Revisit an old **barotropic** quasigeostrophic (QG) model on a beta-plane.

(Hart 1979, Davey 1980, Bretherton & Haidvogel 1976, Holloway 1987, Carnevale & Fredericksen 1987)

A distinctive feature of this model is a “large-scale **barotropic** flow” $U(t)$.

↑
this is
the ACC

Study how momentum is balanced by topographic form stress and investigate the requirements for eddy saturation.



topographic potential vorticity (PV)

$$\eta = \frac{f_0 h}{H}$$

QGPV

$$\nabla^2 \psi + \eta + \beta y$$

total streamfunction $-U(t)y + \psi(x, y, t)$

a barotropic QG model for a mid-ocean region

total streamfunction $-U(t)y + \psi(x, y, t)$

QGPV $\nabla^2\psi + \eta + \beta y$

Material conservation of QGPV

$$\nabla^2\psi_t + U(\nabla^2\psi + \eta)_x + \mathbf{J}(\psi, \nabla^2\psi + \eta) + \beta\psi_x = -\mu \nabla^2\psi + \text{hyper visc.}$$

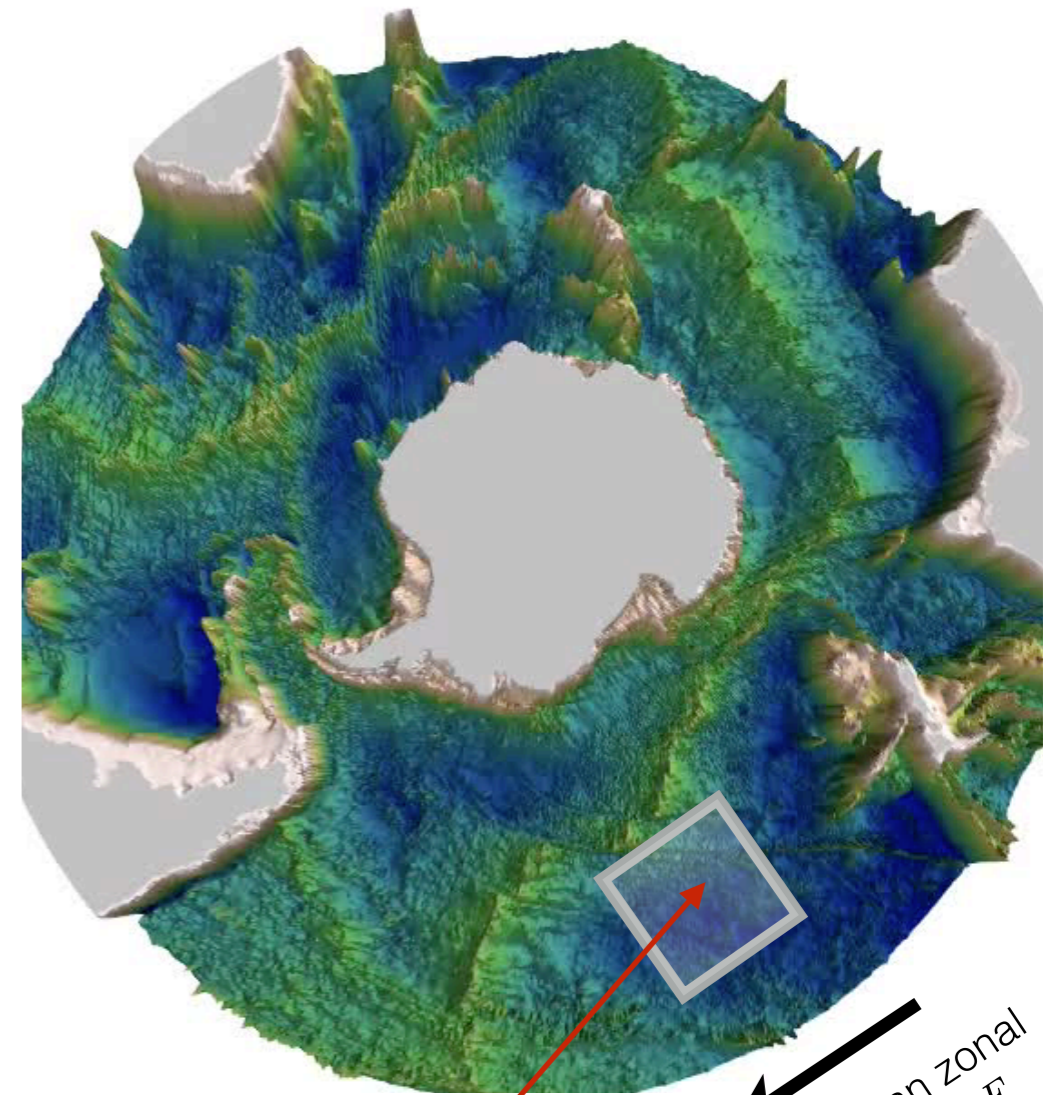
Large-scale zonal momentum

$$U_t = F - \mu U - \langle \psi \eta_x \rangle$$

topographic form stress

$\langle \rangle$ is domain average ; $F = \frac{\tau_s}{\rho_0 H}$ wind stress forcing

periodic boundary conditions



a mid-ocean region size $2\pi L \times 2\pi L$

steady mean zonal wind stress F

the large-scale flow equation: $U_t = F - \mu U - \langle \psi \eta_x \rangle$

zonal angular momentum density: $a(x, y, z, t) = u(x, y, z, t) - \int^y f(y') dy'$

vertically integrated
zonal angular
momentum equation

$$\begin{aligned} \partial_t \int_{-h}^0 a dz + \partial_x \left[\int_{-h}^0 ua + p dz \right] + \partial_y \int_{-h}^0 va dz = \\ = \underbrace{\tau(0)}_{\text{wind stress}} - \underbrace{\tau(-h)}_{\text{bottom drag}} + \underbrace{h_x p(-h)}_{\text{form stress}} \end{aligned}$$

the large-scale flow equation: $U_t = F - \mu U - \langle \psi \eta_x \rangle$

zonal angular momentum density: $a(x, y, z, t) = u(x, y, z, t) - \int^y f(y') dy'$

vertically integrated
zonal angular
momentum equation

$$\begin{aligned} \partial_t \int_{-h}^0 a dz + \cancel{\partial_x \left[\int_{-h}^0 ua + p dz \right] + \partial_y \int_{-h}^0 va dz} = \\ = \underbrace{\tau(0)}_{\text{wind stress}} - \underbrace{\tau(-h)}_{\text{bottom drag}} + \underbrace{h_x p(-h)}_{\text{form stress}} \end{aligned}$$

horizontally integrate,
drop the boundary fluxes,
and divide by the volume

$$U_t = F - \mu U - \langle \psi \eta_x \rangle$$

$$U(t) \stackrel{\text{def}}{=} V^{-1} \iiint u(x, y, z, t) dV$$

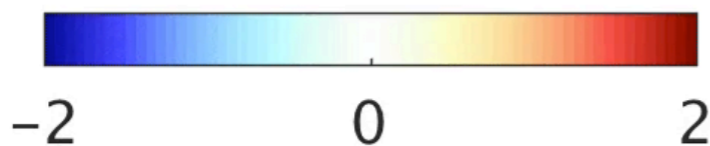
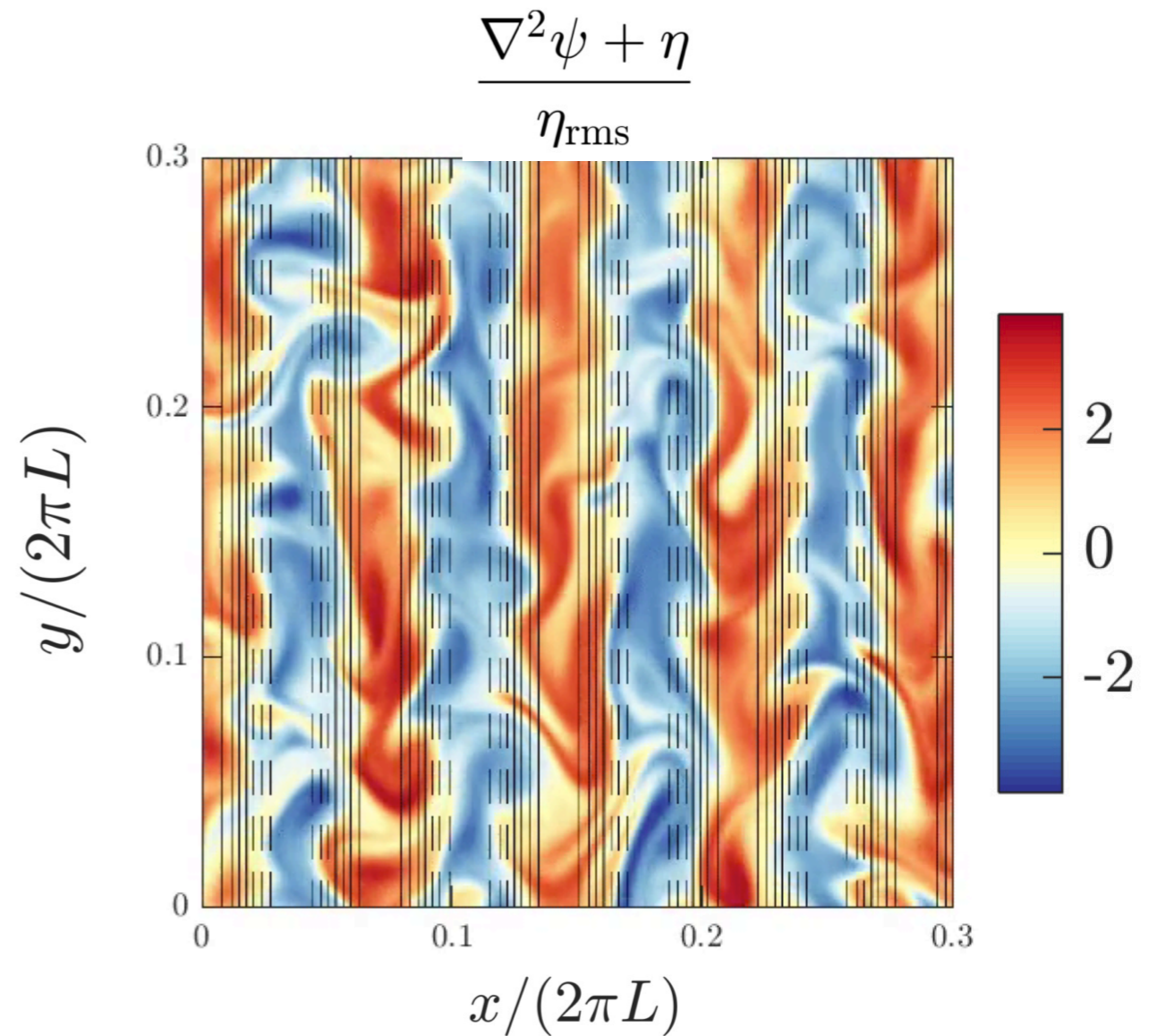
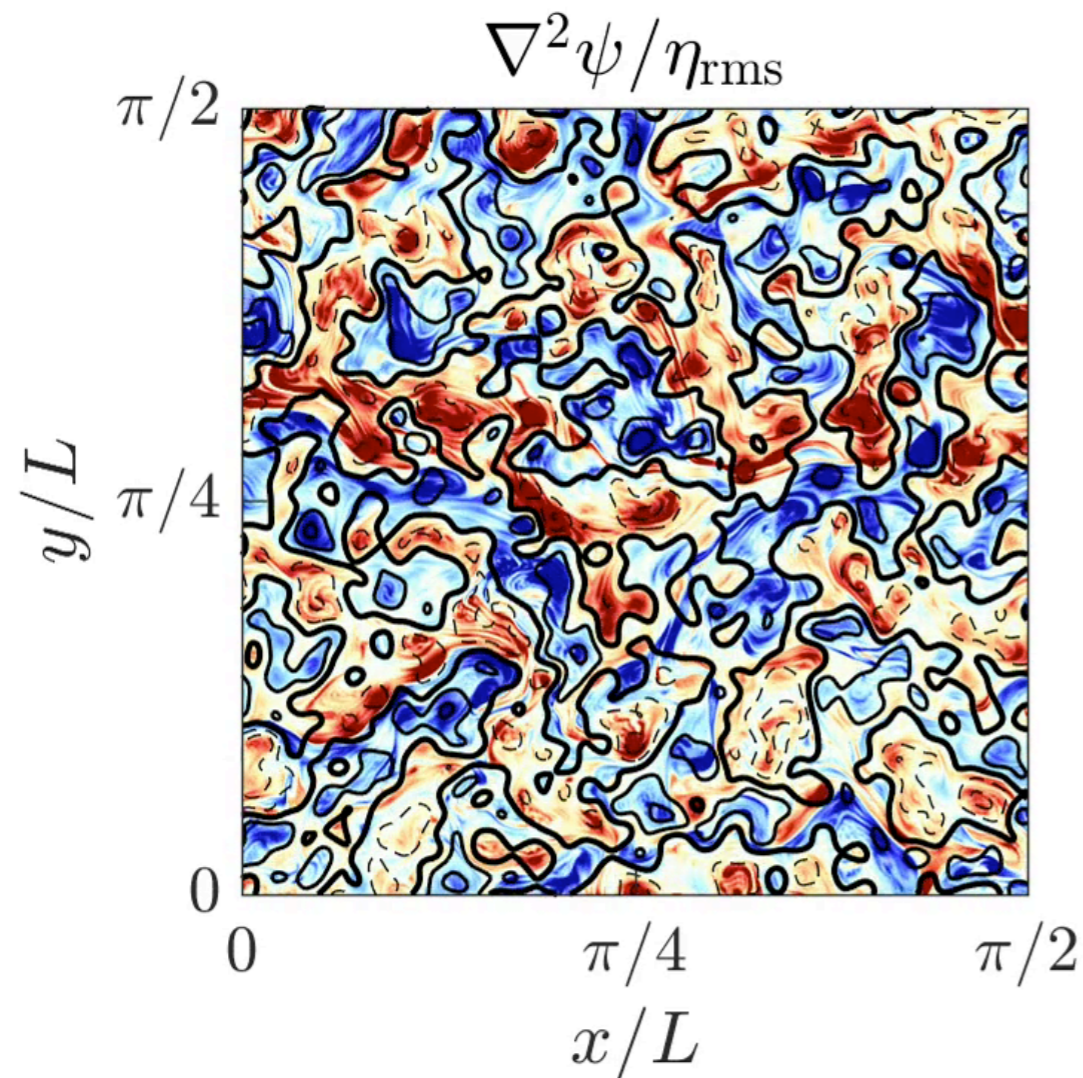
vertical & horizontal integral
over a mid-ocean region
(**not** a zonal average)

this **barotropic** QG model exhibits turbulence and eddies

random topography
with k^{-2} spectrum

$$\mu t = 3.15$$

$$h \propto \cos(mx)$$



Question:

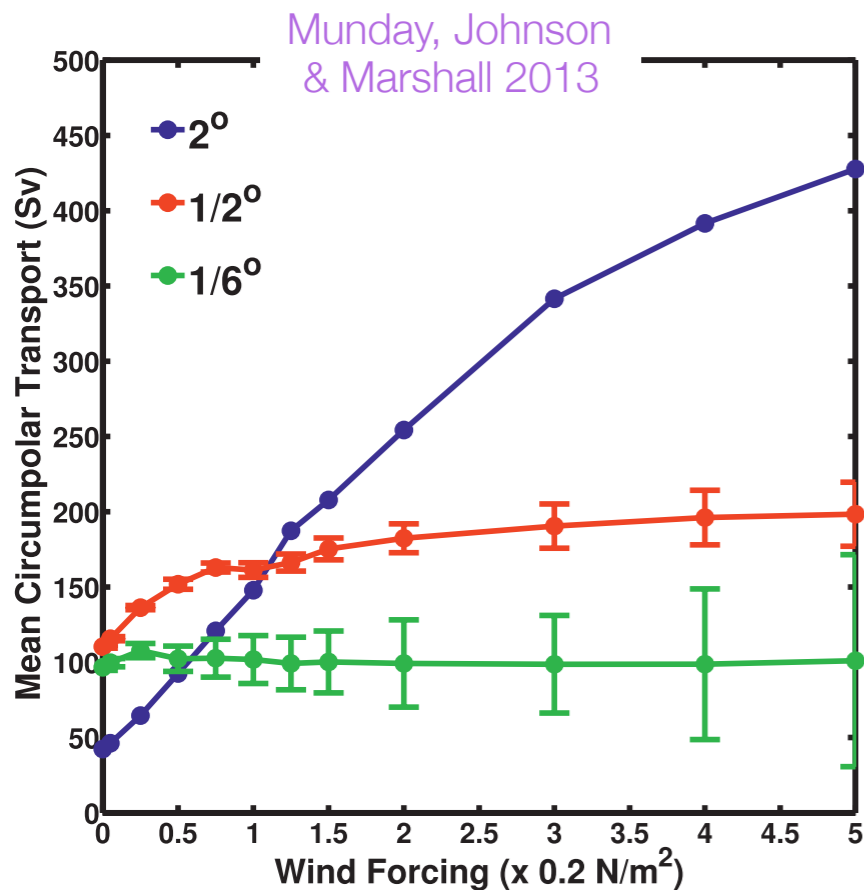
Does this **barotropic** QG model
show eddy saturation?

