# Understanding coherent structure emergence in homogeneously forced turbulence by means of the statistical state dynamics


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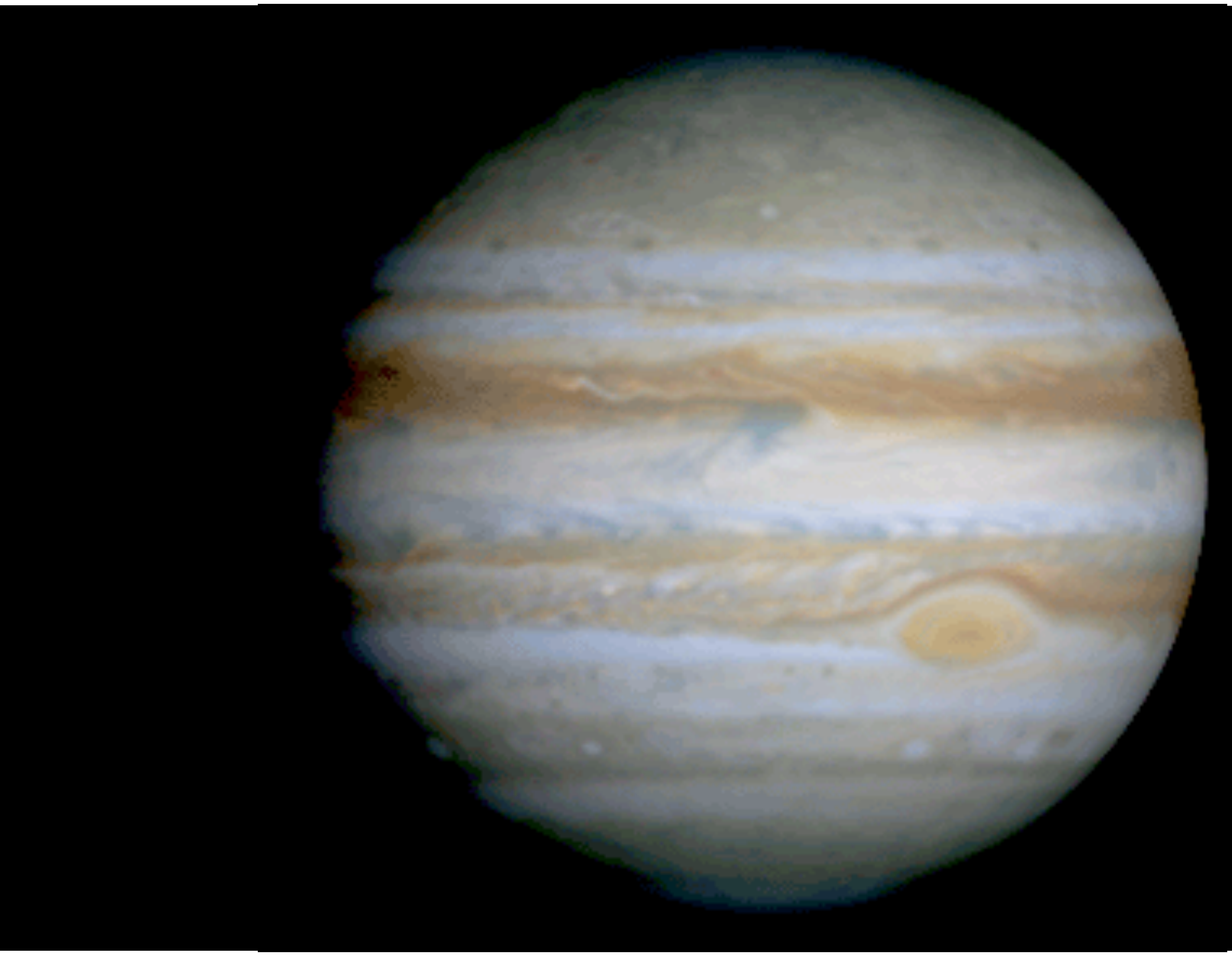
KITP
11 Jan. 2017

# Planetary turbulence

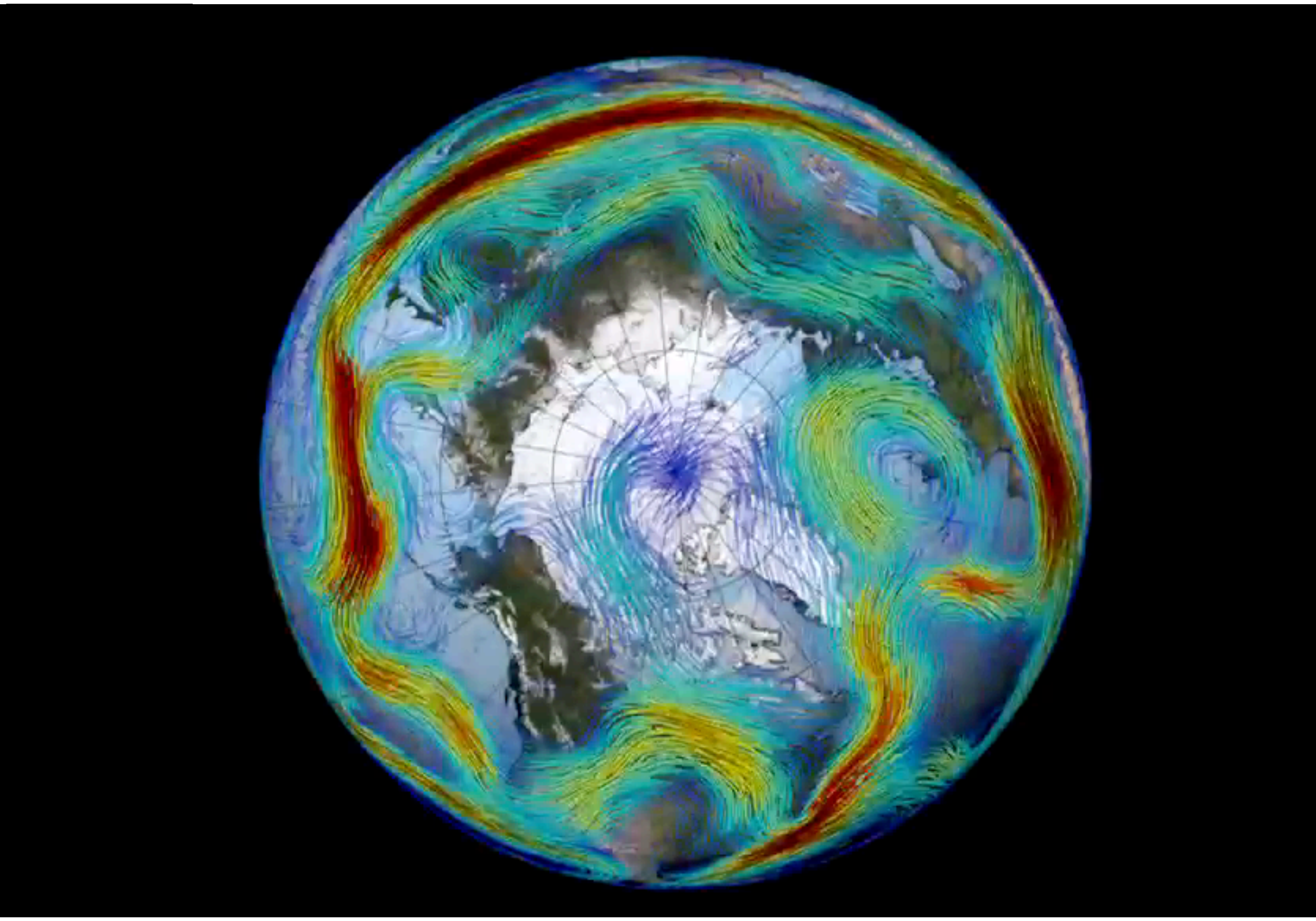
most of the energy of the flow is in large-scale coherent jets and vortices of specific form

not at the largest allowed scale (as inverse cascade might imply)
arrest of the cascade by jets



