Cause-and-effect of linear mechanisms in wall turbulence

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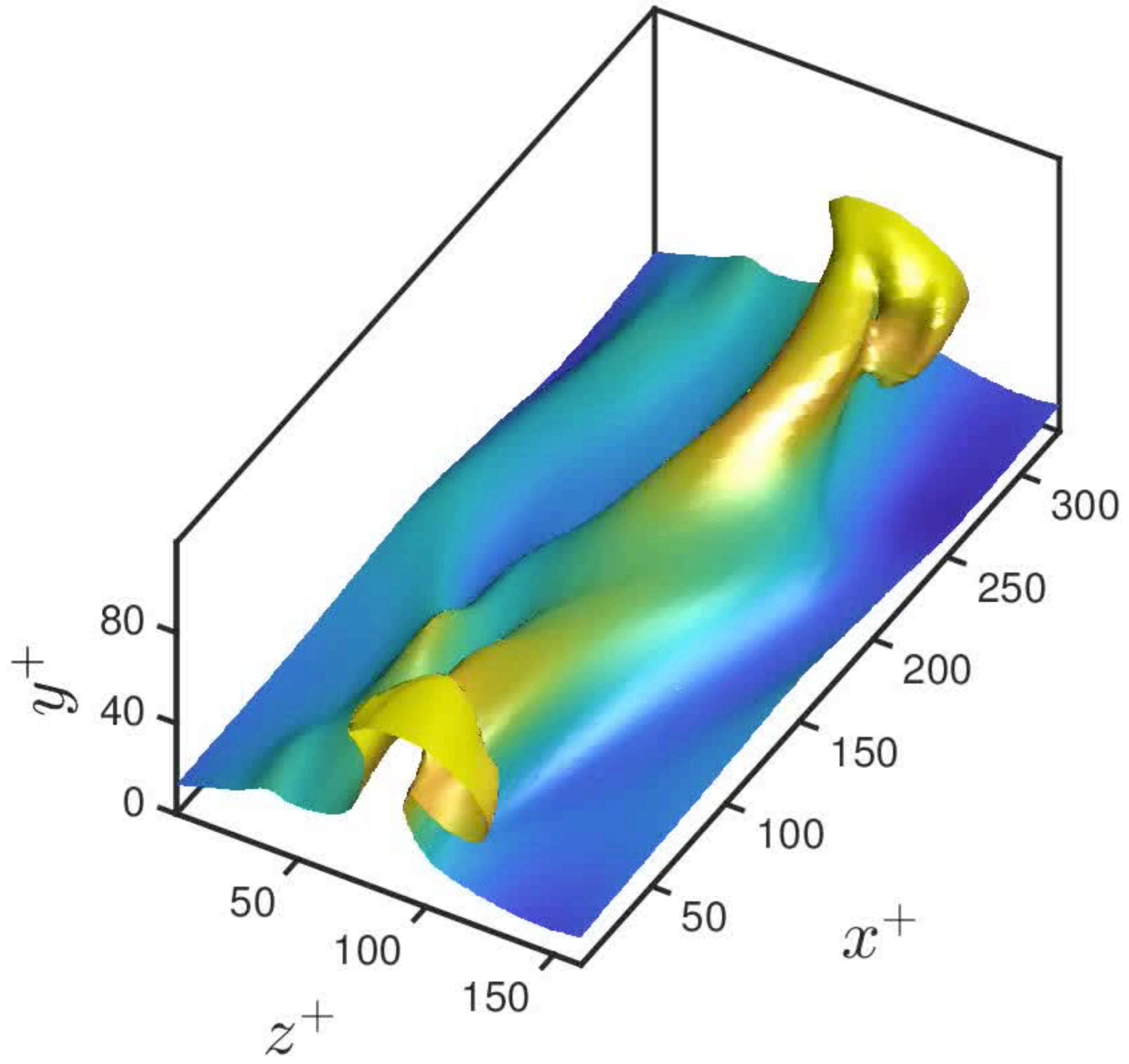