Do we need **baroclinicity**?

Do we even need channel walls?

but first, what is “eddy saturation”?

The *insensitivity* of the total ACC volume transport to wind stress increase.



Eddy saturation is seen in eddy-resolving ocean models.

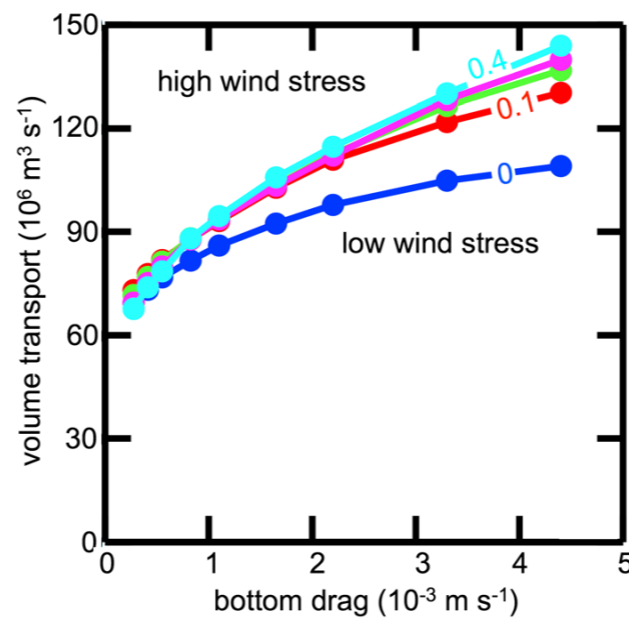
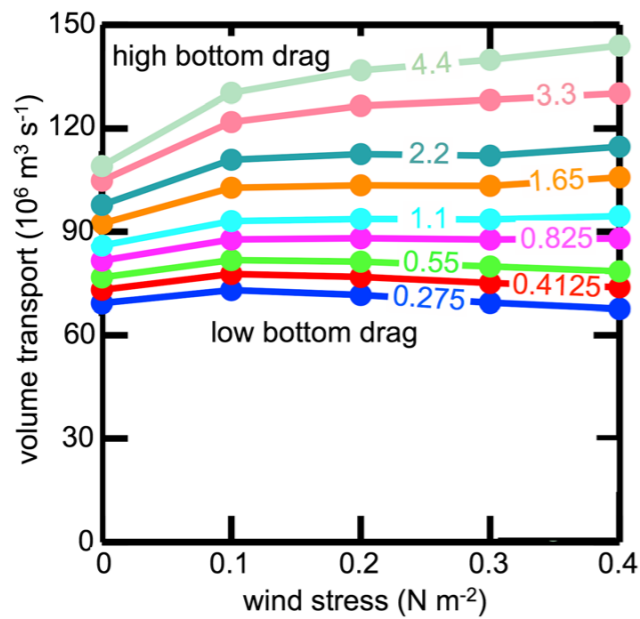
Higher resolution → eddy saturation “occurs”

Eddy saturation was theoretically predicted by [Straub \(1993\)](#) but with an *entirely baroclinic* argument. (based on vertical momentum transfer interfacial eddy form stress)

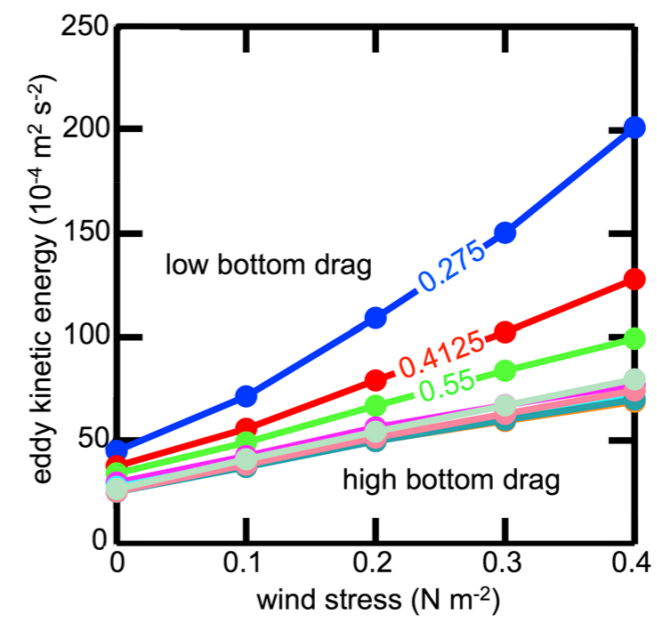
[There are many other examples: [Hallberg & Gnanadesikan 2001](#), [Tansley & Marshall 2001](#), [Hallberg & Gnanadesikan 2006](#), [Hogg et al. 2008](#), [Nadeau & Straub 2009](#), [Farneti et al. 2010](#), [Nadeau & Straub 2012](#), [Meredith et al. 2012](#), [Morisson & Hogg 2013](#), [Abernathy & Cessi 2014](#), [Farneti et al. 2015](#), [Nadeau & Ferrari 2015](#), [Marshall et al. 2016](#).]

yet more eddy saturation

Marshall, Ambaum,
Maddison, Munday
& Novak 2016



increasing the bottom drag
increases the transport

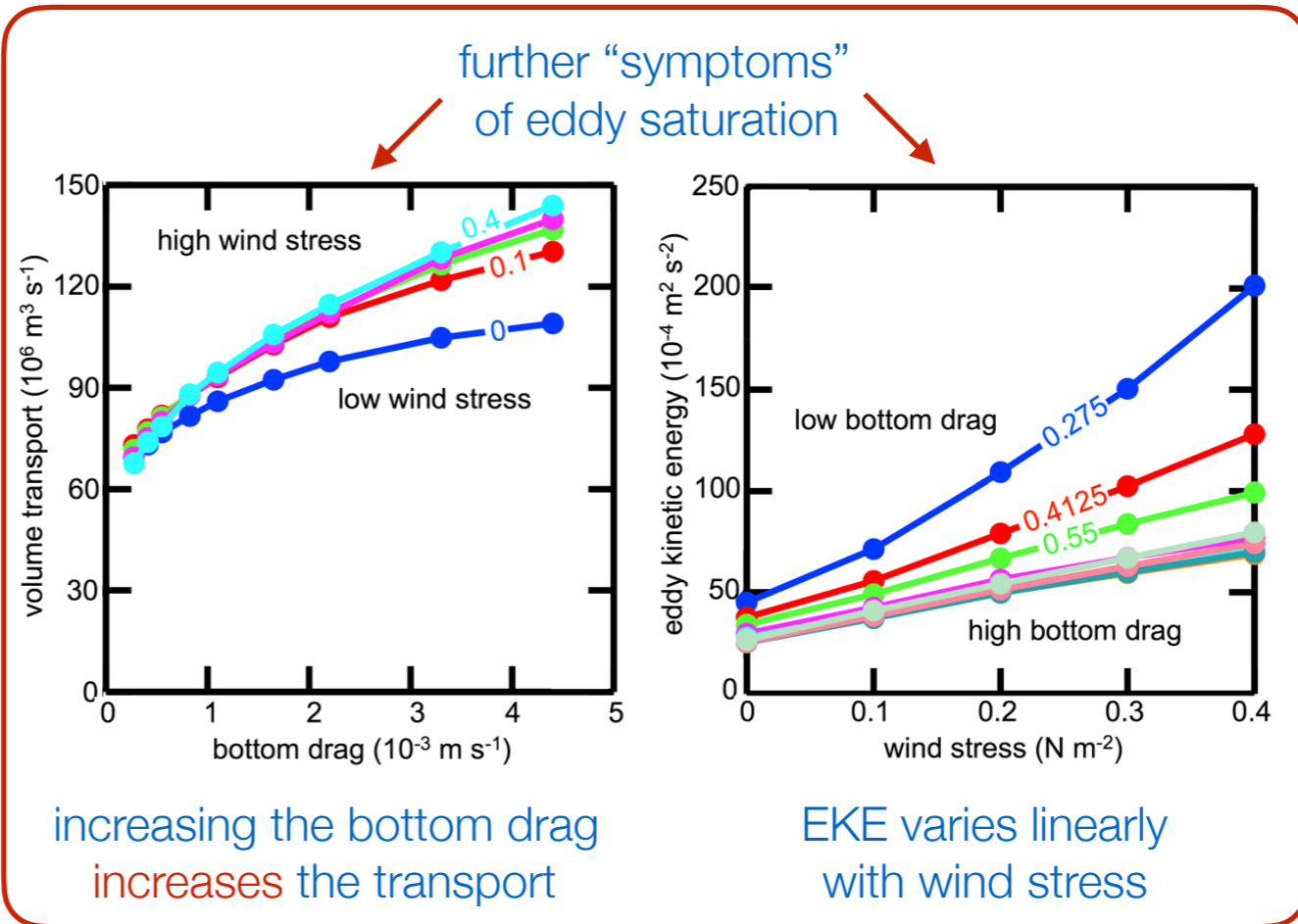
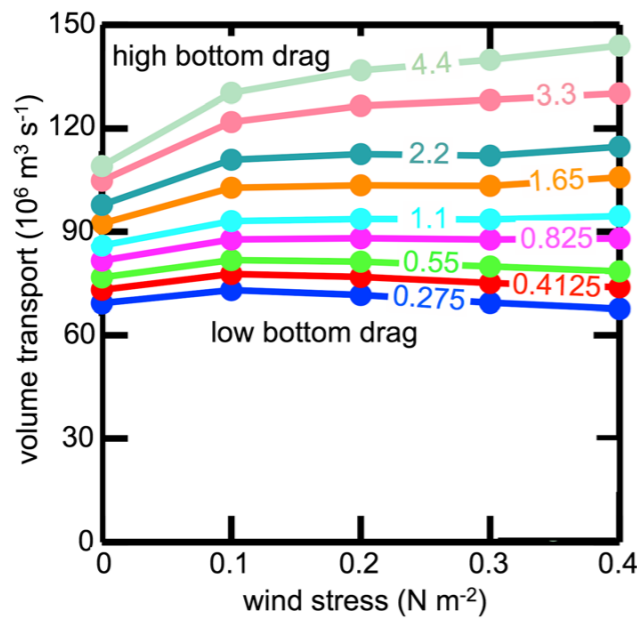


EKE varies linearly
with wind stress

[See also: [Hogg & Blundell 2006](#), [Nadeau & Straub 2012](#), and [Nadeau & Ferrari 2015](#).]

yet more eddy saturation

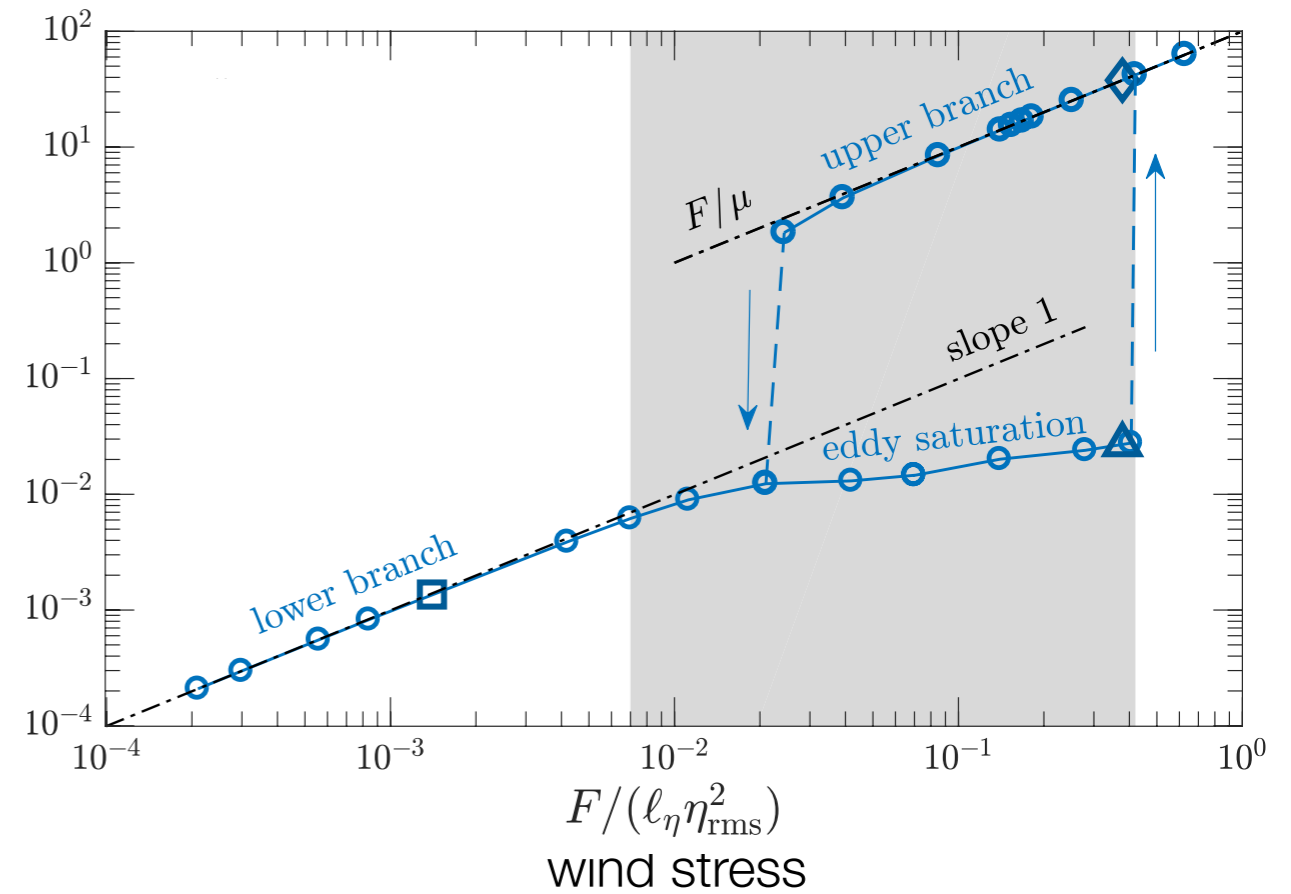
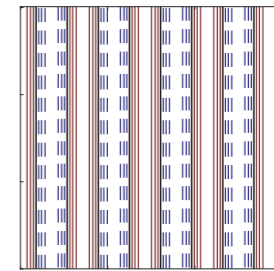
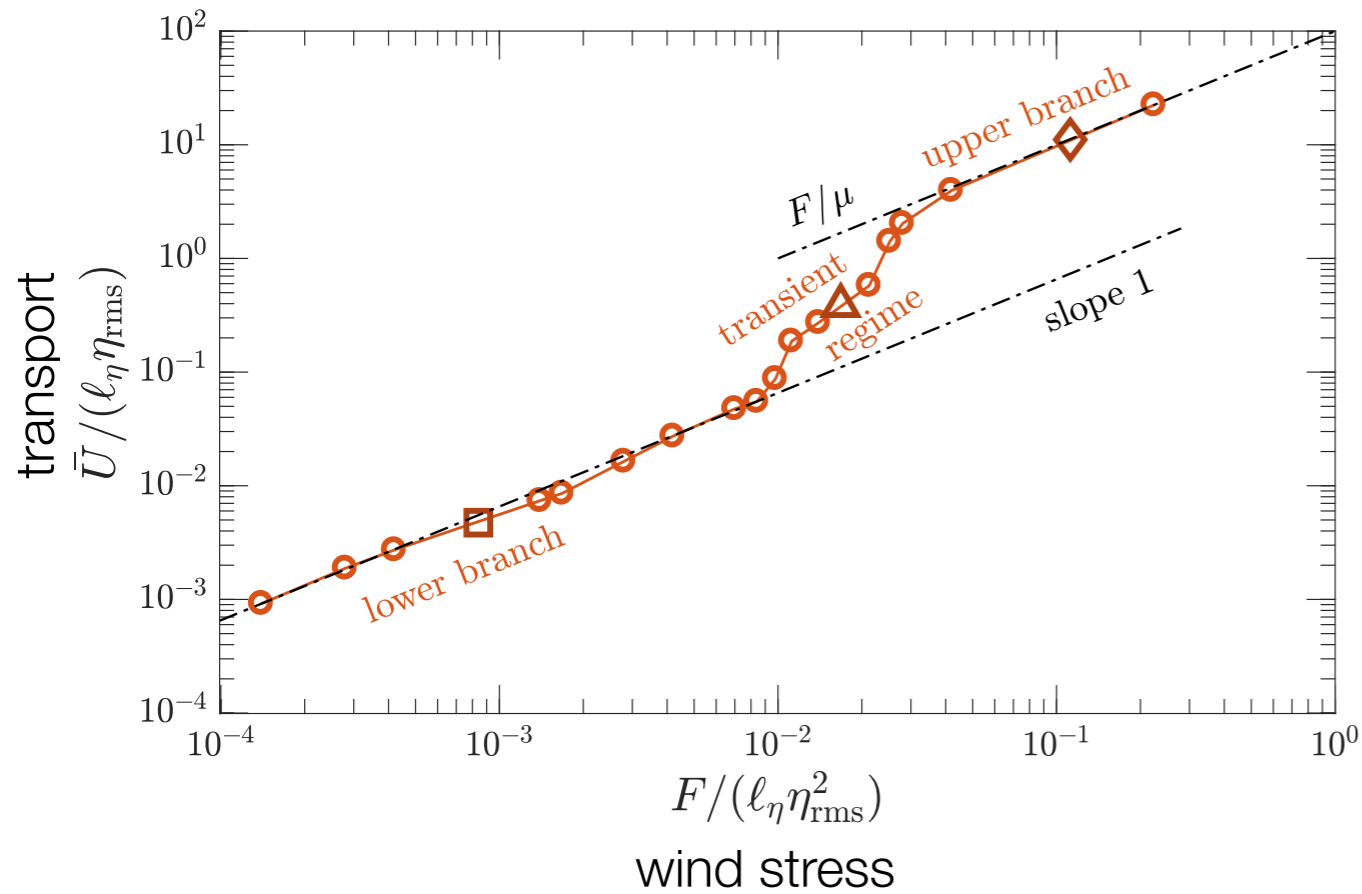
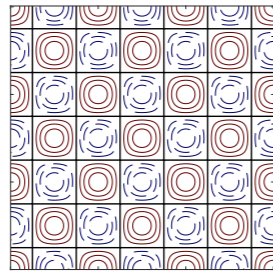
Marshall, Ambaum,
Maddison, Munday
& Novak 2016