banded Jovian jets

NASA/Cassini Jupiter Images



polar front jet

NASA/Goddard Space Flight Center

# Boundary layer turbulence

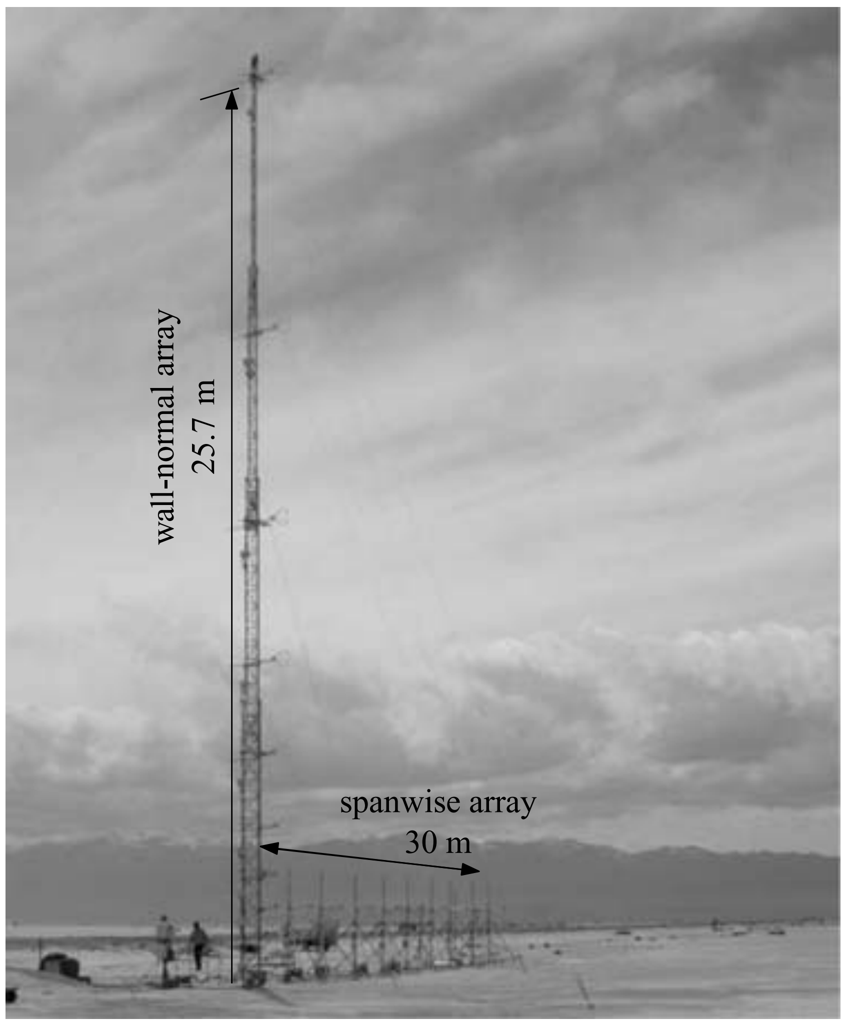
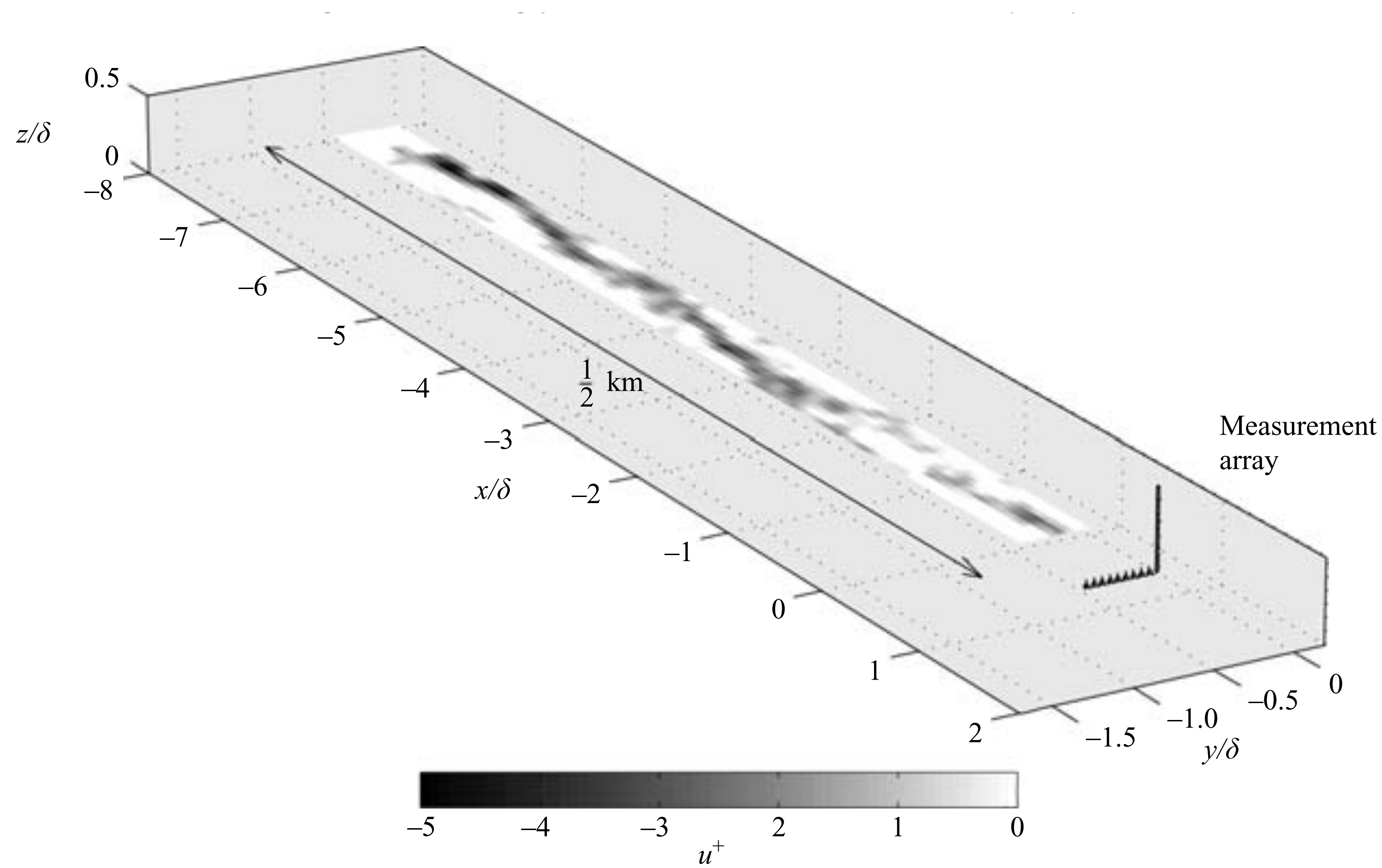
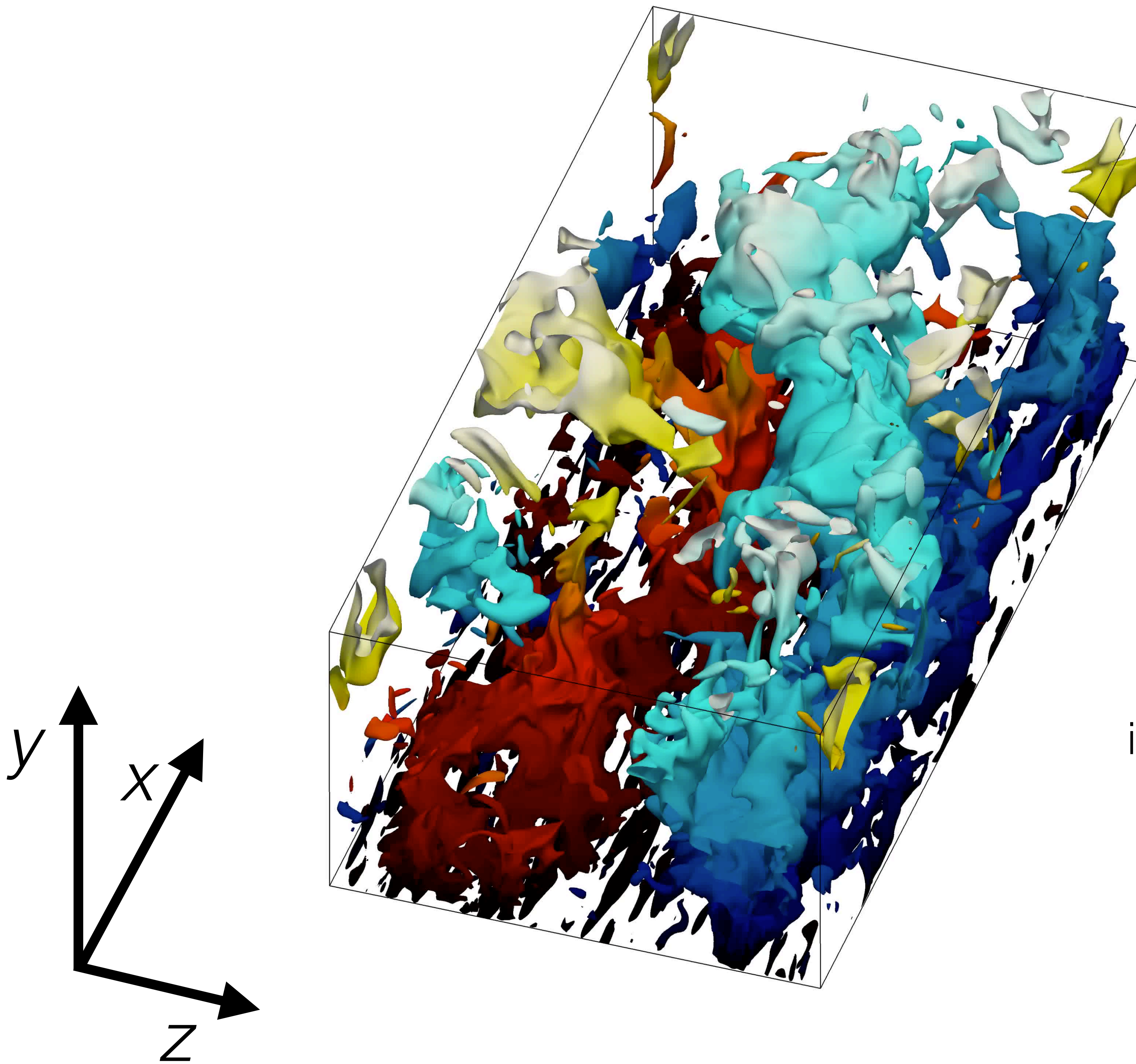



FIGURE 12. View of the measurement array installed at the SLTEST site.




Hutchins & Marusic 2007

# Wall-bounded turbulence




high and low speed
streak isocontours
in Poiseuille turbulence
at $Re_\tau = 950$

Credit: A Lozano-Durán

The problem to be addressed:

Understand how these *specific* structures arise and how are they maintained

# Claims

**I.** The underlying dynamics of structure formation lies in the interaction of turbulent eddies with mean flows

**II.** Often, structure formation has analytic expression *only* in the Statistical State Dynamics (SSD/DSS)
(the dynamics that govern the statistics of the flow rather than the dynamics governing single flow realizations)

**III.** Because of (**I**) a second-order closure of the SSD is adequate

# Statistical State Dynamics (SSD)

**1.** split the flow variables into: ⟨mean⟩ + eddy ′

$$\boldsymbol{u}(\boldsymbol{x},t) \ = \ \langle \boldsymbol{u}(\boldsymbol{x},t)\rangle \ + \ \boldsymbol{u}'(\boldsymbol{x},t)$$

**2.** form the hierarchy of same-time statistical moments/cumulants

$$\underbrace{\langle \boldsymbol{u}(\boldsymbol{x}_a,t)\rangle}_{=C^{(1)}_a} \ , \ \underbrace{\langle \boldsymbol{u}'(\boldsymbol{x}_a,t)\boldsymbol{u}'(\boldsymbol{x}_b,t)\rangle}_{=C^{(2)}_{ab}} \ , \ \underbrace{\langle \boldsymbol{u}'(\boldsymbol{x}_a,t)\boldsymbol{u}'(\boldsymbol{x}_b,t)\boldsymbol{u}'(\boldsymbol{x}_c,t)\rangle}_{=C^{(3)}_{abc}} \ , \ \dots$$

**3.** find how each one of the moments/cumulants evolve

$$\partial_t C^{(1)}_a = \mathcal{F}_1\left(C^{(1)}_a \ , \ C^{(2)}_{ab}\right)$$

$$\partial_t C^{(2)}_{ab} = \mathcal{F}_2\left(C^{(1)}_a \ , \ C^{(2)}_{ab} \ , \ C^{(3)}_{abc}\right)$$

$$\partial_t C^{(3)}_{abc} = \mathcal{F}_3\left(C^{(1)}_a \ , \ C^{(2)}_{ab} \ , \ C^{(3)}_{abc} \ , \ C^{(4)}_{abcd}\right) \quad , \text{etc} \dots$$

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**4. S3T/CE2**: closure at second-order

# Remarks on SSD — What is novel here?

$$\partial_t C_a^{(1)} = \mathcal{F}_1 \left( C_a^{(1)} \ , \ C_{ab}^{(2)} \right)$$

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# Remarks on SSD — What is novel here?