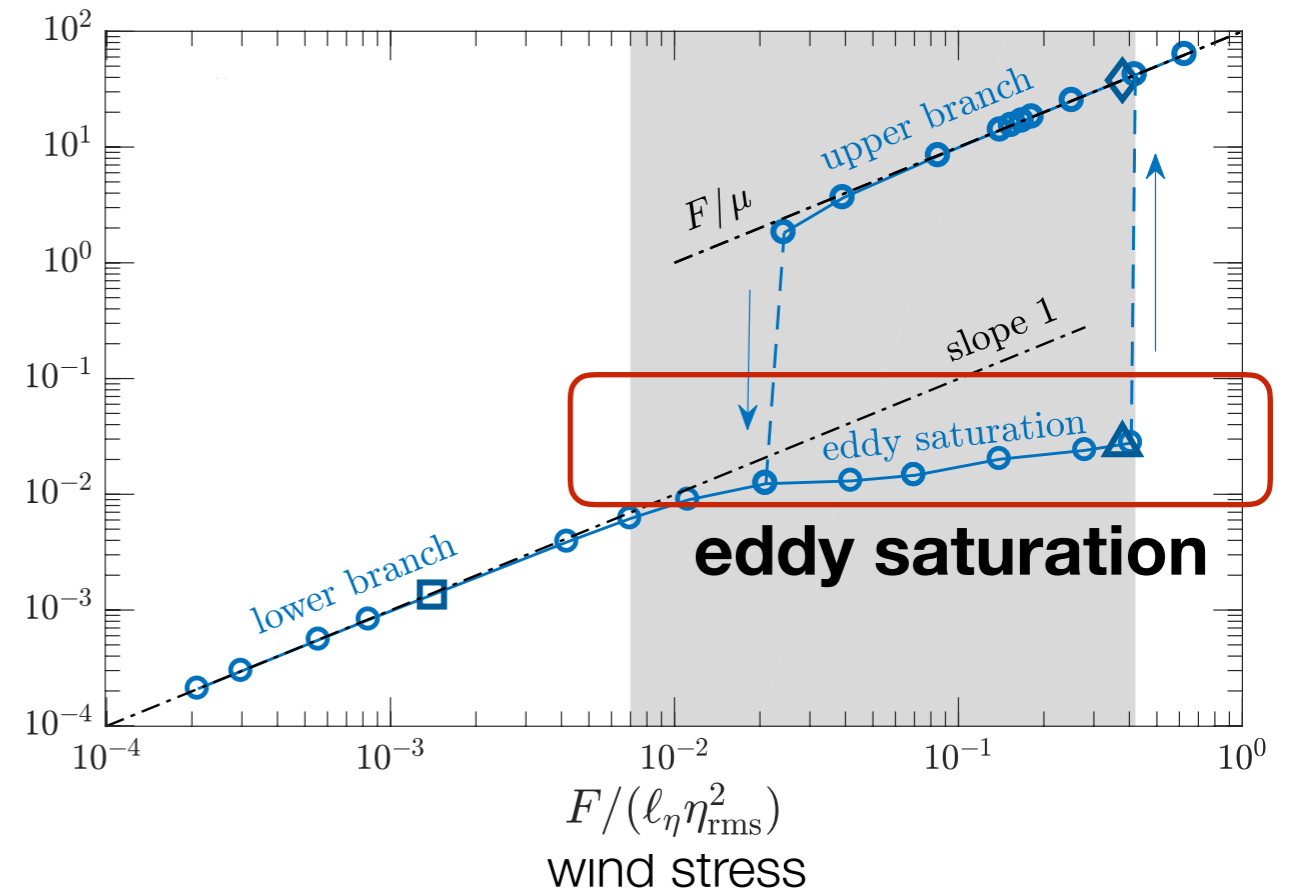
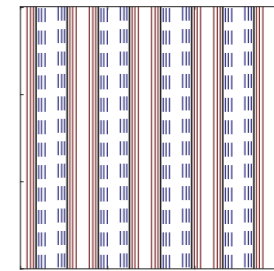
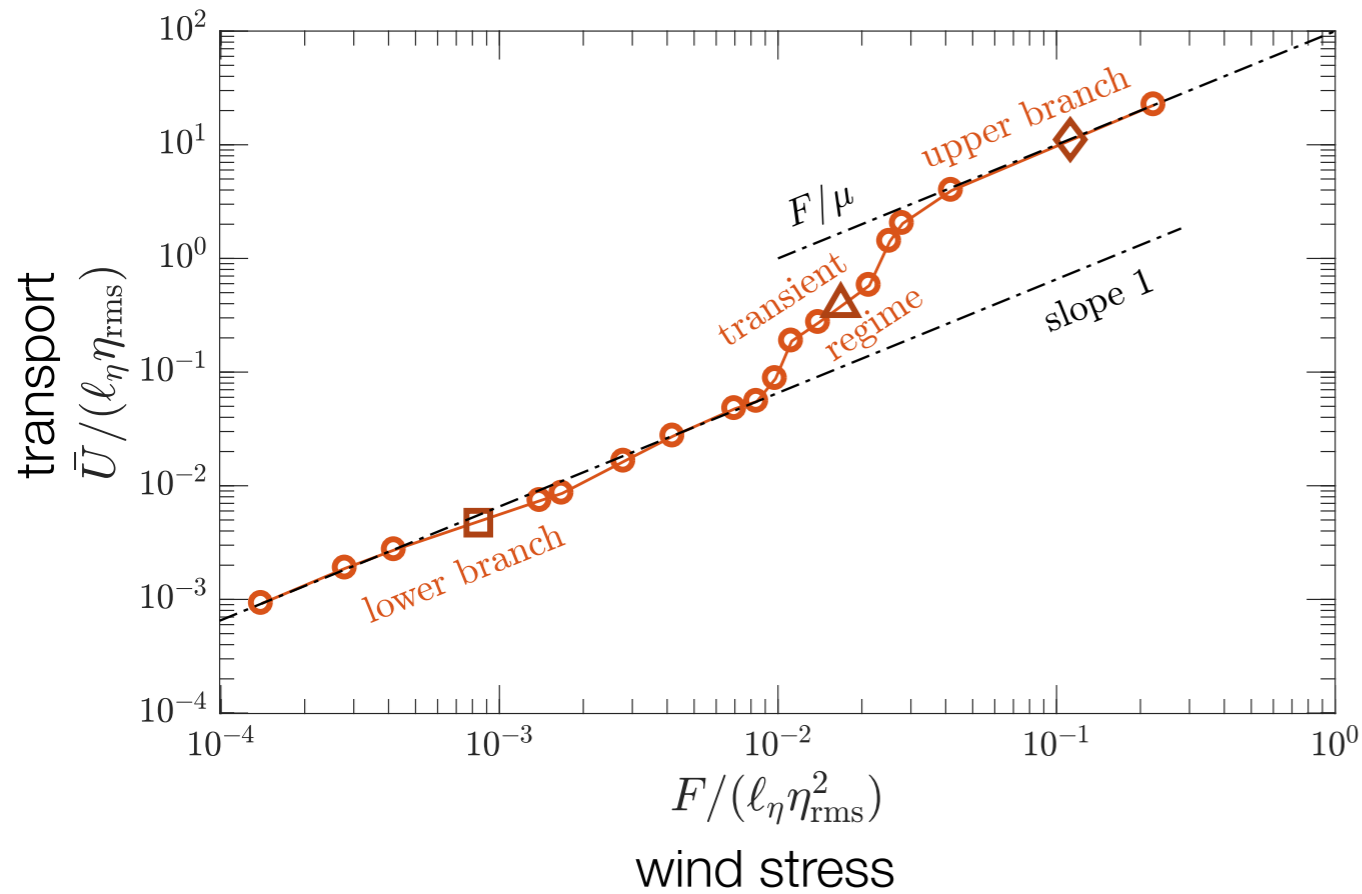
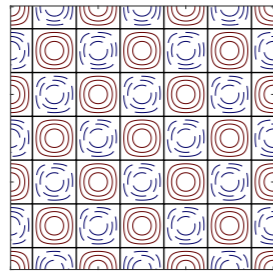
[See also: [Hogg & Blundell 2006](#), [Nadeau & Straub 2012](#), and [Nadeau & Ferrari 2015](#).]

how does the transport vary with wind stress
in this **barotropic** QG model?

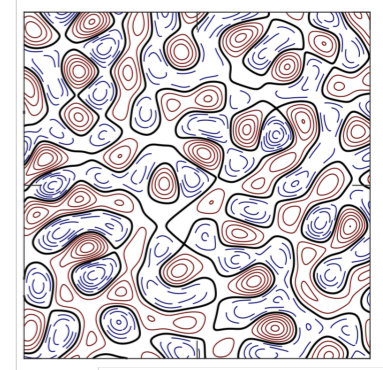
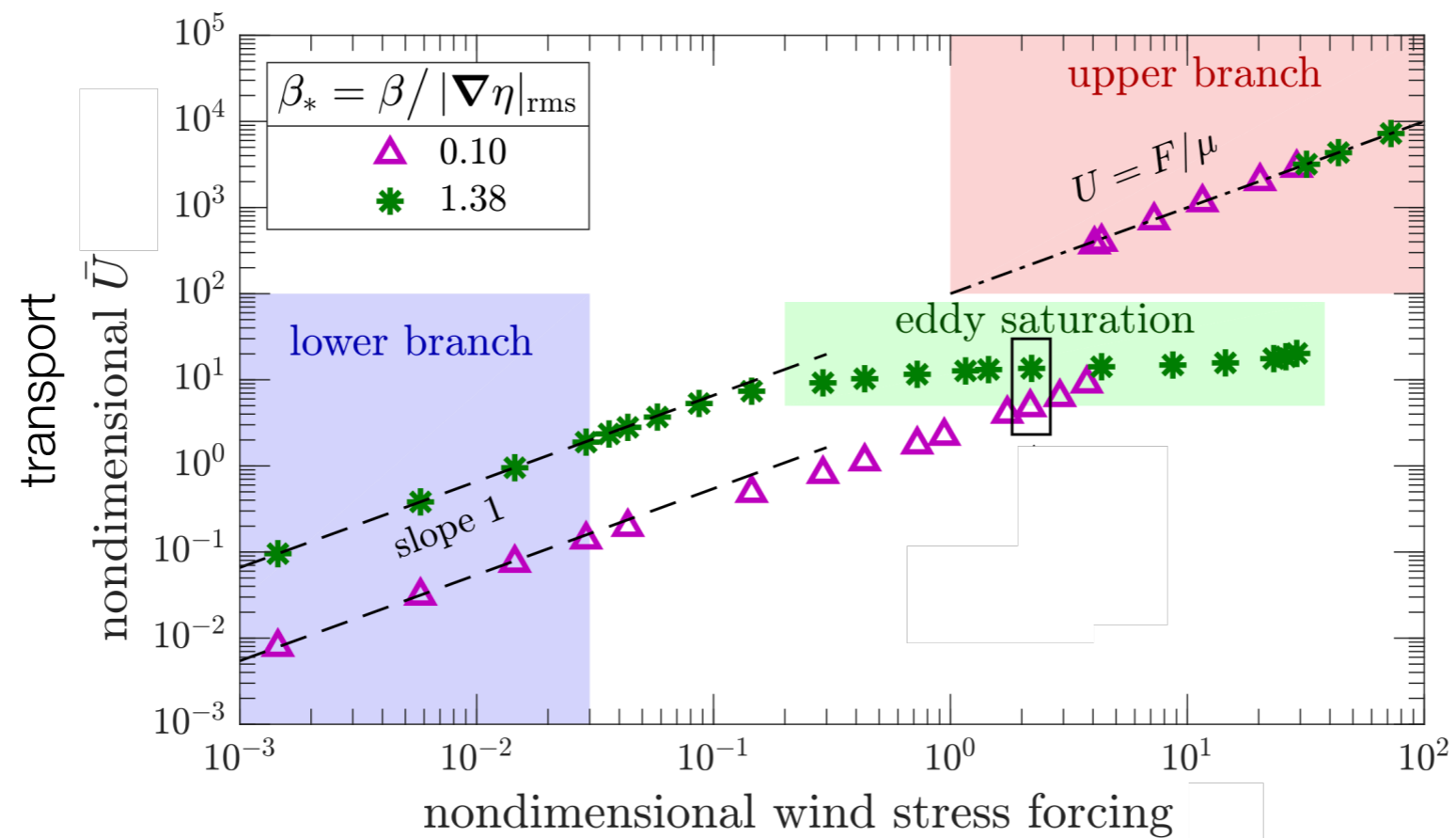
all parameters same, different topography