Usually (inspired by homogeneous isotropic turbulence) people took $\langle \boldsymbol{u}(\boldsymbol{x},t)\rangle = 0$

$$\partial_t C^{(1)}_{a} = \mathcal{F}_1\left(C^{(1)}_{a}\ ,\ C^{(2)}_{ab}\right)$$

but this is fundamental for structure formation (claim (**I**))

$$\partial_t C^{(2)}_{ab} = \mathcal{F}_2\left(C^{(1)}_{a}\ ,\ C^{(2)}_{ab}\ ,\ C^{(3)}_{abc}\right)$$

$$\partial_t C^{(3)}_{abc} = \mathcal{F}_3\left(C^{(1)}_{a}\ ,\ C^{(2)}_{ab}\ ,\ C^{(3)}_{abc}\ ,\ C^{(4)}_{abcd}\right) \quad \text{, etc ...}$$

# Remarks on SSD — What is novel here?

Usually (inspired by homogeneous isotropic turbulence) people took $\langle \boldsymbol{u}(\boldsymbol{x}, t) \rangle \;=\; 0$

Main effort/interest was to obtain the equilibrium statistics: $\partial_t = 0$

$$\partial_t C_a^{(1)} = \mathcal{F}_1 \left( C_a^{(1)} \;,\; C_{ab}^{(2)} \right)$$

but this is fundamental for structure formation (claim (**I**))

$$0 \;= \mathcal{F}_2 \left( C_a^{(1)} \;,\; C_{ab}^{(2)} \;,\; C_{abc}^{(3)} \right)$$

$$0 \;= \mathcal{F}_3 \left( C_a^{(1)} \;,\; C_{ab}^{(2)} \;,\; C_{abc}^{(3)} \;,\; C_{abcd}^{(4)} \right) \quad , \text{etc} \;...$$

# Remarks on SSD — What is novel here?

$$\partial_t C_a^{(1)} = \mathcal{F}_1\left(C_a^{(1)}\ ,\ C_{ab}^{(2)}\right)$$
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By studying the *dynamics* of the statistics new phenomena arise that are either not present or are obscured in single flow realizations

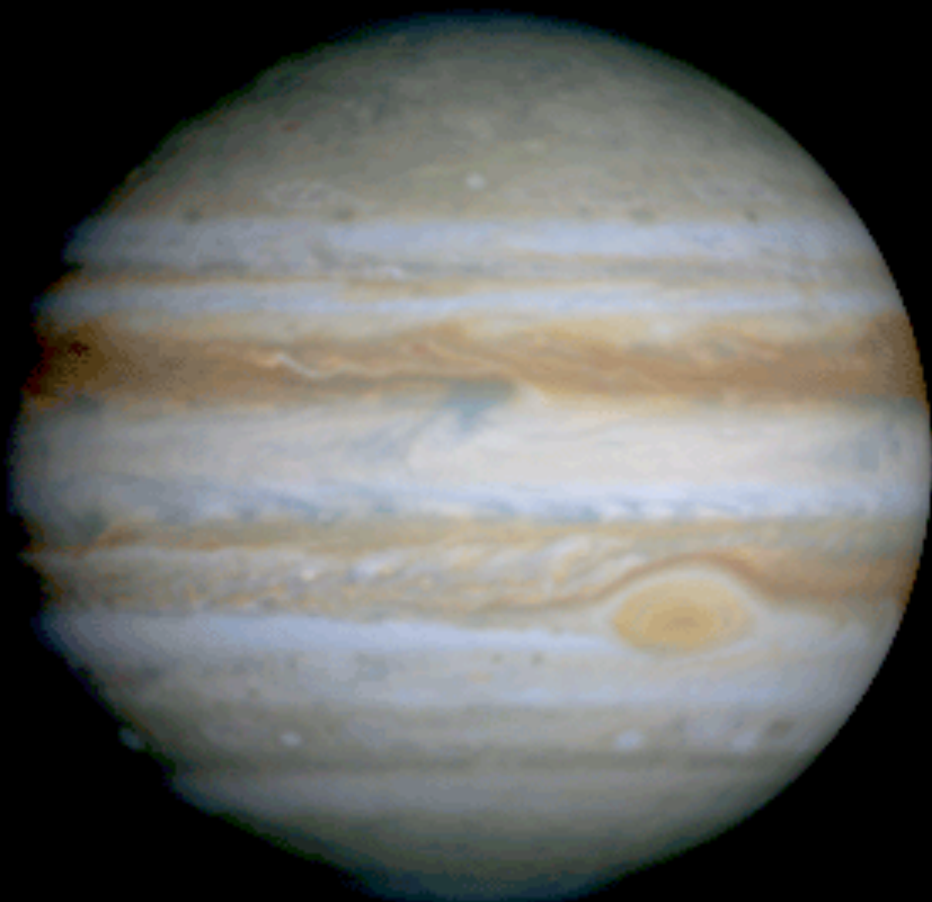
I will show that within the framework of SSD we understand:

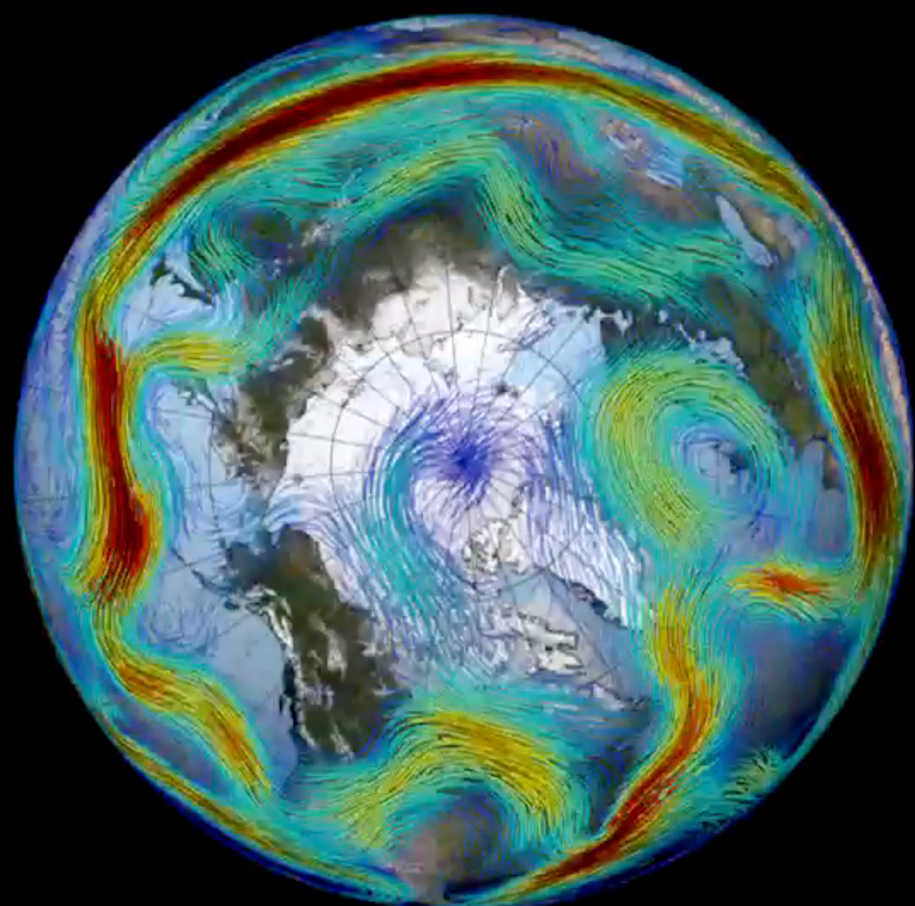
**A.** Jet/large-scale wave emerge in planetary turbulence
as an instability of the SSD
(this shows that SSD capture the mechanism)

**B.** Roll/streak structures
in pre-transitional free-stream Couette turbulence
emerge as an instability of the SSD

**A.**

# Jet/Large-scale wave emergence in planetary turbulence





Farrell & Ioannou 2003, 2007; Srinivasan & Young 2012; Tobias & Marston 2013; NCC, Farrell & Ioannou 2014, 2016
Bakas, NCC & Ioannou 2015, Bakas & Ioannou 2013, 2014; Parker & Krommes 2013, 2014, Marston, Tobias, Chini, 2016;
Ait-Chaalal, Schneider, Meyer, Marston 2016