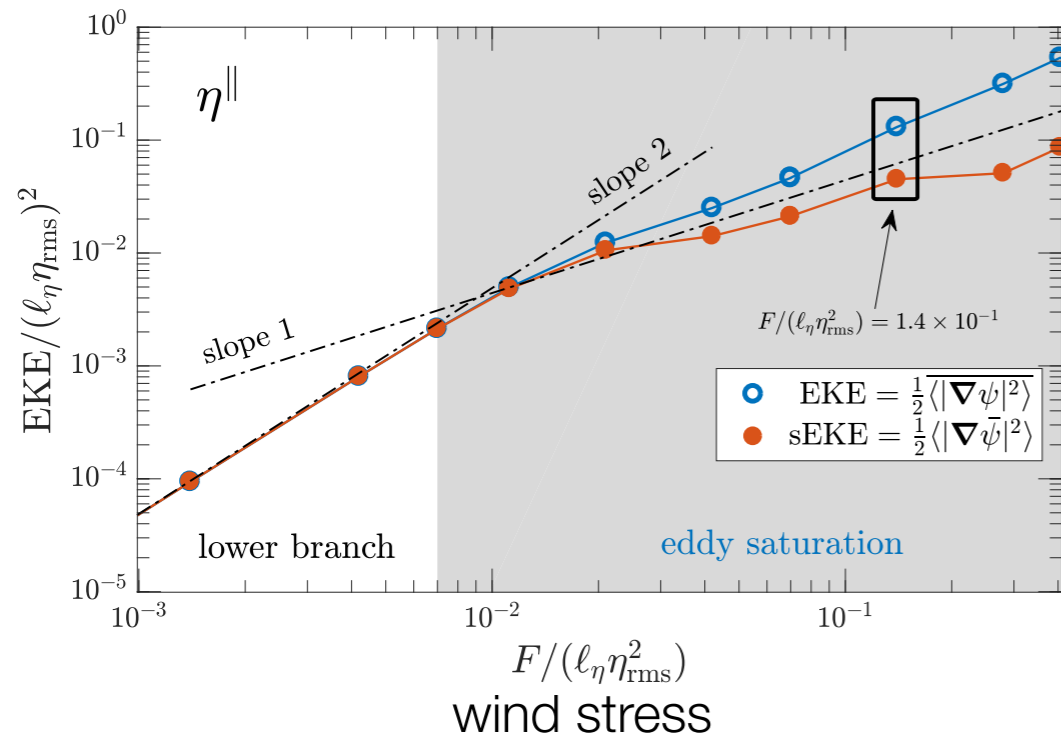
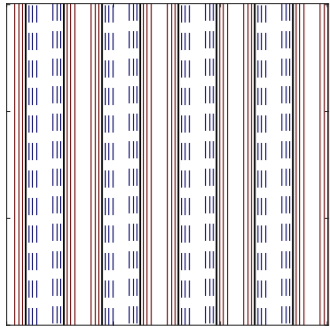
all parameters same, different topography



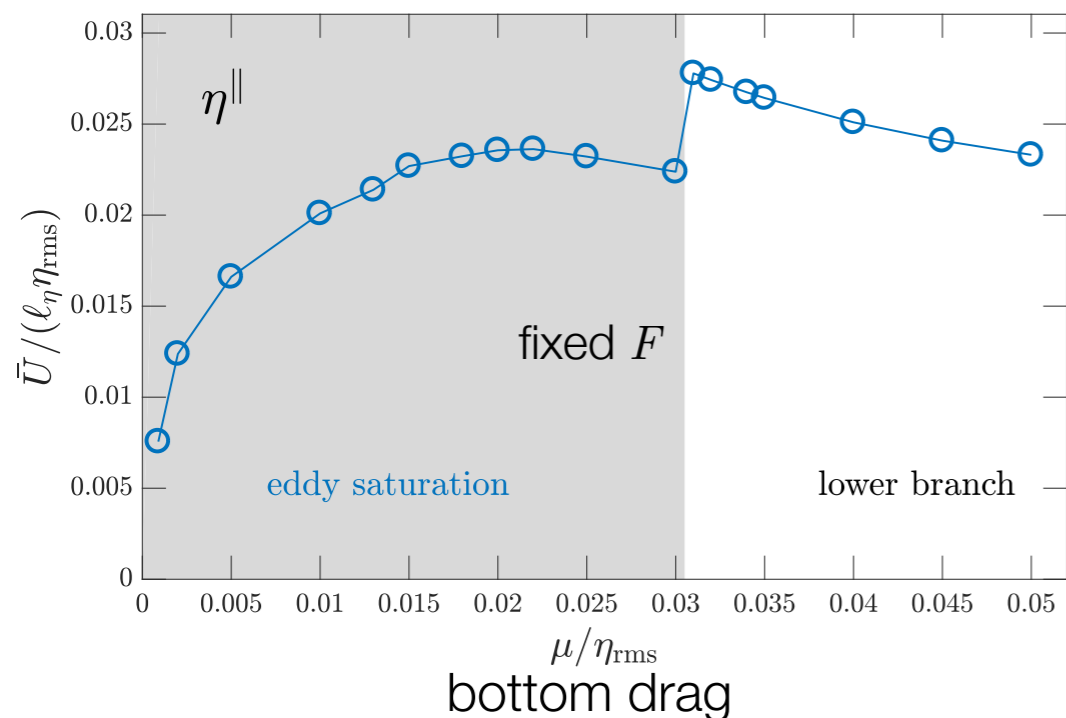
all parameters same, different value of β



some further “symptoms” of eddy saturation



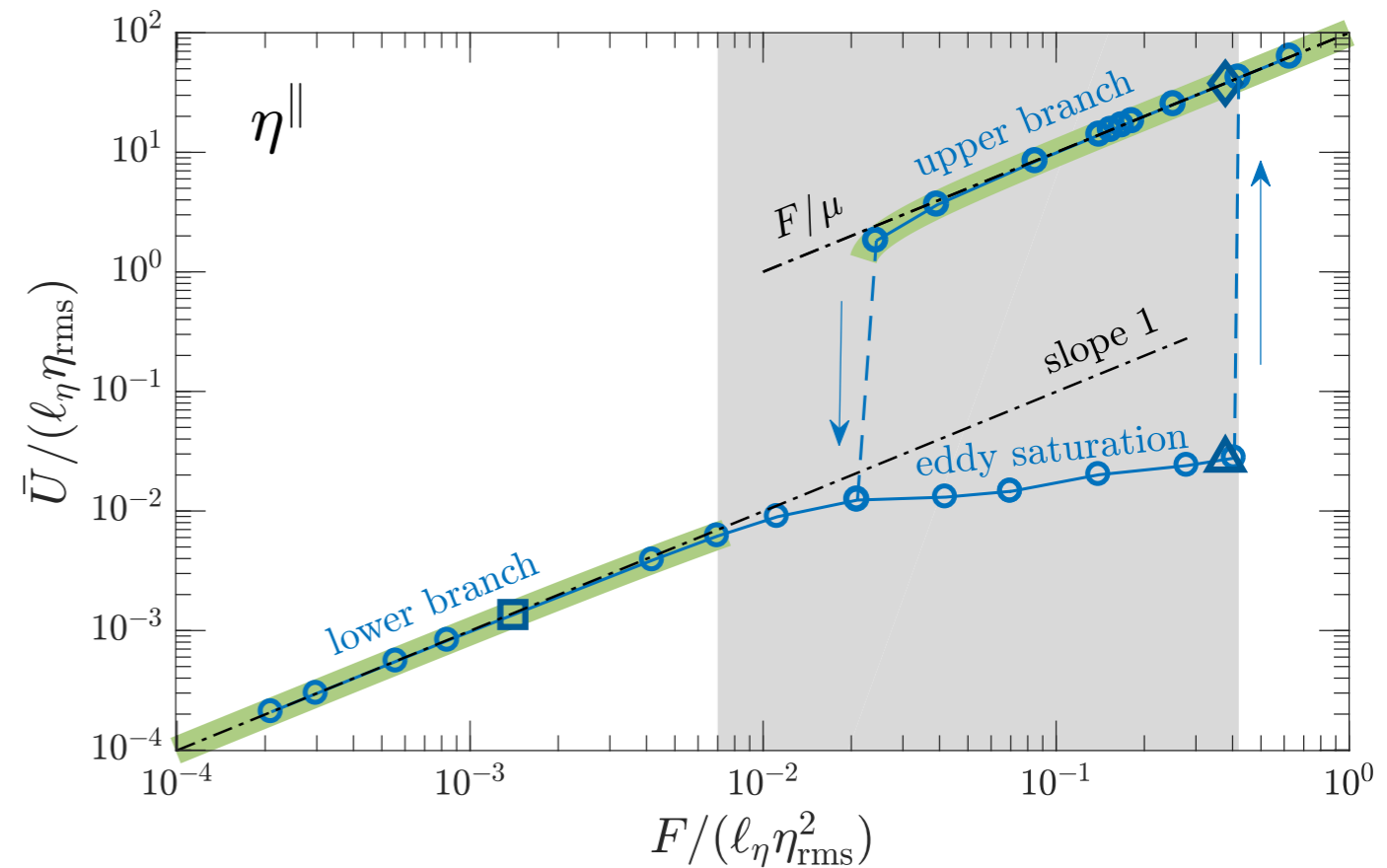
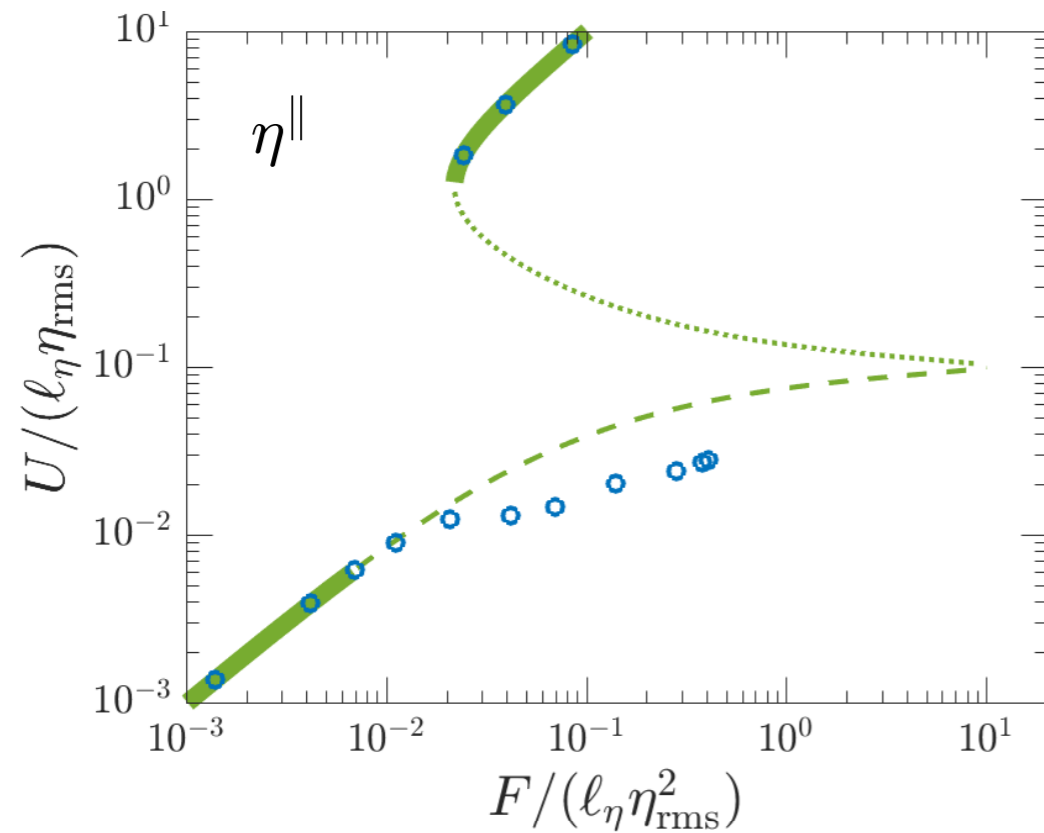
EKE grows roughly linearly
with wind stress



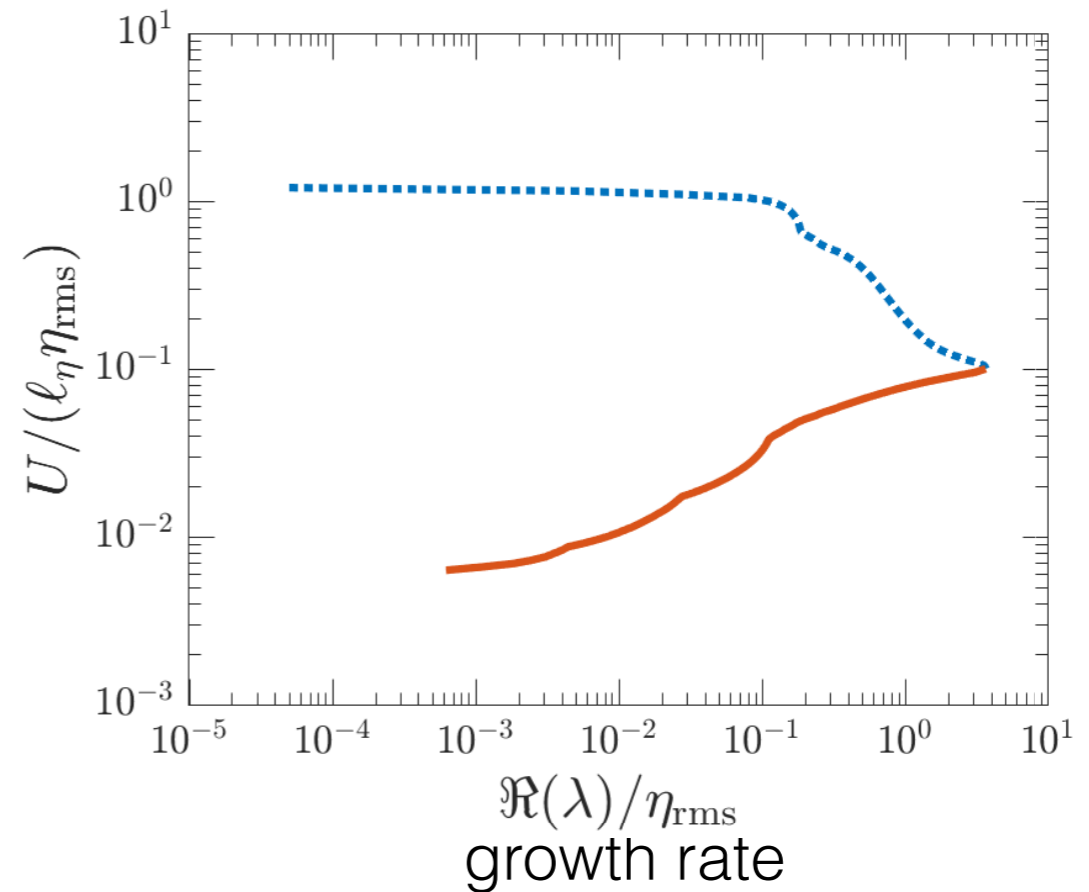
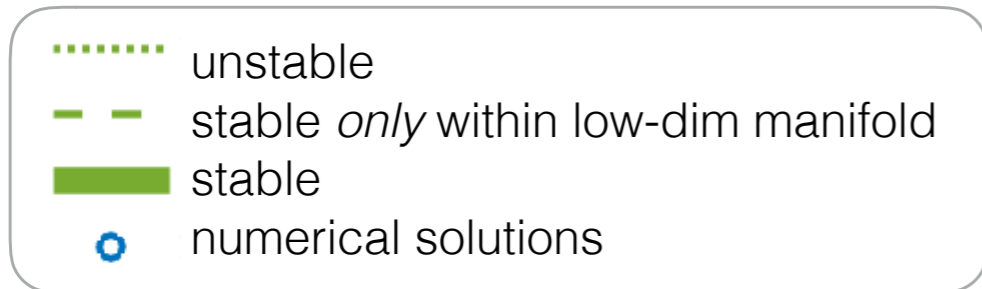
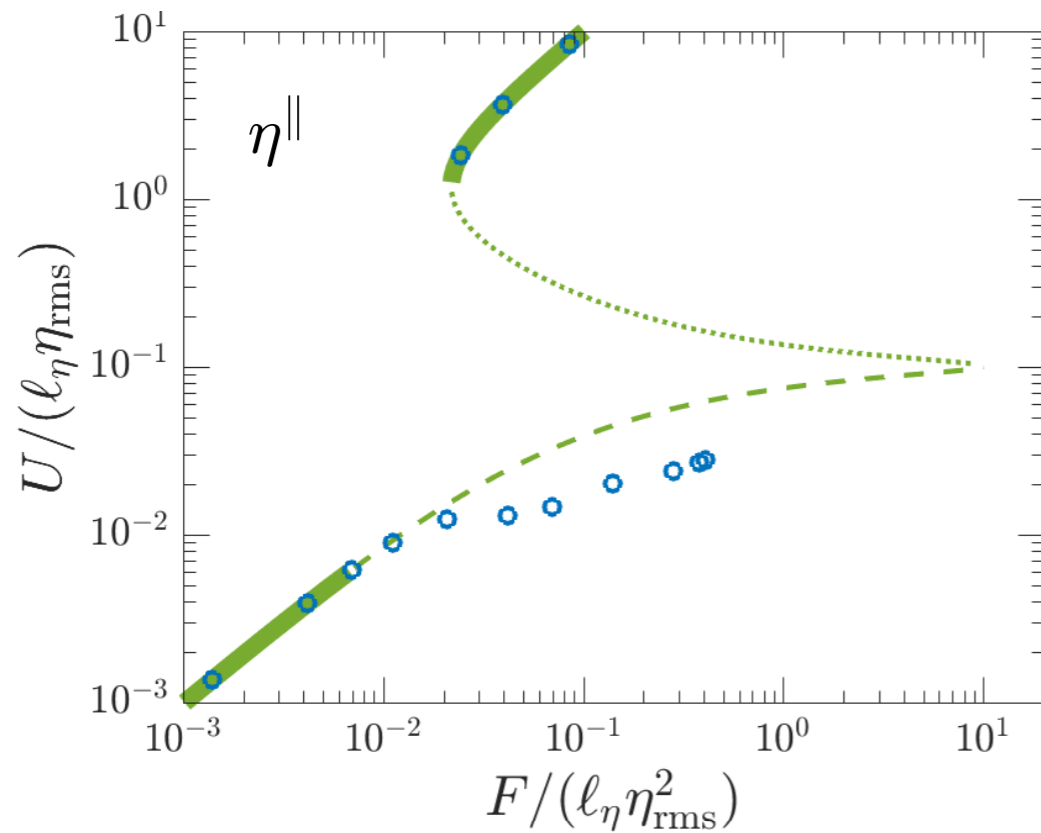
transport grows
with increasing bottom drag