# barotropic vorticity equation on a $\beta$-plane

$$\partial_t \zeta + \boldsymbol{u} \cdot \boldsymbol{\nabla}(\zeta + \beta y) = -r\zeta + \sqrt{\varepsilon}\xi$$

linear dissipation at rate *r*

stochastic forcing

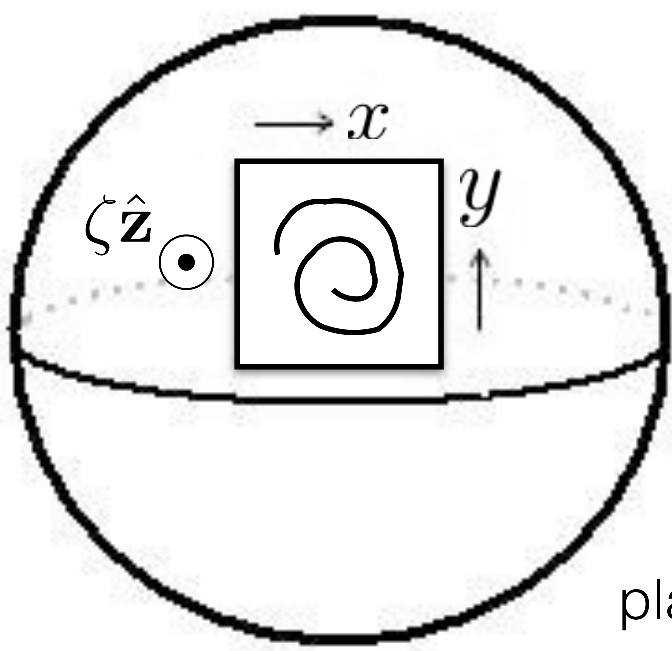
$$\boldsymbol{\nabla} \cdot \boldsymbol{u} = 0$$

$$\boldsymbol{u} = (u, v) = (-\partial_y \psi, \partial_x \psi)$$

$$\zeta = (\boldsymbol{\nabla} \times \boldsymbol{u}) \cdot \hat{\mathbf{z}} = \Delta\psi$$

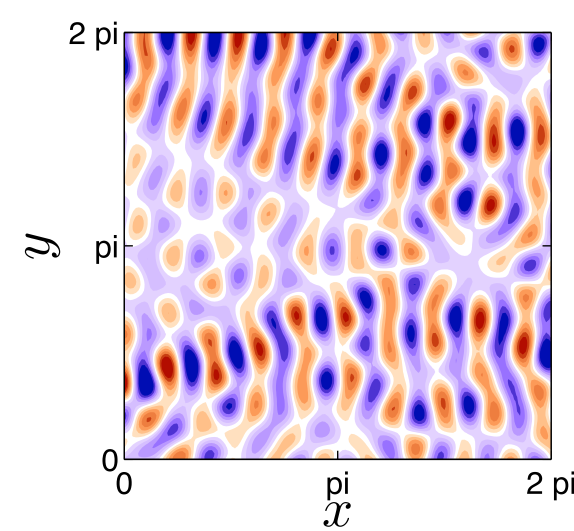
zero mean
white in time
&
statistically homogeneous

$$\langle \xi(\boldsymbol{x}_a, t_a)\xi(\boldsymbol{x}_b, t_b)\rangle = Q(\boldsymbol{x}_a - \boldsymbol{x}_b)\,\delta(t_a - t_b)$$




$\beta$: gradient of planetary vorticity

$\xi(\boldsymbol{x}, t)$




anisotropic Earth-like forcing modeling energy injected to the barotropic mode by baroclinic instability

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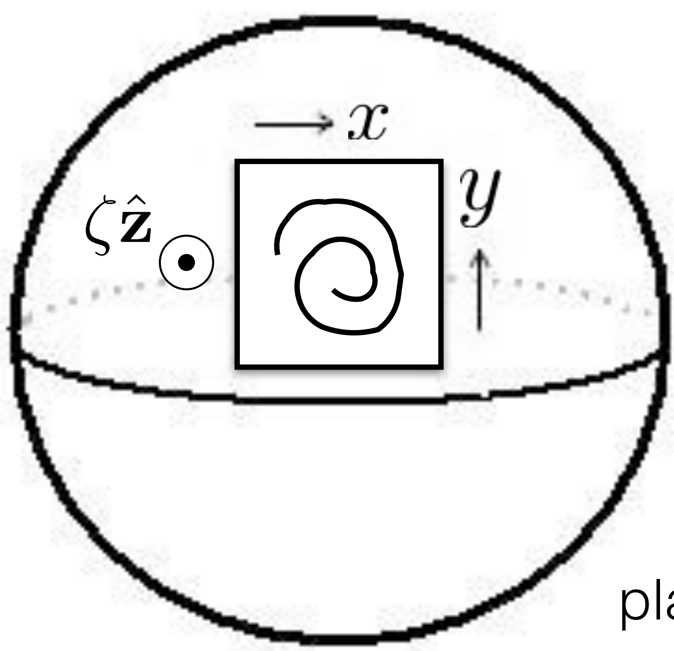


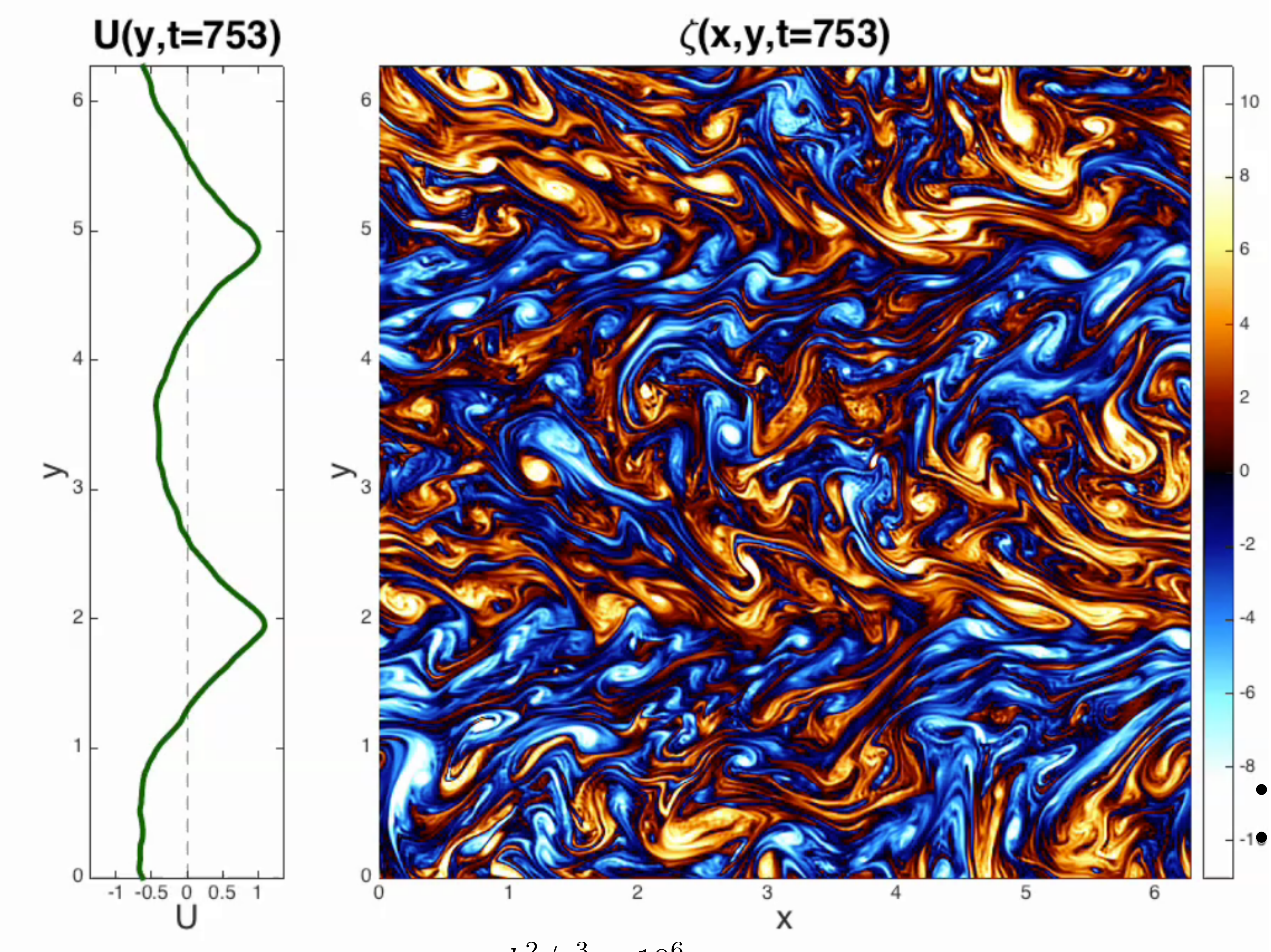

*β:* gradient of planetary vorticity

two non-dimensional parameters

$\varepsilon k_f^2 / r^3$

$\beta / (k_f r)$

# barotropic $\beta$-plane turbulence exhibits large-scale structure formation




$$\varepsilon k_f^2/r^3 = 10^6$$

$$\beta/(k_f r) = 67$$

http://www.youtube.com/watch?v=ZtzW25NooDk

statistically homogeneous forcing

(no inhomogeneity is imposed by the forcing)

any random flow inhomogeneities organize the turbulence in a manner so that they are reinforced

we observe:

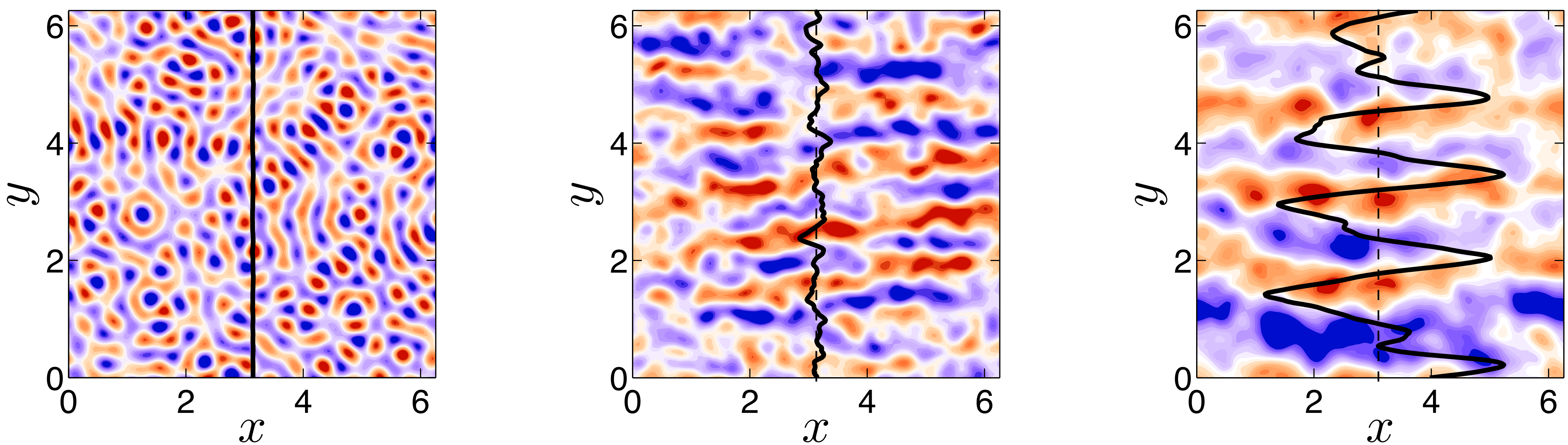
- jet emerge
- jets appear to change much slower compared to the eddies
- jet have a particular structure

# various $\beta$-plane turbulence flows at statistically steady state:

## homogeneous — traveling waves — zonal jets

$\beta/(k_f r) = 67$

$\varepsilon k_f^2/r^3 \ = 10^2$ $\qquad 5 \times 10^3 \qquad 5 \times 10^4$




this suggests that there is some kind of transition as $\varepsilon$ is increased

[ snapshots of the streamfunction $\psi(\mathbf{x},t)$ with instantaneous zonal mean zonal flow $U(y,t)$ ]

# S3T closure of SSD

take the $\langle$mean$\rangle$ as a zonal mean
under the ergodic assumption that

$\langle$mean$\rangle$ = ensemble average over forcing realizations

$$Z(\mathbf{x}, t) = \langle \zeta(\mathbf{x}, t) \rangle \quad , \qquad C^{(2)}_{ab} = \langle \zeta'(\mathbf{x}_a, t) \zeta'(\mathbf{x}_b, t) \rangle$$

1st cumulant

2nd cumulant

# S3T closure of SSD

$$\partial_t Z + \mathbf{U}\cdot\boldsymbol{\nabla}(Z+\beta y) = \mathcal{R}(C^{(2)}_{ab}) - rZ$$

$$\partial_t C^{(2)}_{ab} = \left[\mathcal{A}(\mathbf{U})_a + \mathcal{A}(\mathbf{U})_b\right] C^{(2)}_{ab} + \varepsilon Q_{ab}$$

with

$$\mathbf{U} \stackrel{\text{def}}{=} (-\partial_y, \partial_x)\Delta^{-1} Z$$

$$C^{(2)}_{ab} \stackrel{\text{def}}{=} \langle \zeta'(\mathbf{x}_a, t)\zeta'(\mathbf{x}_b, t)\rangle$$

$$Q_{ab} \stackrel{\text{def}}{=} Q(\mathbf{x}_a - \mathbf{x}_b)$$ ⟶ the spatial covariance of the statistically homogeneous stochastic forcing

$$\mathcal{R}(C^{(2)}_{ab}) \stackrel{\text{def}}{=} -\langle \boldsymbol{u}'\cdot\boldsymbol{\nabla}\zeta'\rangle = \boldsymbol{\nabla}\cdot\left[\frac{\hat{\mathbf{z}}}{2}\times\left(\boldsymbol{\nabla}_a\Delta_a^{-1} + \boldsymbol{\nabla}_b\Delta_b^{-1}\right)C^{(2)}_{ab}\right]_{a=b}$$

(the Reynolds stresses are given as a linear function of *C*)

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$$\partial_t Z + \mathbf{U} \cdot \boldsymbol{\nabla}(Z + \beta y) = \mathcal{R}(C^{(2)}_{ab}) - rZ$$

$$\partial_t C^{(2)}_{ab} = [\mathcal{A}(\mathbf{U})_a + \mathcal{A}(\mathbf{U})_b]\, C^{(2)}_{ab} + \varepsilon Q_{ab}$$

neglect of third cumulant
is *equivalent* with
neglect of the eddy—eddy term in eddy equation in the EOM

(⟶ PainInNeck-term Tobias was talking about on Monday)

Note: The dynamics of the 1st & 2nd cumulants is necessarily quasi-linear (Herring 1963)

# S3T closure of SSD

$$\partial_t Z + \mathbf{U} \cdot \boldsymbol{\nabla}(Z + \beta y) = \mathcal{R}(C^{(2)}_{ab}) - rZ$$

$$\partial_t C^{(2)}_{ab} = \left[\mathcal{A}(\mathbf{U})_a + \mathcal{A}(\mathbf{U})_b\right] C^{(2)}_{ab} + \varepsilon Q_{ab}$$

The S3T system

- nonlinear
- autonomous, deterministic (central limit theorem)
- admits fixed point solutions $\left(\mathbf{U}^e(\mathbf{x}), C^e(\mathbf{x}_a, \mathbf{x}_b)\right)$
- associated perturbation equations used to determine stability of these fixed points

# S3T equilibria for homogeneous forcing

$$\mathbf{U}^e = 0 \quad , \quad C^e(\mathbf{x}_a - \mathbf{x}_b) = \frac{\varepsilon\, Q}{2r}$$

(for any $\varepsilon$, $\beta$ and homogeneous $Q$)

zero mean flow + non-zero second-order eddy statistics

$$\mathbf{U}^e(\mathbf{x}) = \Big(U^e(y), 0\Big) \quad , \quad C^e(x_a - x_b, y_a, y_b)$$

zonal jet mean flow + non-zero zonally homogeneous 2nd-order eddy statistics

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zonal jet mean flow + non-zero zonally homogeneous 2nd-order eddy statistics

Perturbations about these equilibria are governed by:

hydrodynamic stability

$$\partial_t \delta Z = \mathcal{A}(\mathbf{U}^e)\,\delta Z + \mathcal{R}(\delta C)$$

we linearized about a turbulent state!

$$\partial_t \delta C_{ab} = [\mathcal{A}_a(\mathbf{U}^e) + \mathcal{A}_b(\mathbf{U}^e)]\,\delta C_{ab} + (\delta\mathcal{A}_a + \delta\mathcal{A}_b) C^e_{ab}$$

$$\delta\mathcal{A} \overset{\text{def}}{=} \mathcal{A}(\mathbf{U}^e + \delta\mathbf{U}) - \mathcal{A}(\mathbf{U}^e)$$

eigenanalysis of this system determines the stability of $\Big(\mathbf{U}^e(\mathbf{x}), C^e(\mathbf{x}_a, \mathbf{x}_b)\Big)$

# Consider the homogeneous turbulent equilibrium:

$$\mathbf{U}^e = 0 \quad , \quad C^e(\mathbf{x}_a - \mathbf{x}_b) = \frac{\varepsilon\, Q}{2r}$$

(for any $\varepsilon$, $\beta$ and homogeneous $Q$)

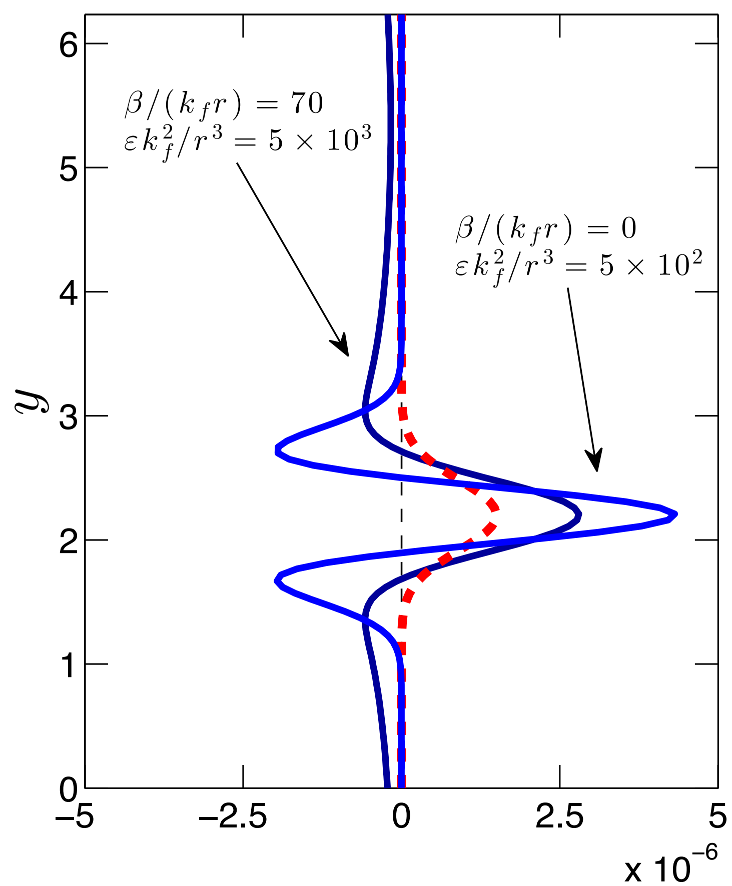
zero mean flow + non-zero second-order eddy statistics

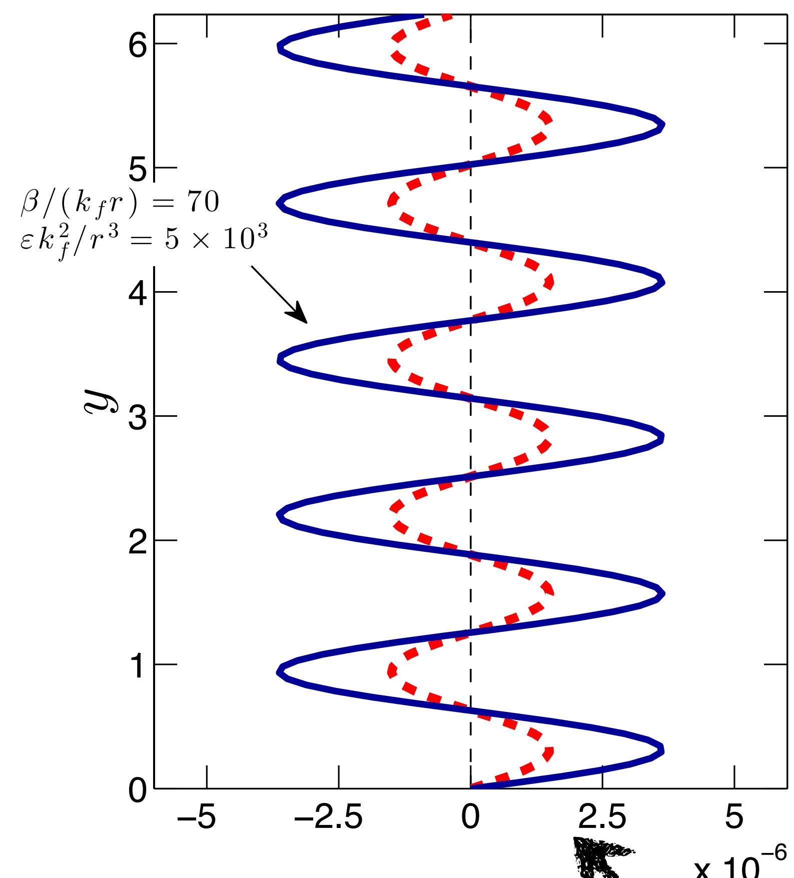
How does the state with *no mean flow* becomes unstable?