what produces eddy saturated states
in this **barotropic** QG model?

stability analysis for $\eta^{\parallel} = \eta_0 \cos(mx)$

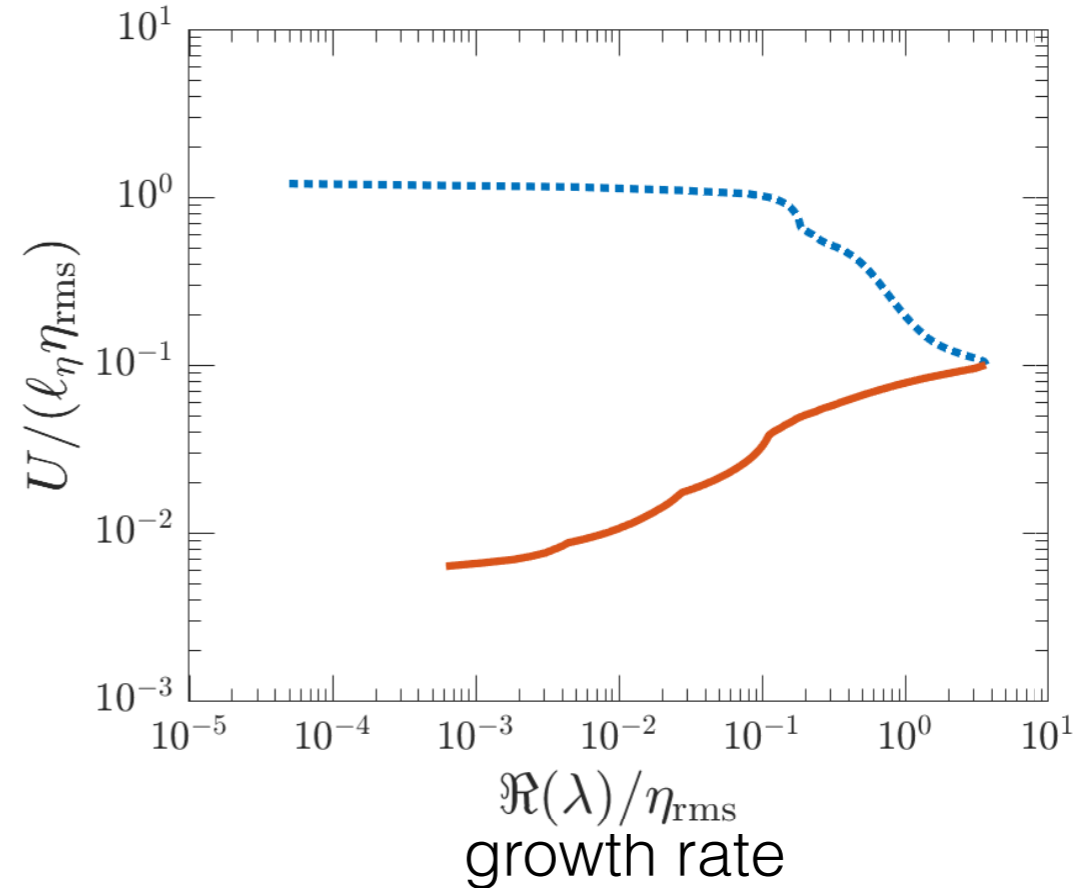
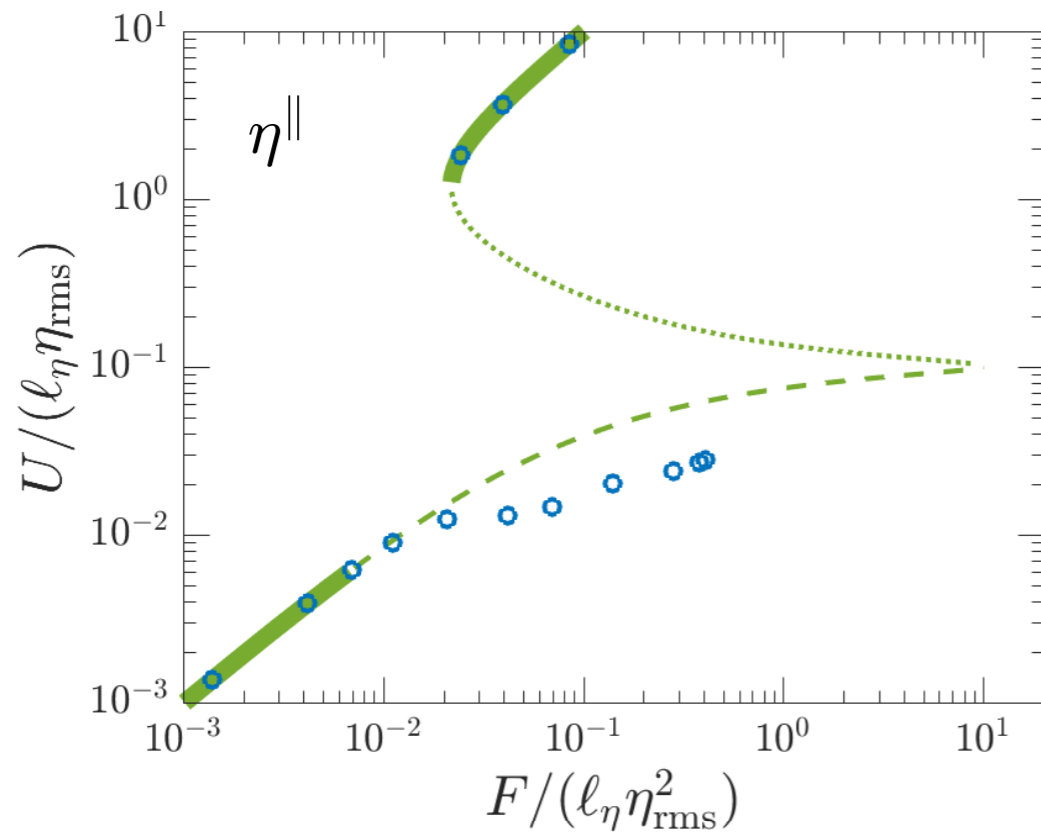


stability analysis for $\eta^{\parallel} = \eta_0 \cos(mx)$



Max instability growth rate increases $\sim 10^4$ times with a 10-fold increase in U !

stability analysis for $\eta^{\parallel} = \eta_0 \cos(mx)$



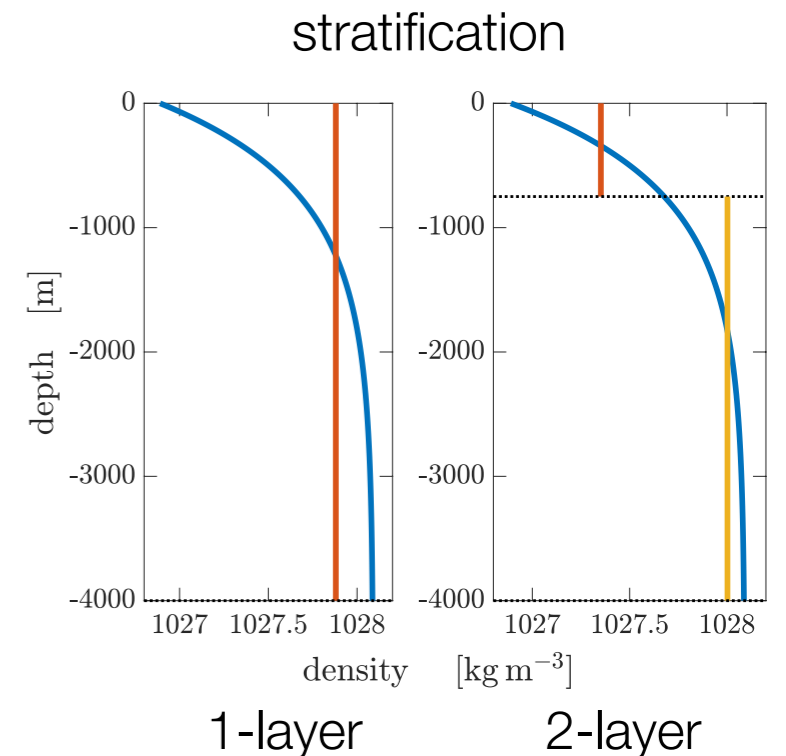
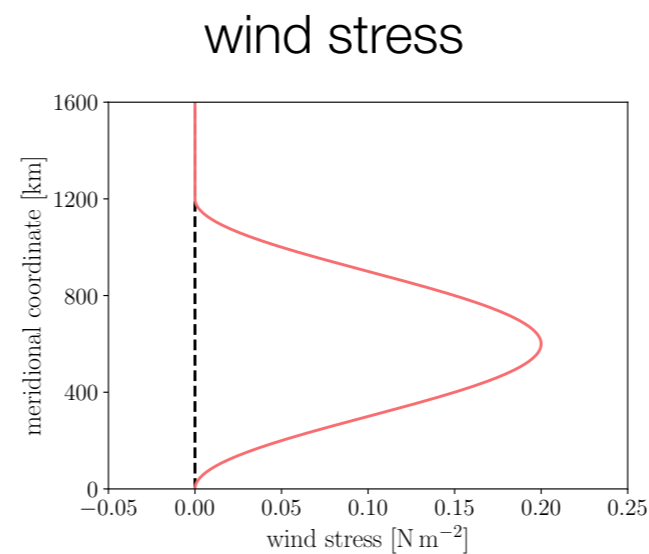
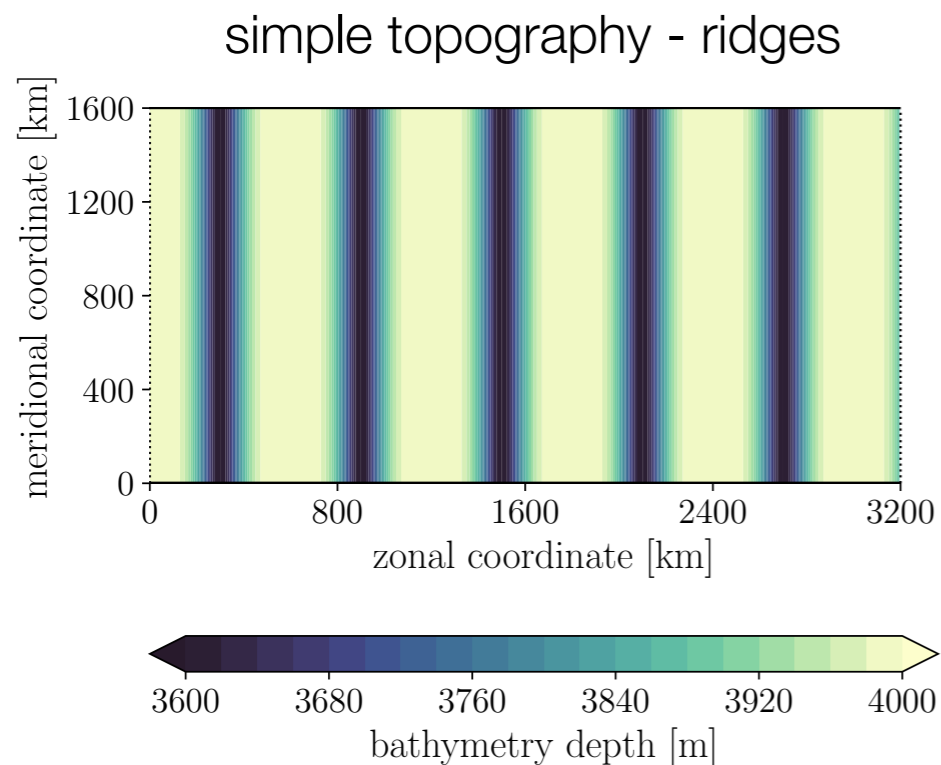
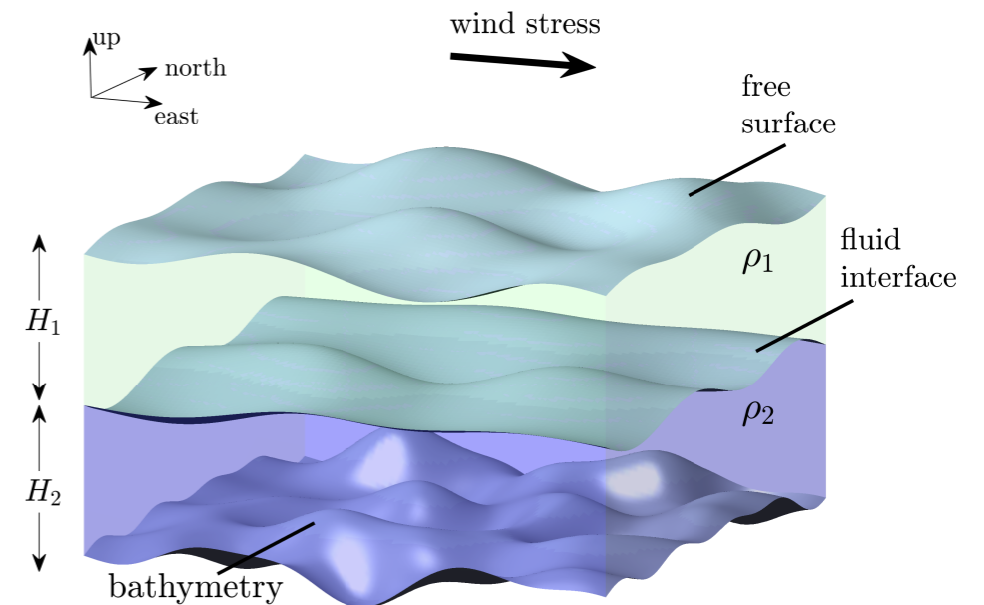
Max instability growth rate increases $\sim 10^4$ times with a 10-fold increase in U !

Minor changes in $U \longrightarrow$ large transient energy production.
 Transient eddies balance most of the momentum imparted by $F \longrightarrow$ eddy saturation.
 (Similarly as in the **baroclinic** scenario.)

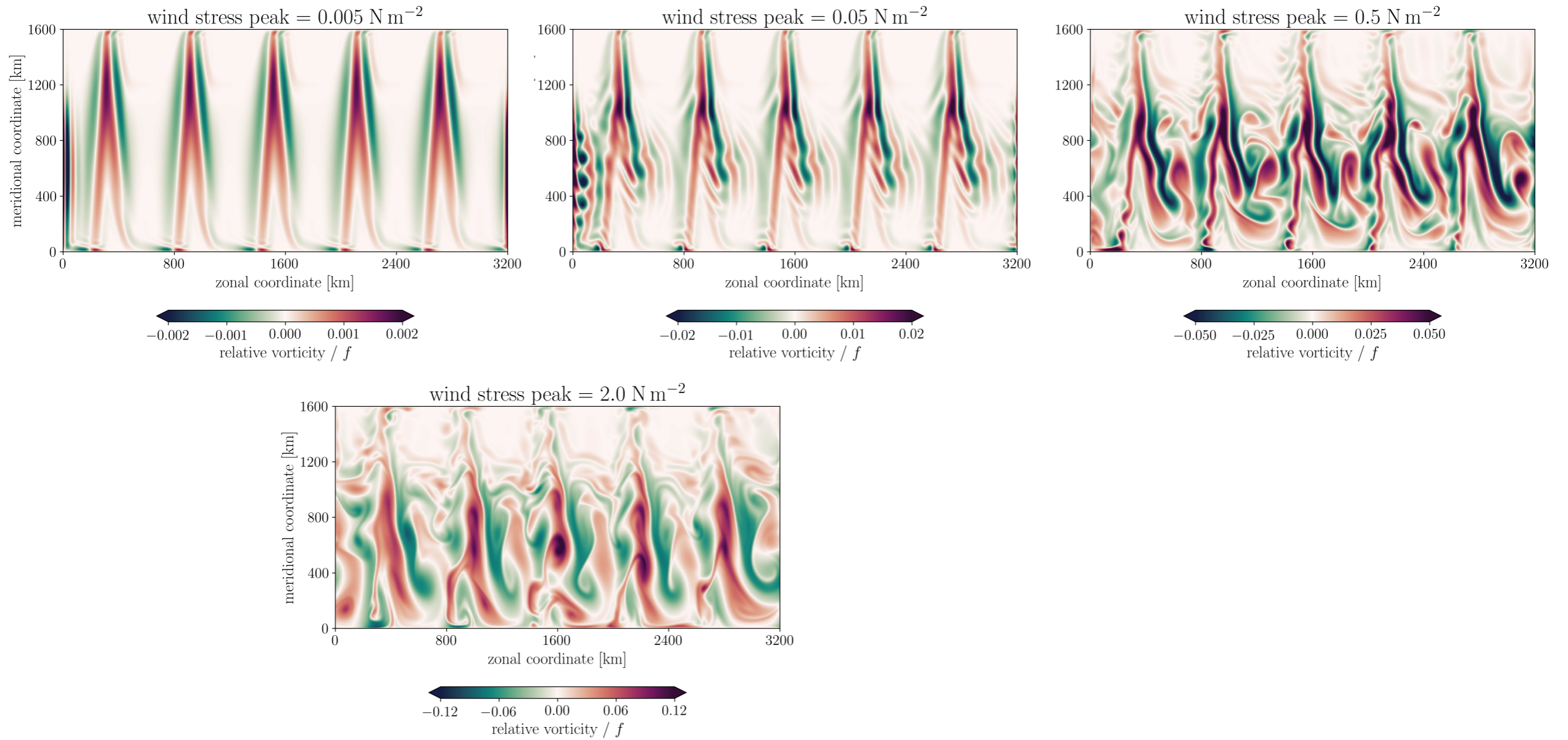
let's change page now

a setup with both BT and BC eddy saturations

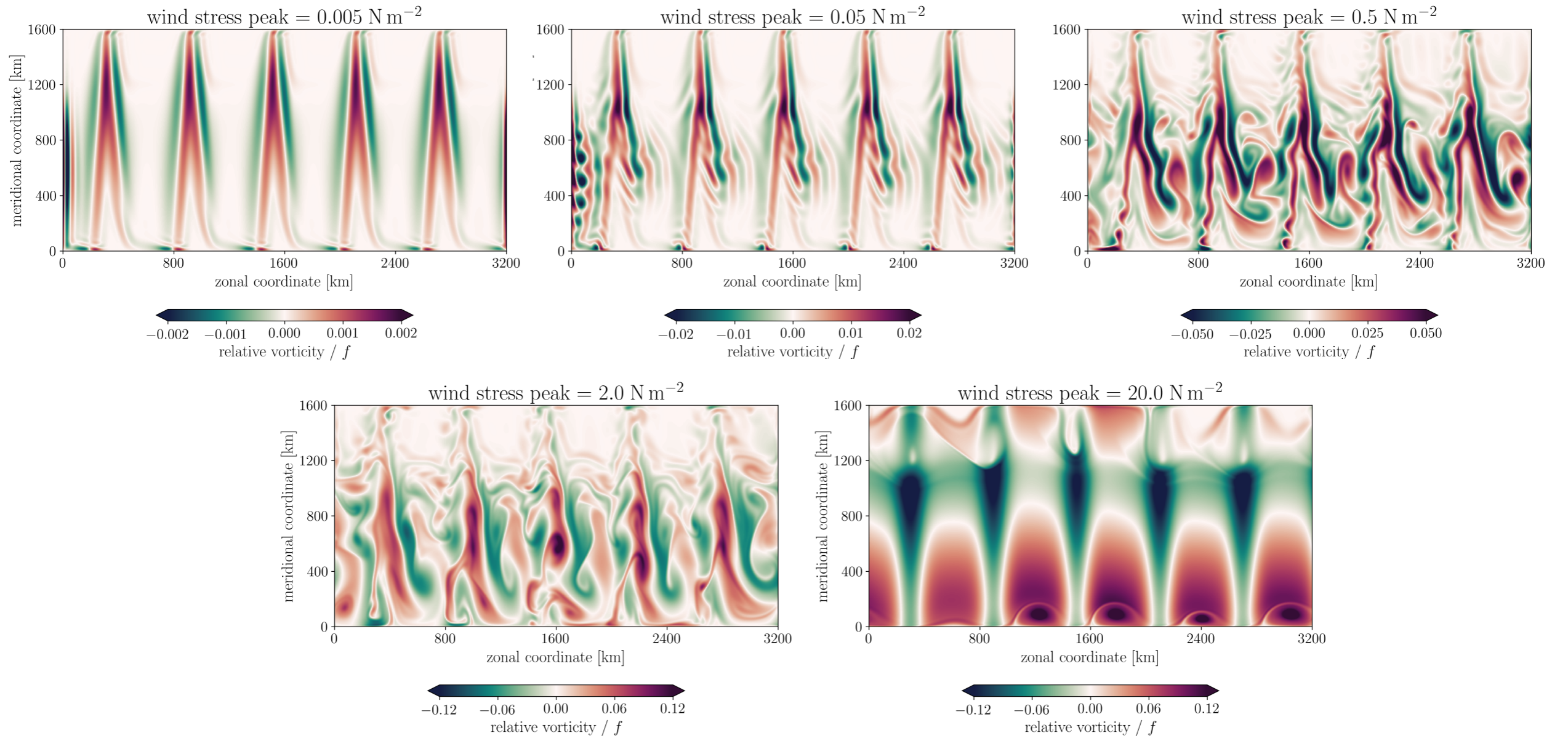
- Idealized re-entrant channel with "bumpy" bottom
- $L_x = 3200$ km, $L_y = 1600$ km, and $H = 4$ km
- beta-plane with Southern Ocean parameters
- Modest stratification (few fluid layers of constant ρ)
- 1st Rossby radius of deformation: 15.7 km (for >1 layers)
- Modular Ocean Model v6 (MOM6) in isopycnal mode



flow structure for 1-layer configuration

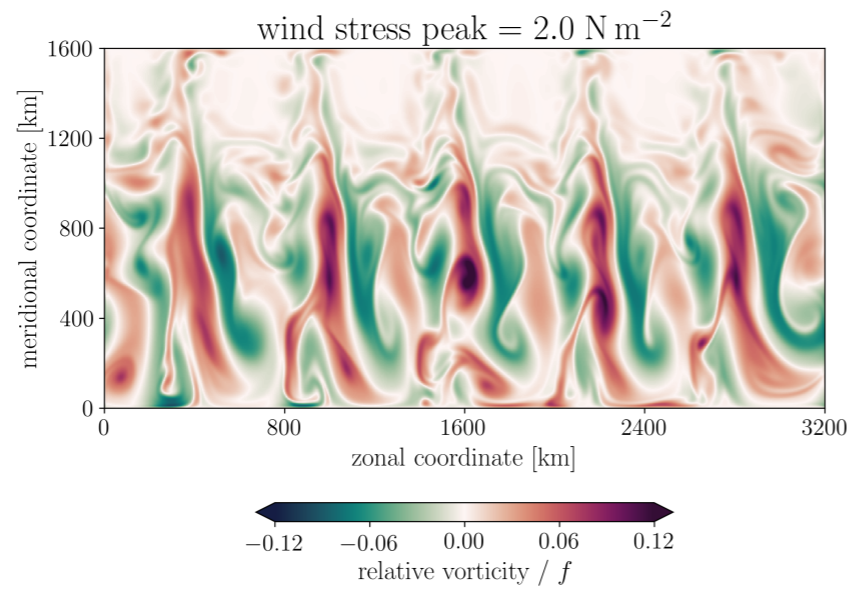


flow structure for 1-layer configuration

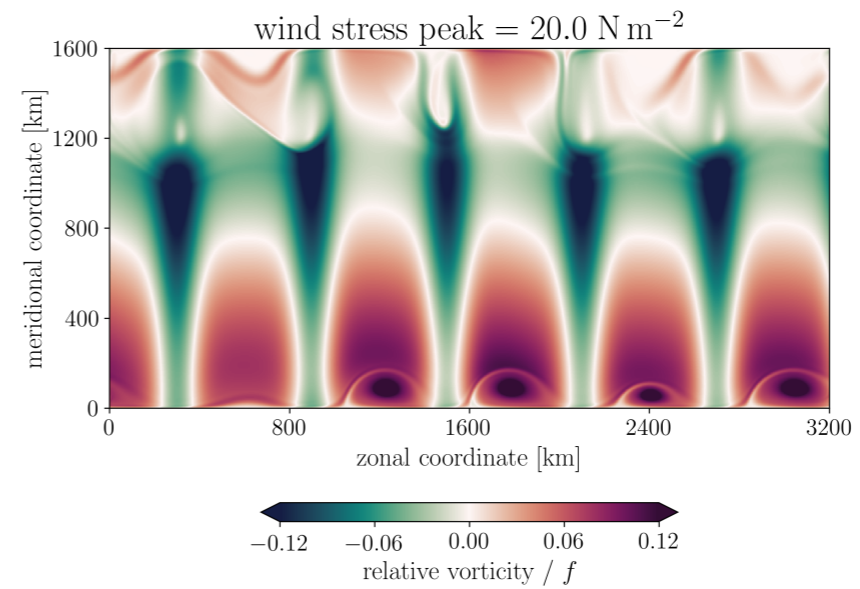


flow structure for 1-layer configuration

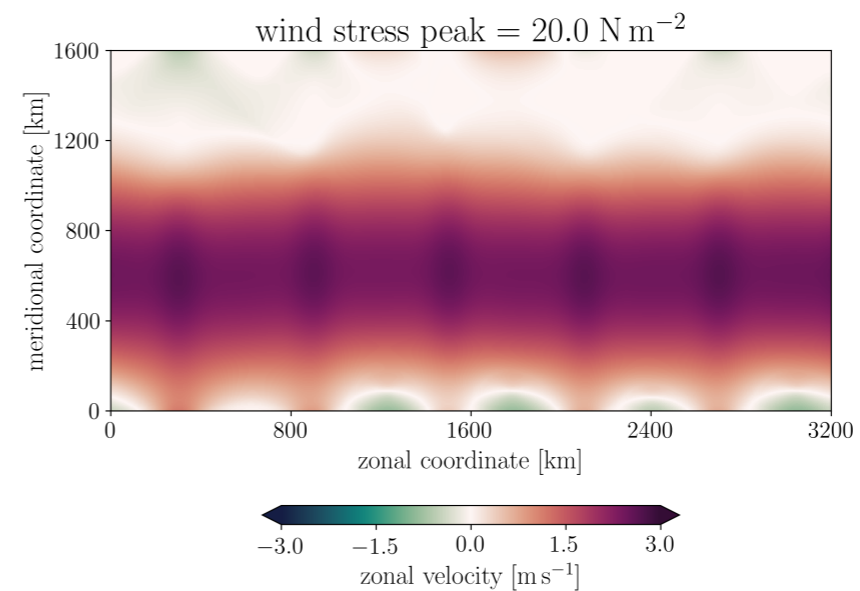
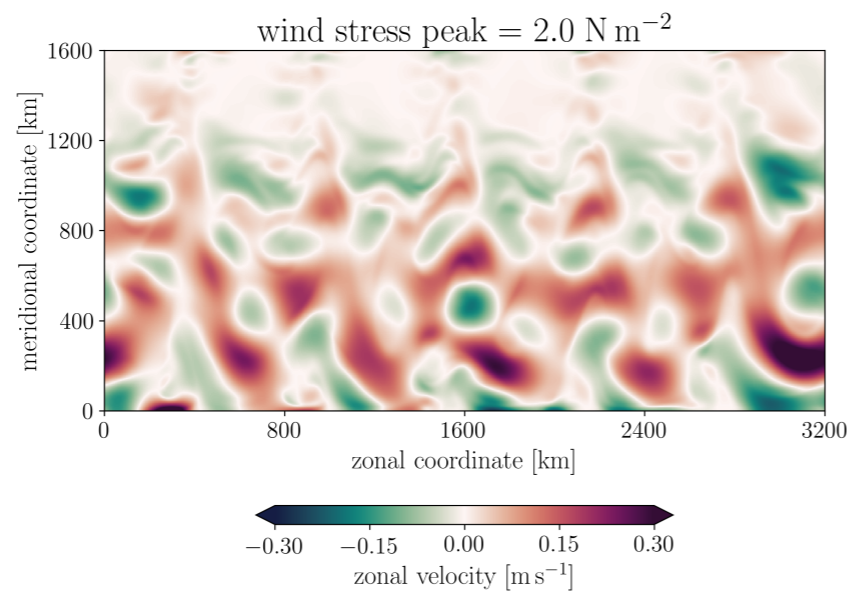
relative vorticity →