# proof of concept

An infinitesimal mean flow $\delta U$ distorts the turbulence in a manner so as to produce Reynolds stresses R($\delta C$) that reinforce the $\delta U$ itself

$$\partial_t \delta Z = \mathcal{A}(\ 0\ )\, \delta Z + \mathcal{R}(\delta C)$$



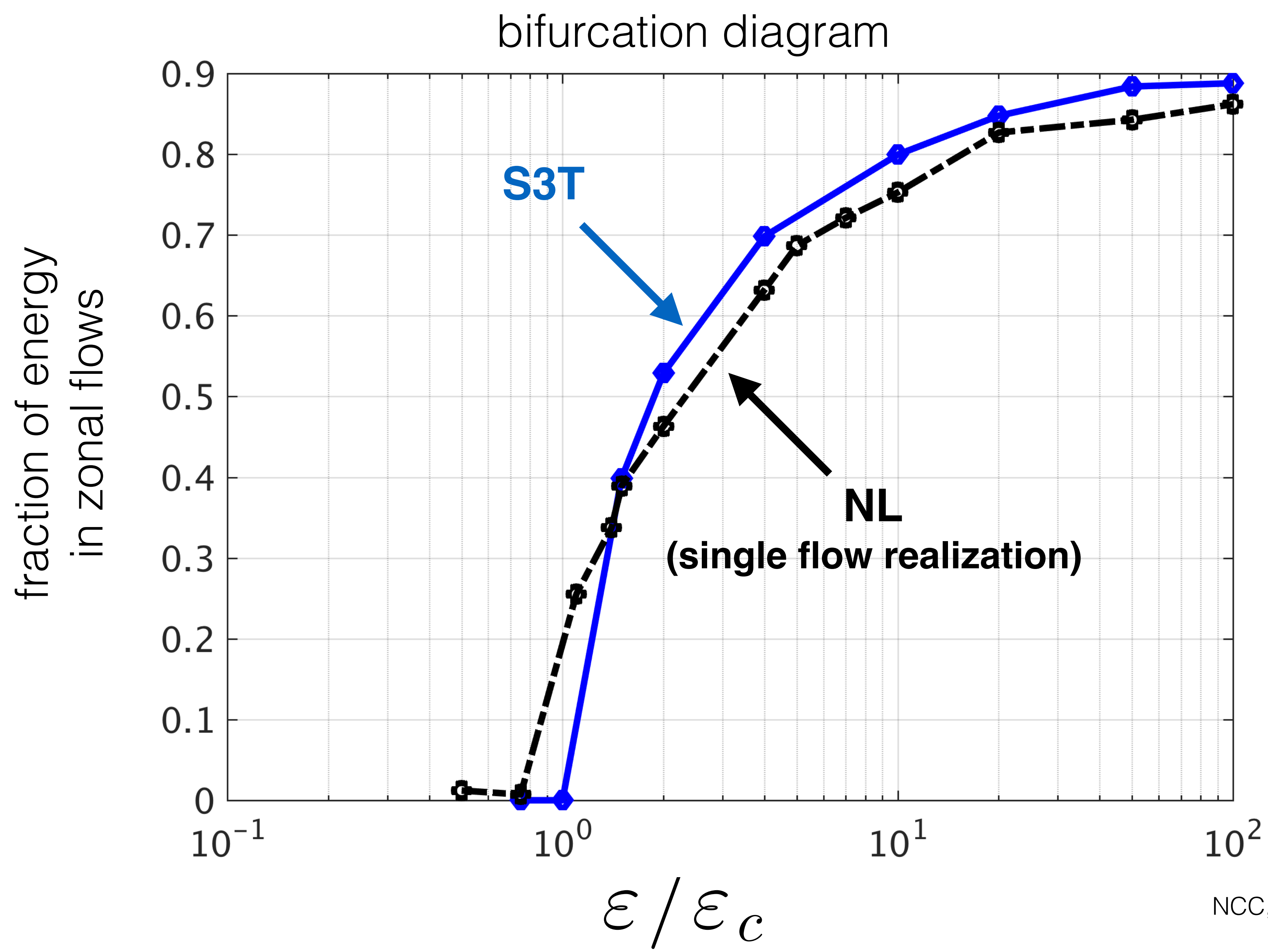




$$(--)\ \delta U \quad , \quad (—)\ \mathcal{R}(\delta C)$$

S3T eigenfunction

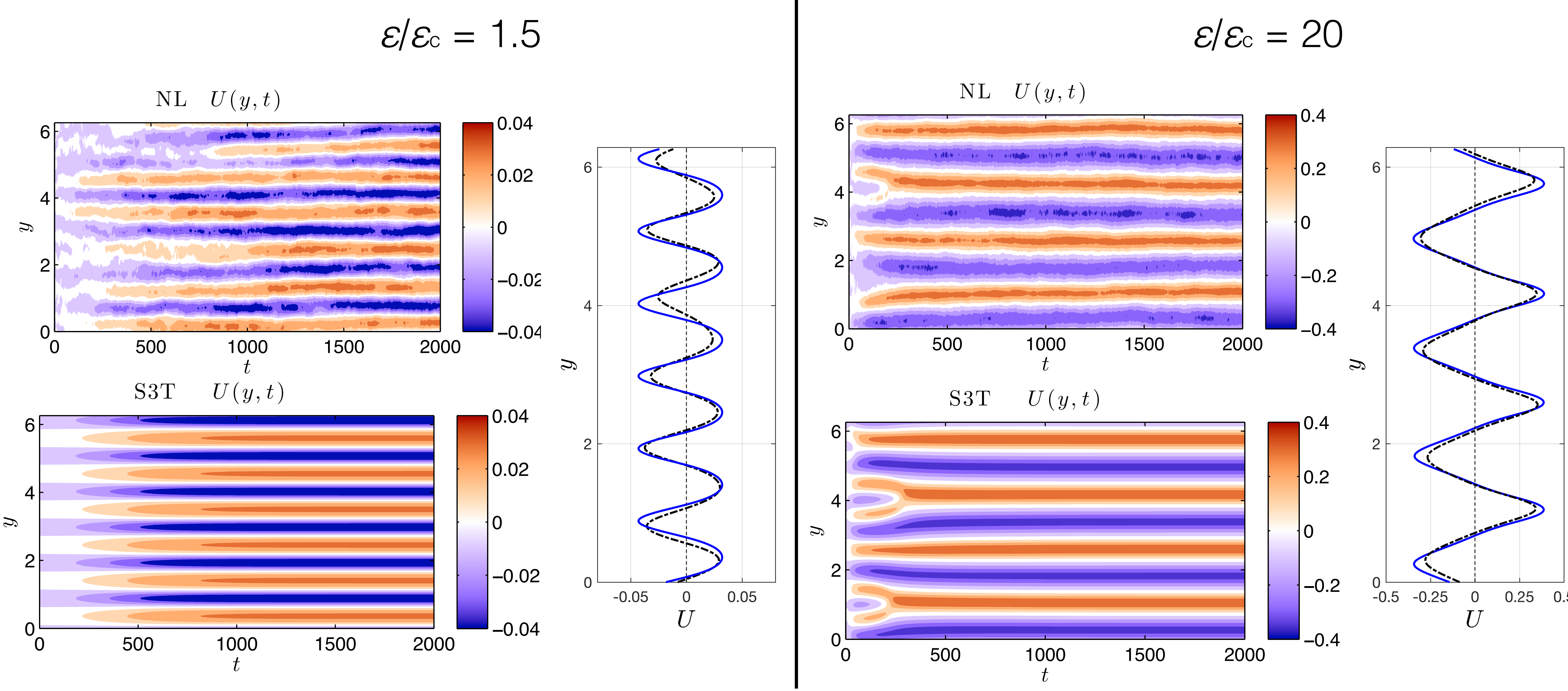
Farrell & Ioannou 2007
Srinivasan & Young 2012
NCC, Farrell & Ioannou 2014
Bakas, NCC & Ioannou 2015

# Verification of S3T predictions for the jet formation bifurcation




NCC, Farrell & Ioannou 2014

# Verification of the S3T predictions for the structure of the finite amplitude jet equilibria

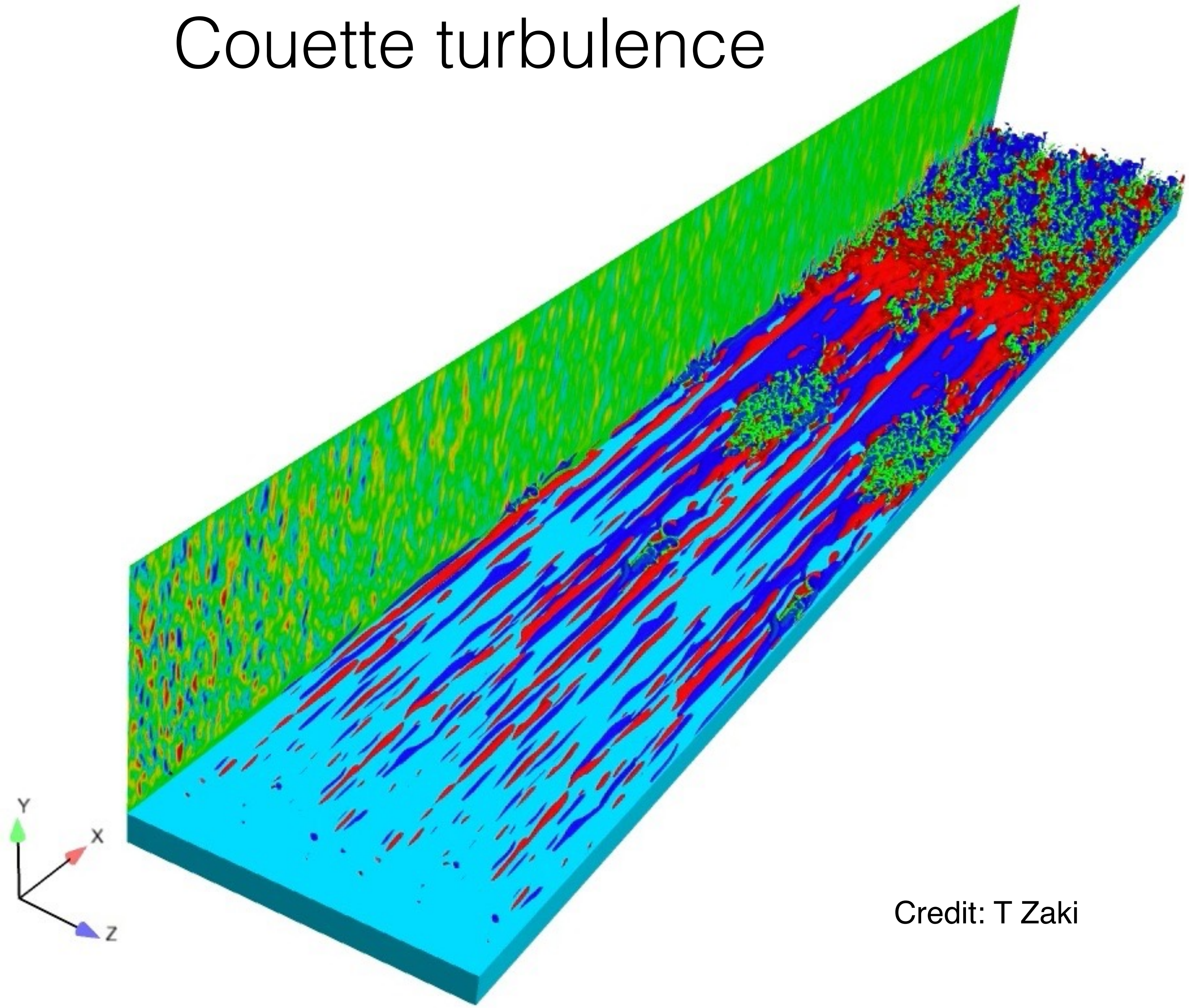



NCC, Farrell & Ioannou 2014

S3T instabilities grow and reach finite amplitude
to produce new inhomogeneous S3T equilibria

**B.**

# Roll/streak formation in pre-transitional free-stream Couette turbulence




Credit: T Zaki

Farrell, Ioannou & Nikolaidis (2016) Instability of the roll/streak structure induced by free-stream turbulence in pre-transitional Couette flow, *Phys. Rev. Fluids.* (sub judice, arXiv:1607.05018)

# roll/streak formation in free-stream Couette turbulence

flow = streamwise mean + perturbations

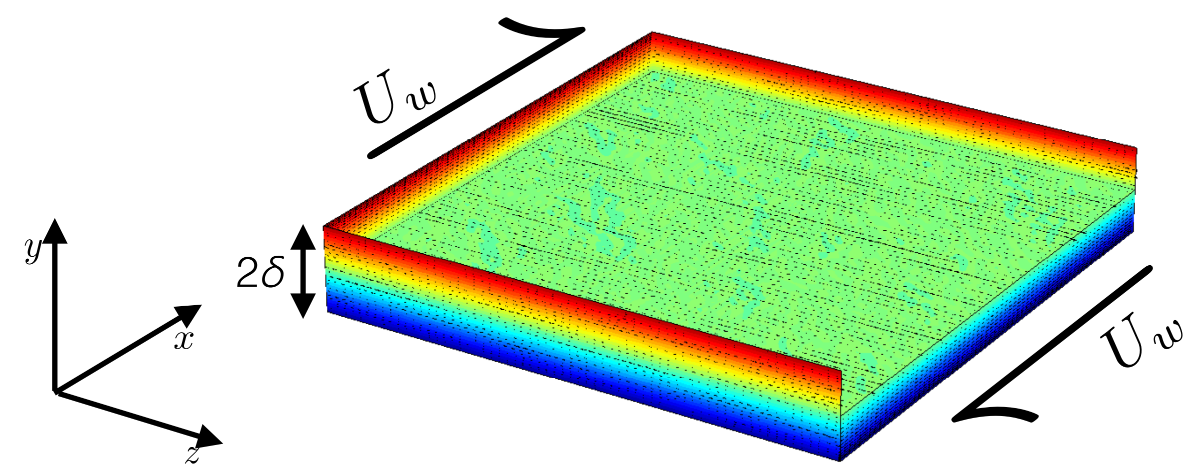
$$\boldsymbol{u} = \mathbf{U} + \boldsymbol{u}'$$

$$\partial_t \mathbf{U} + \mathbf{U}\cdot\boldsymbol{\nabla}\mathbf{U} + \boldsymbol{\nabla}P - \frac{1}{Re}\Delta\mathbf{U} = -\langle \boldsymbol{u}'\cdot\boldsymbol{\nabla}\boldsymbol{u}'\rangle$$

$$\partial_t \boldsymbol{u}' + \mathbf{U}\cdot\boldsymbol{\nabla}\boldsymbol{u}' + \boldsymbol{u}'\cdot\boldsymbol{\nabla}\mathbf{U} + \boldsymbol{\nabla}p' - \frac{1}{Re}\Delta\boldsymbol{u}' = -\left(\boldsymbol{u}'\cdot\boldsymbol{\nabla}\boldsymbol{u}' - \langle \boldsymbol{u}'\cdot\boldsymbol{\nabla}\boldsymbol{u}'\rangle\right) + \sqrt{\varepsilon}\boldsymbol{\xi}$$

$$\boldsymbol{\nabla}\cdot\mathbf{U} = \boldsymbol{\nabla}\cdot\boldsymbol{u}' = \boldsymbol{\nabla}\cdot\boldsymbol{\xi} = 0$$

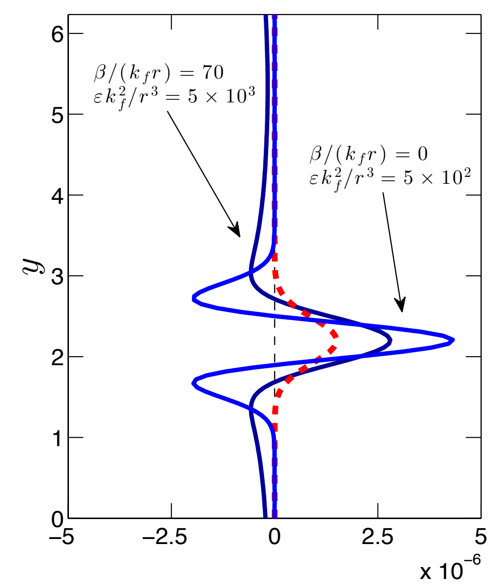
$$Re = \frac{U_w \delta}{\nu}$$




Credit: V Thomas

# proof of concept

2D problem

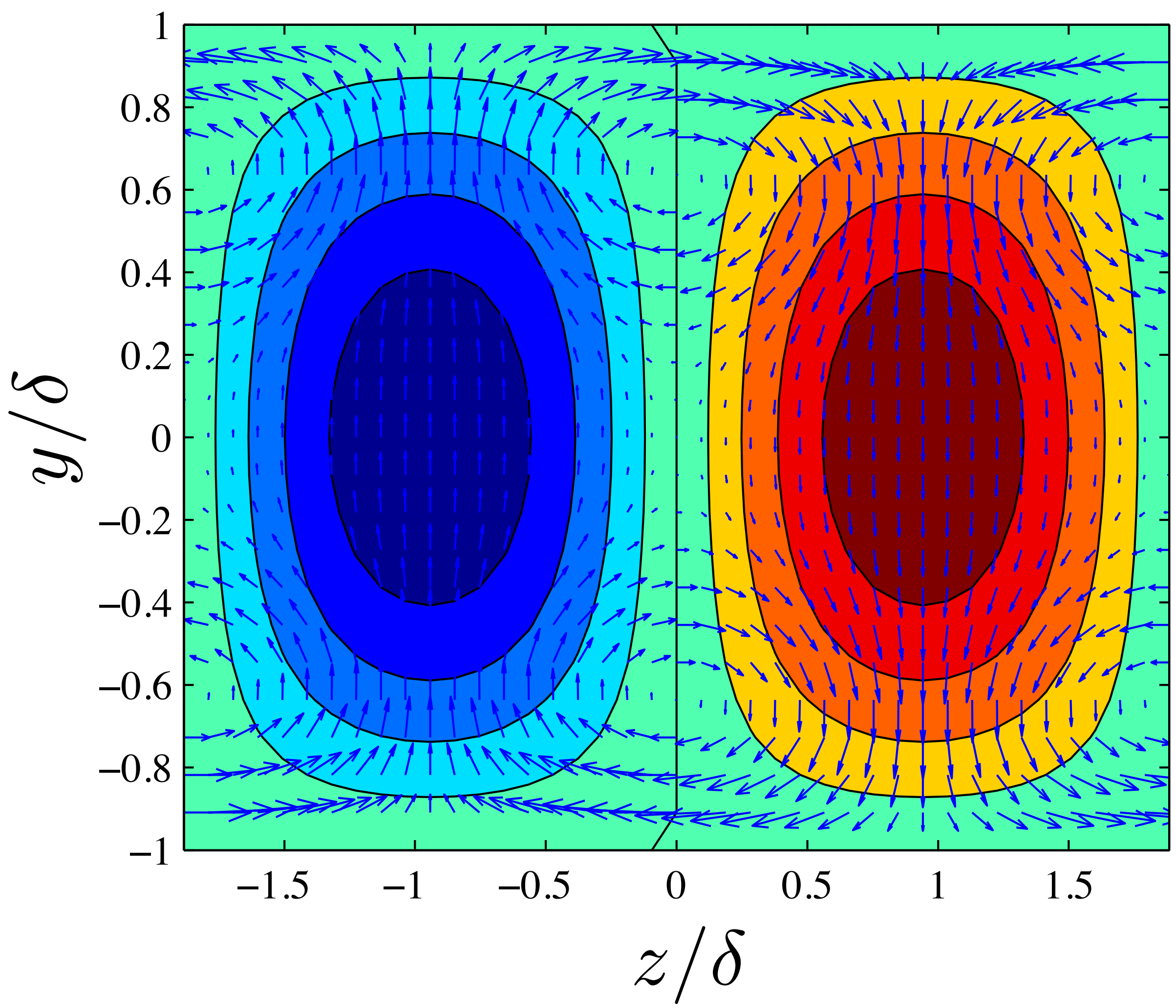



$(--)\ \delta U \quad , \quad (—)\ \mathcal{R}(\delta C)$

Analogously, in the 3D problem
infinitesimal mean flows organize the turbulent Reynolds stresses
so as to reinforce the very same mean flow

# proof of concept

1. Perturb a shear flow by an infinitesimal streak in the presence of turbulence
2. Calculate the response of the turbulence and the Reynolds stresses the are produce.