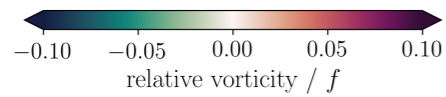
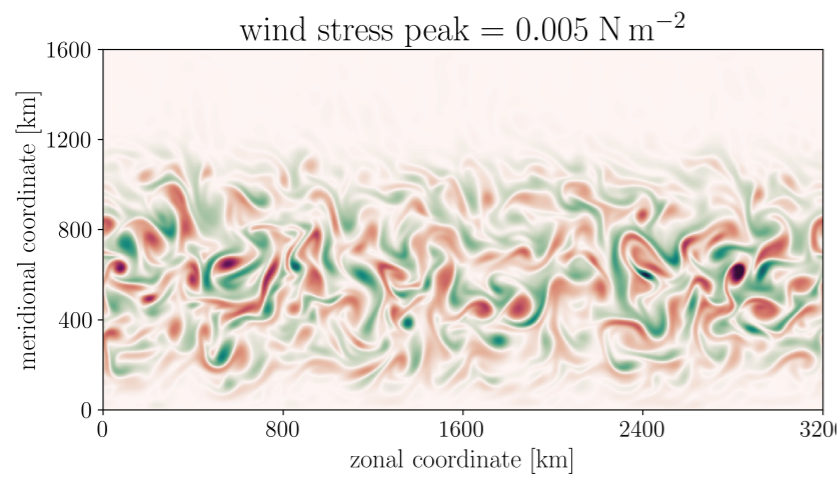
upper branch
(flow barely sees the topography)



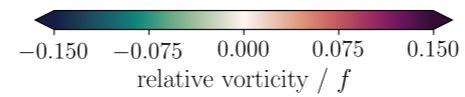
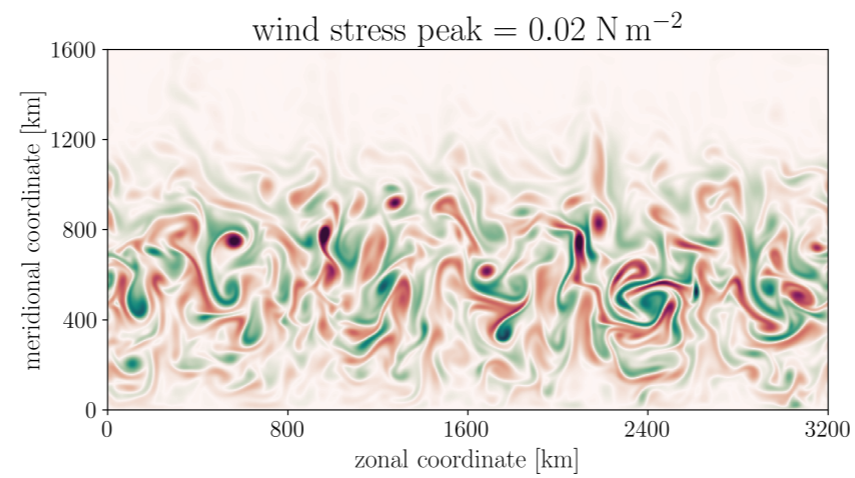
zonal flow →



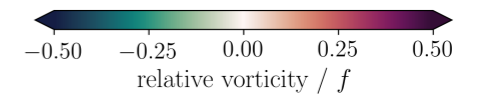
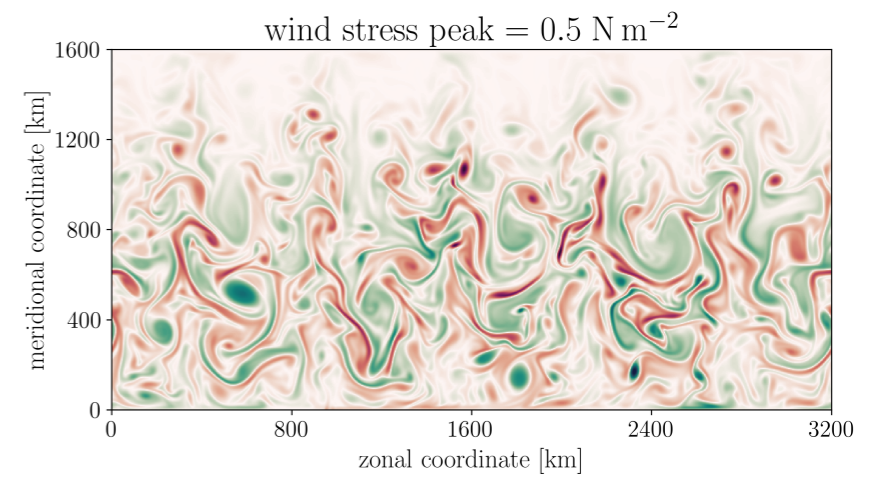
flow structure for 2-layer configuration



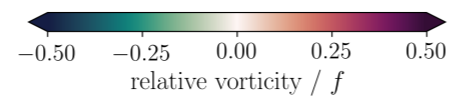
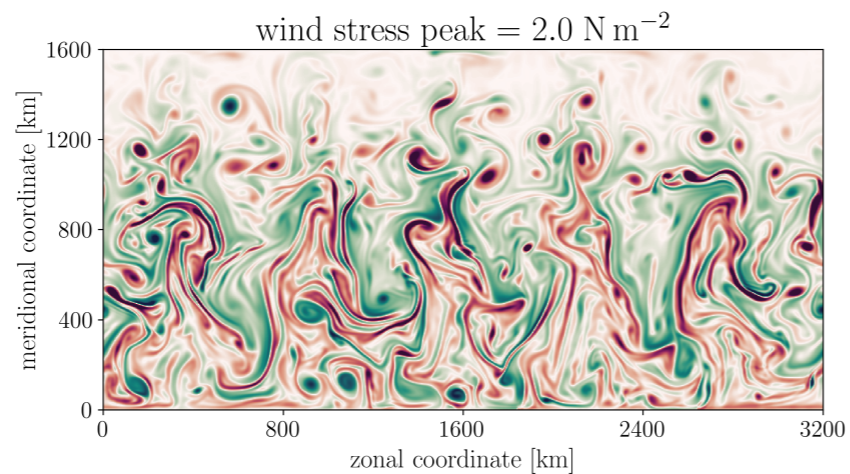
baroclinic eddies
(~200yr spinup)



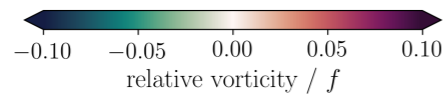
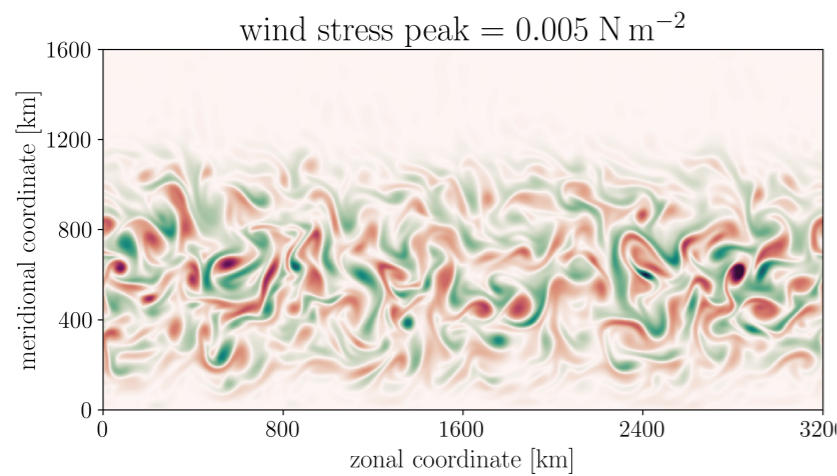
baroclinic eddies
(~50yr spinup)



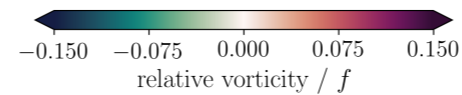
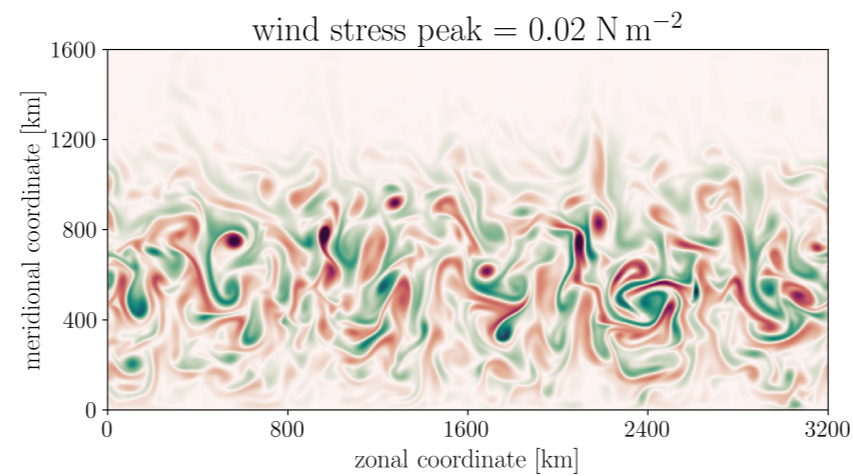
flow starts "seeing"
the topography



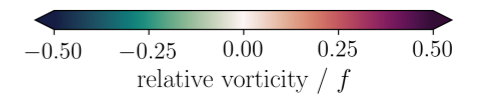
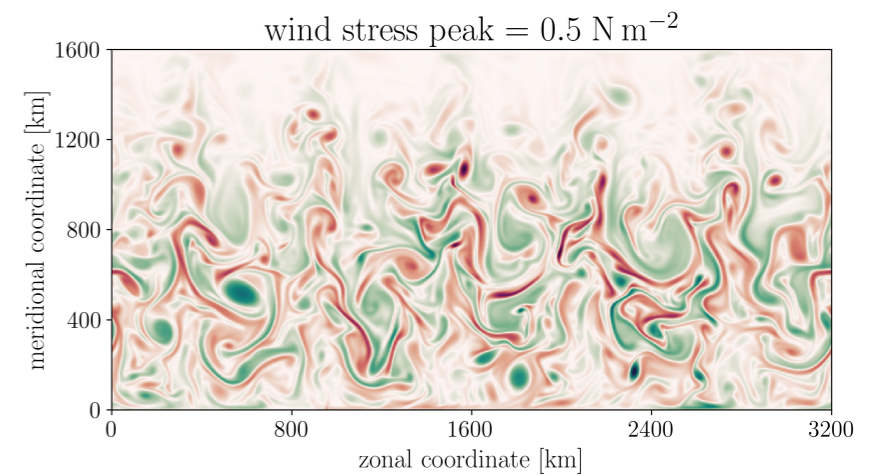
flow structure for 2-layer configuration



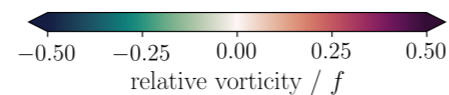
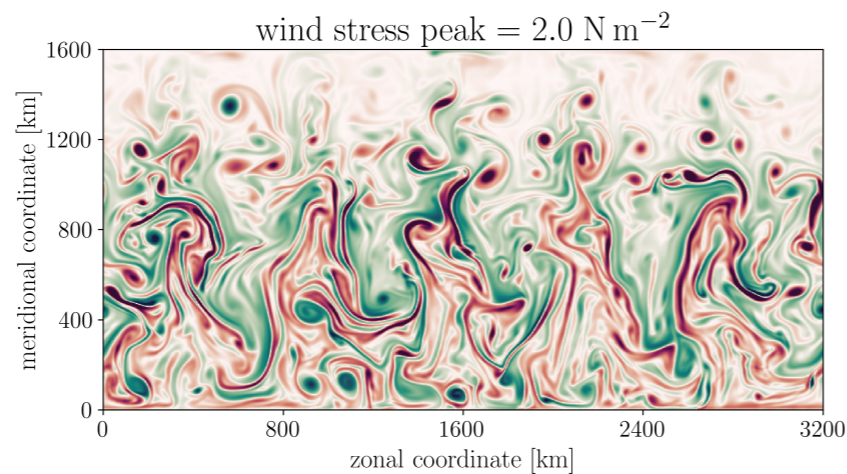
baroclinic eddies
(~200yr spinup)



baroclinic eddies
(~50yr spinup)



flow starts "seeing"
the topography

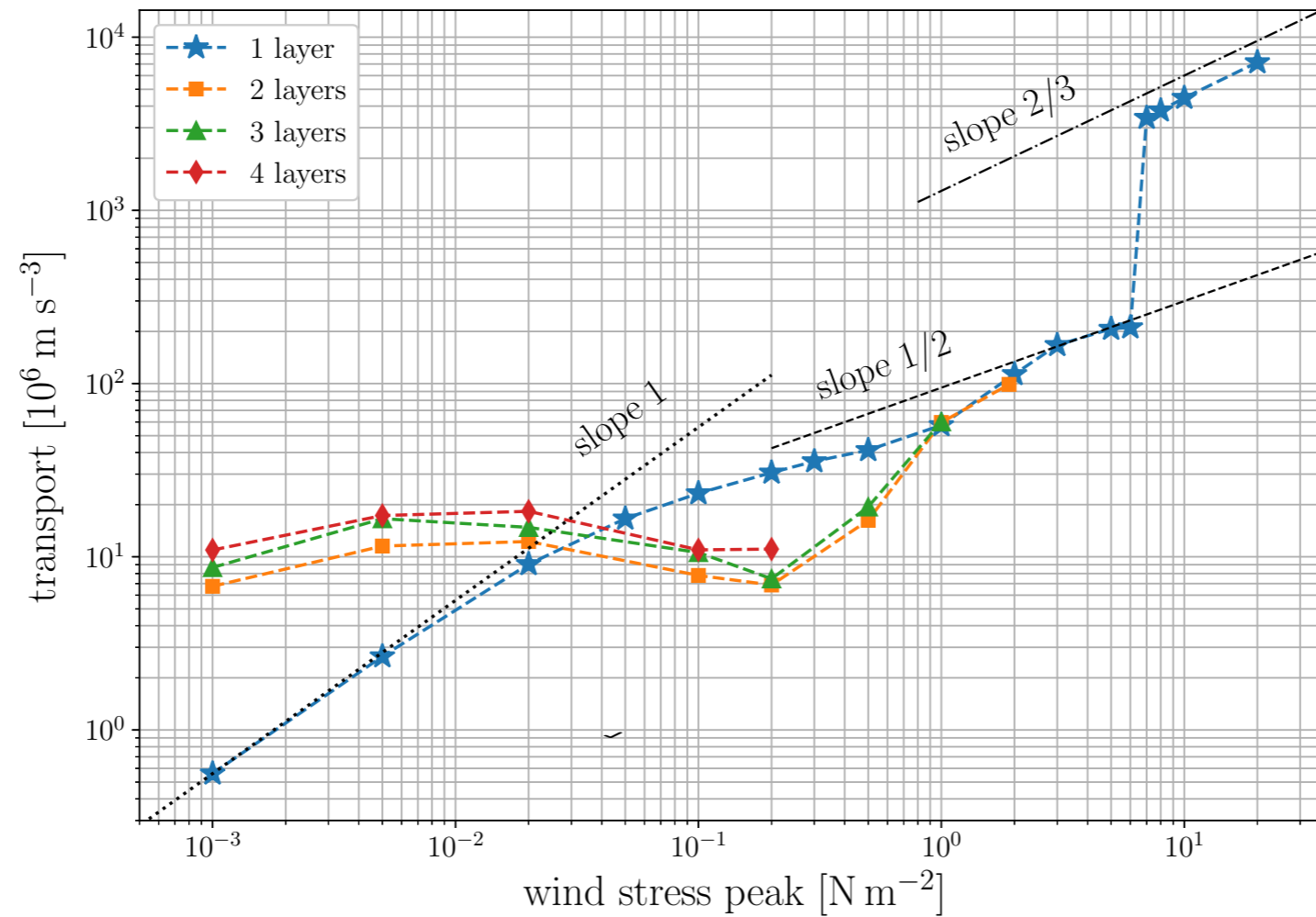


**For higher wind stress values
code blows up
(blow-up related with outcropping)**

Help! Anybody knows MOM6 here? :)

how does the transport vary with wind stress
in this **primitive-equations** model?