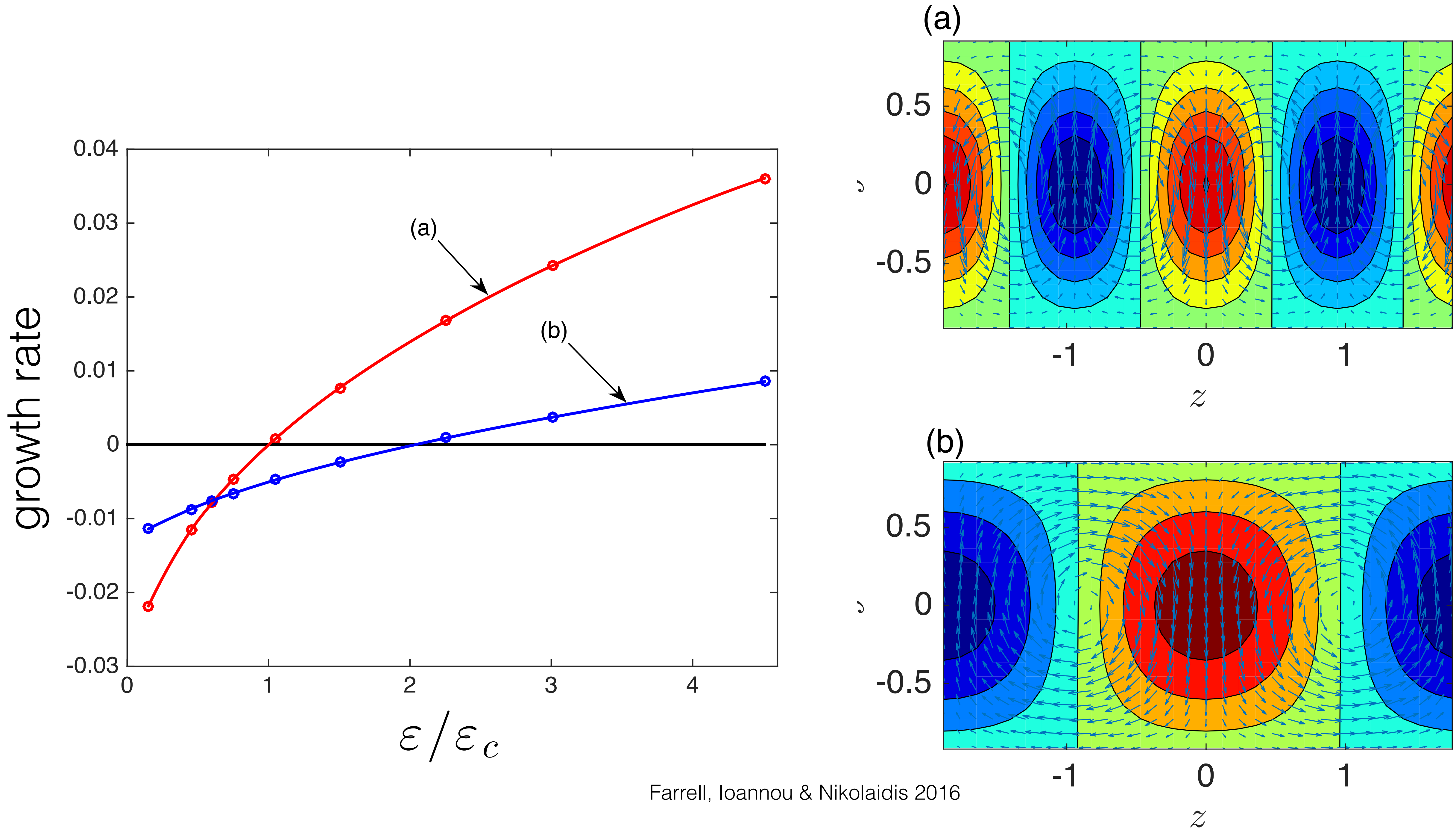

Farrell & Ioannou 2012

minimal channel
*Re*=400

it turns out that the stresses force a roll $(V, W)$
*exactly* such as to amplify the streak

**Interpretation**: turbulent Reynolds stresses are organized by the streak to force a roll circulation configured to amplify the streak

# eigenvalues/eigenmodes of the least stable S3T roll/streak modes

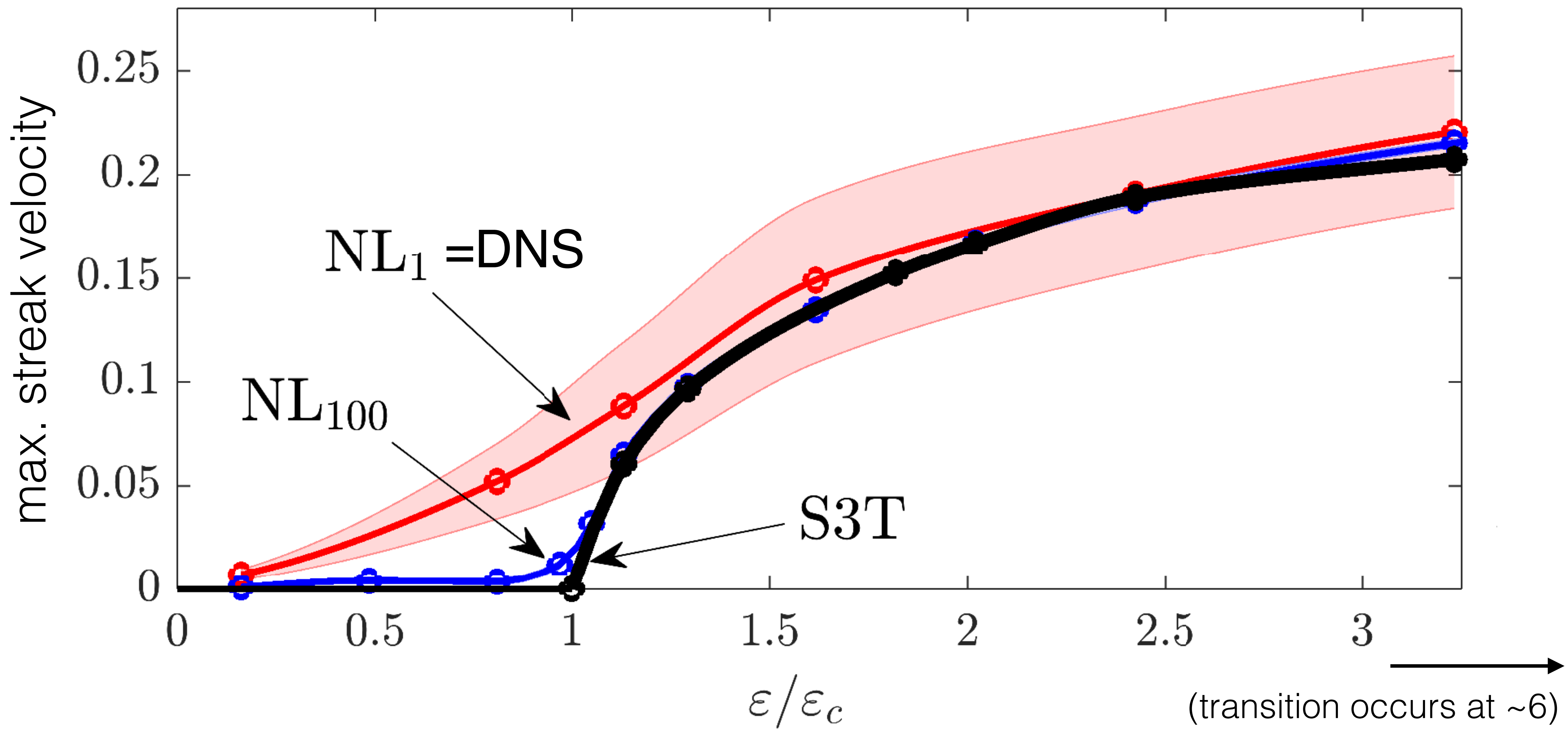



Farrell, Ioannou & Nikolaidis 2016

minimal channel: $L_x = 1.75\pi$, $L_z = 1.2\pi$ , $Re = 400$, stochastic excitation at $k_x = 2\pi/L_x$

$\varepsilon_C$ sustains turbulence with energy 0.14% of the Couette flow energy.

# bifurcation structure




Farrell, Ioannou & Nikolaidis 2016

minimal channel
*Re*=400

# Conclusions

- S3T generalizes the hydrodynamic stability of Rayleigh and allow us to study the stability of turbulent flows
- The emergence of coherent structures in a variety of flow settings is (analytically) predicted as an instability of the turbulent state
- S3T also predicts the final inhomogeneous turbulent state at which the system bifurcates to after the homogeneous state becomes unstable
- This is a first tool that enables us to determine the tipping points of the climate (climate = statistical turbulent equilibrium state)

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# thanks