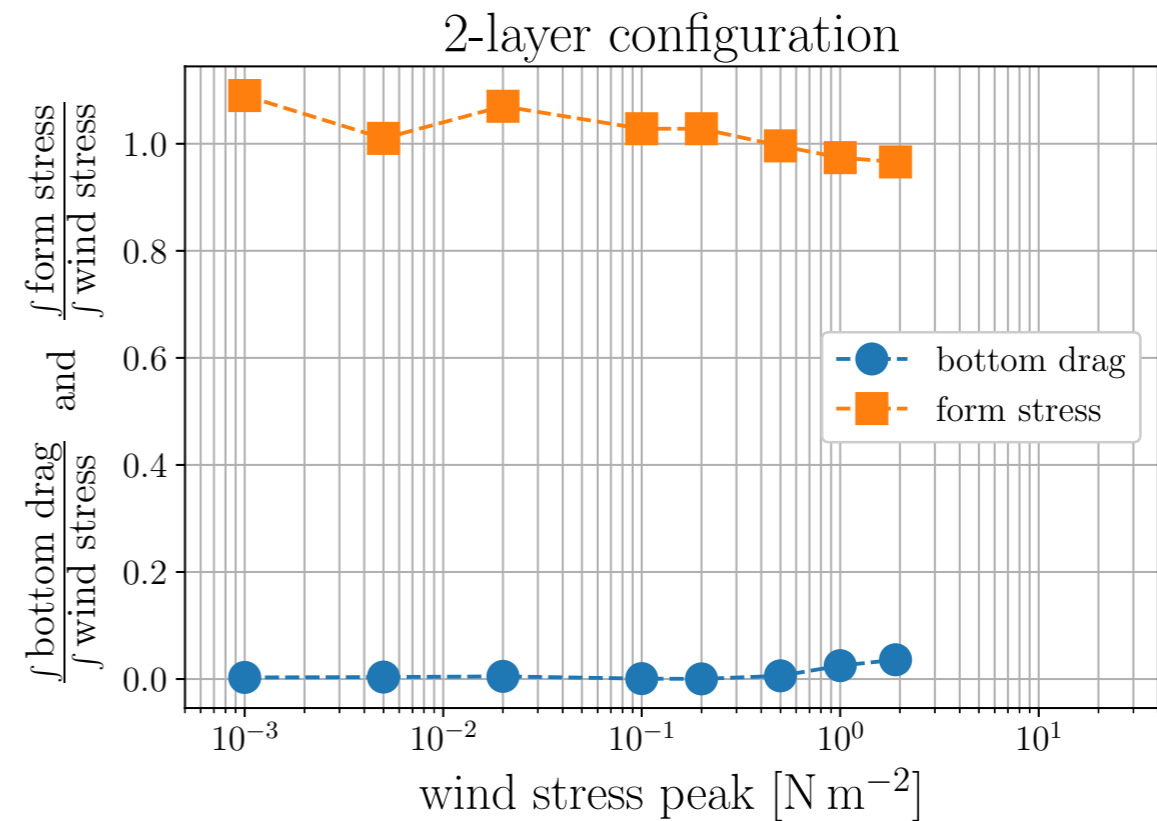
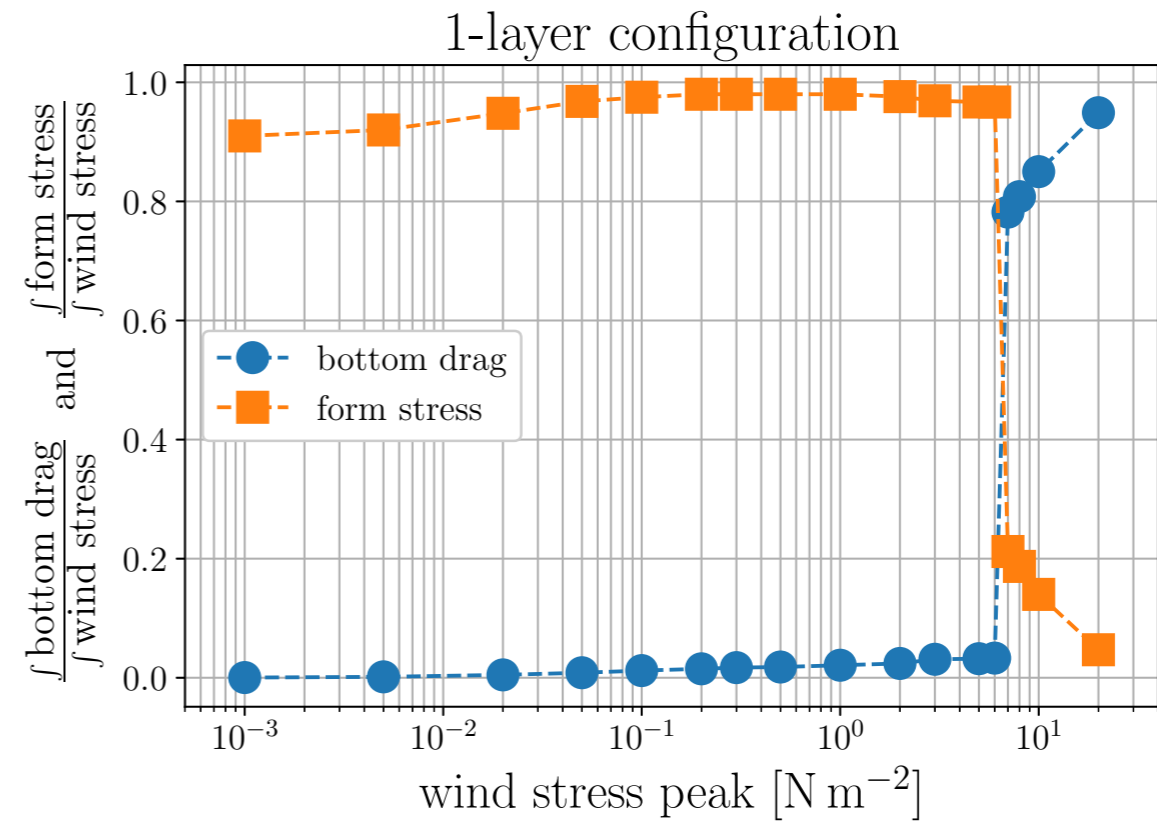
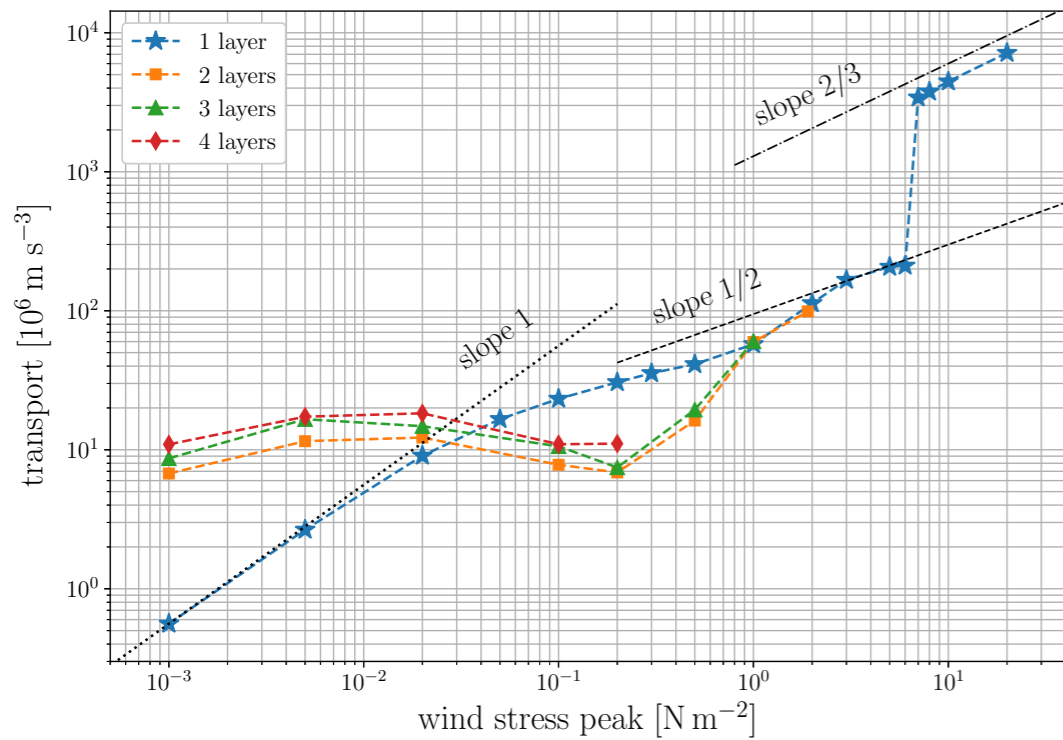
transport Vs wind stress



Baroclinic cases show strong eddy saturation.

The single-layer case **also** shows insensitivity to wind stress (transport grows only about 10-fold over 100-fold wind stress increase)

how is the momentum balanced?



conclusions

This **barotropic** QG model shows eddy saturation.
This is surprising! All previous arguments were based on **baroclinicity**.

The **barotropic**—topographic instability is able to produce transient eddies in this model in a similar manner as **baroclinic** instability.

Barotropic eddy saturation "survives" in a primitive-equations multilayer channel model.

Is there a similar flow-transition bifurcation in baroclinic dynamics as in barotropic dynamics?

(can you help me with MOM6 blowups?)

conclusions

This **barotropic** QG model shows eddy saturation.
This is surprising! All previous arguments were based on **baroclinicity**.

The **barotropic**—topographic instability is able to produce transient eddies in this model in a similar manner as **baroclinic** instability.

Barotropic eddy saturation "survives" in a primitive-equations multilayer channel model.

Is there a similar flow-transition bifurcation in baroclinic dynamics as in barotropic dynamics?

(can you help me with MOM6 blowups?)

This changes the way we view eddy saturation and highlights the role of topographically-induced eddies.

thank you

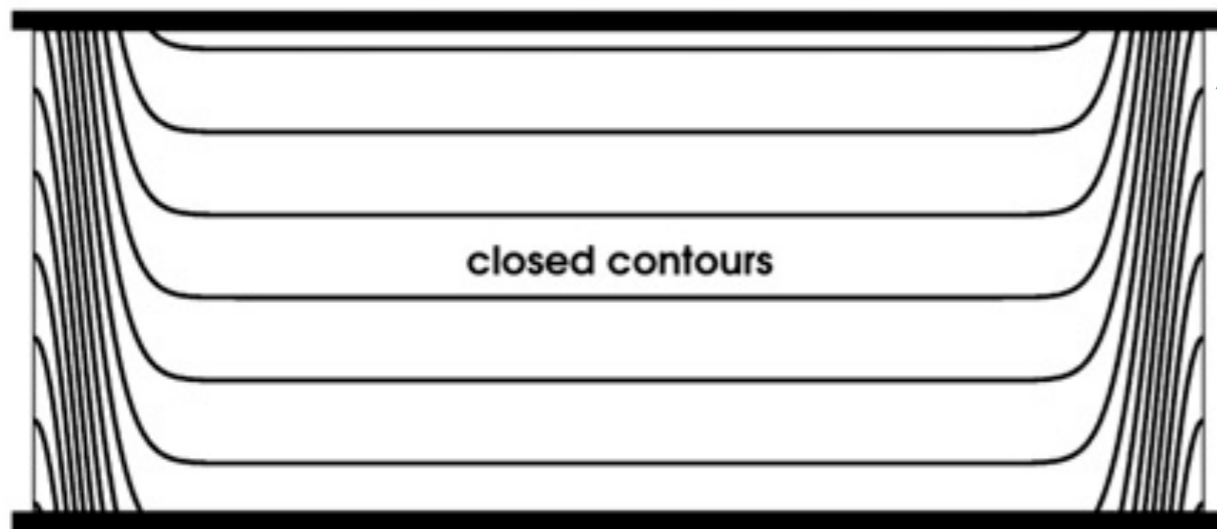
Constantinou and Young (2017). Beta-plane turbulence above monoscale topography. *J. Fluid Mech.*, **827**, 415-447.
Constantinou (2018). A barotropic model of eddy saturation. *J. Phys. Oceanogr.* **48 (2)**, 397-411.

extra slides

characterizing geostrophic contours

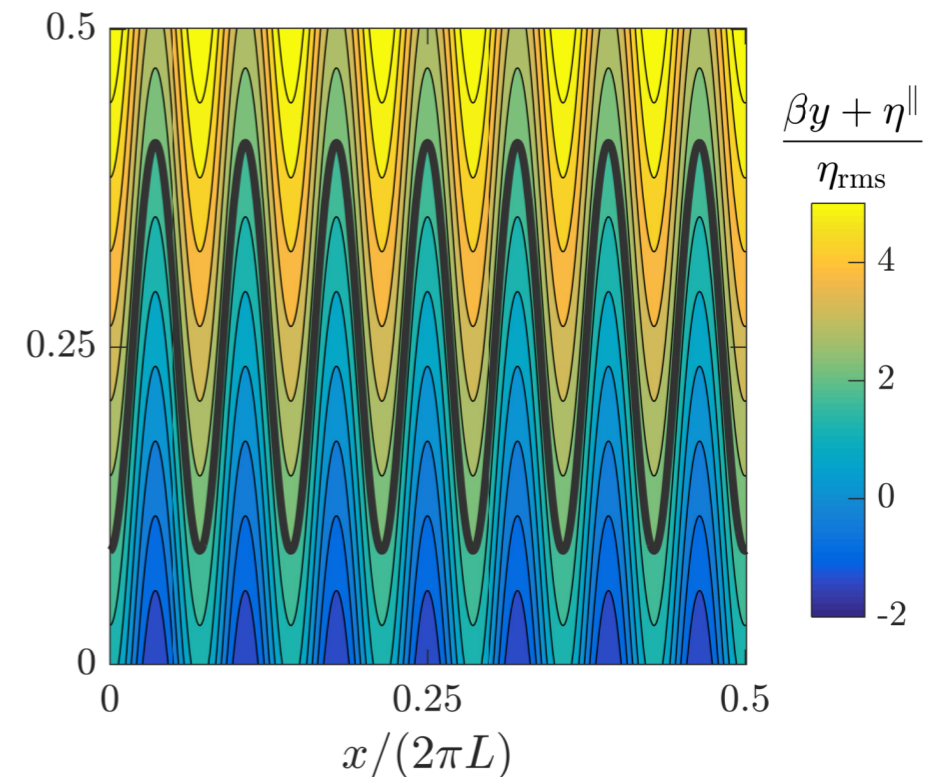
$$\beta y + \eta(x, y)$$

closed/blocked geostrophic contours



Nadeau & Ferrari 2015

open geostrophic contours



Constantinou 2017
Constantinou & Young 2017

without channel walls
both geostrophic contours look alike

decomposing the ACC transport

the time-mean zonal flow:

$$\bar{u}(x, y, z) = \underbrace{\bar{u}(x, y, z) - \bar{u}_{\text{bot}}(x, y)}_{\stackrel{\text{def}}{=} \bar{u}_{\text{tw}}(x, y, z)} + \bar{u}_{\text{bot}}(x, y)$$

“thermal wind” flow

bottom flow

$$\partial_z \bar{u} = -\partial_y \bar{b}$$

$$\underbrace{\int_{-H}^0 dz \int dy \int \frac{dx}{L_x} \bar{u}}_{\stackrel{\text{def}}{=} T_{\text{ACC}}} = \underbrace{\int dy \int \frac{dx}{L_x} \bar{u}_{\text{bot}}}_{\stackrel{\text{def}}{=} T_{\text{bot}}} + \underbrace{\int_{-H}^0 dz \int dy \int \frac{dx}{L_x} \bar{u}_{\text{tw}}}_{\stackrel{\text{def}}{=} T_{\text{tw}}}$$

total
transport

bottom

“thermal wind”

not included in the
barotropic QG model