thanks to

Adrián Lozano-Durán
Marios-Andreas Nikolaidis
Michael Karp

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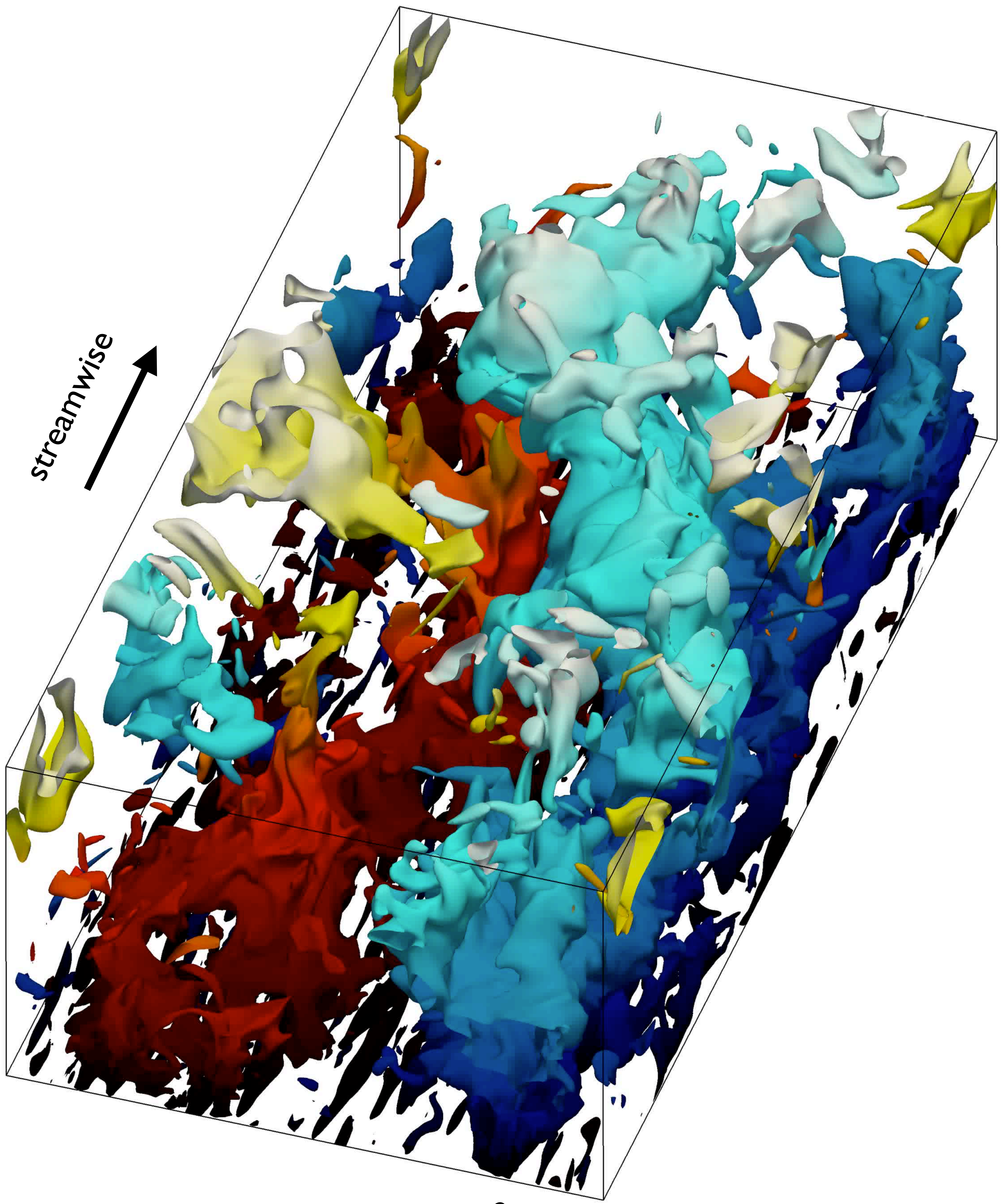
Monash University
October 2020

`Lozano-Duran et al. (2020)` *JFM*
`(in press; arXiv:2005.05303)`

Coherent structures in wall-turbulence



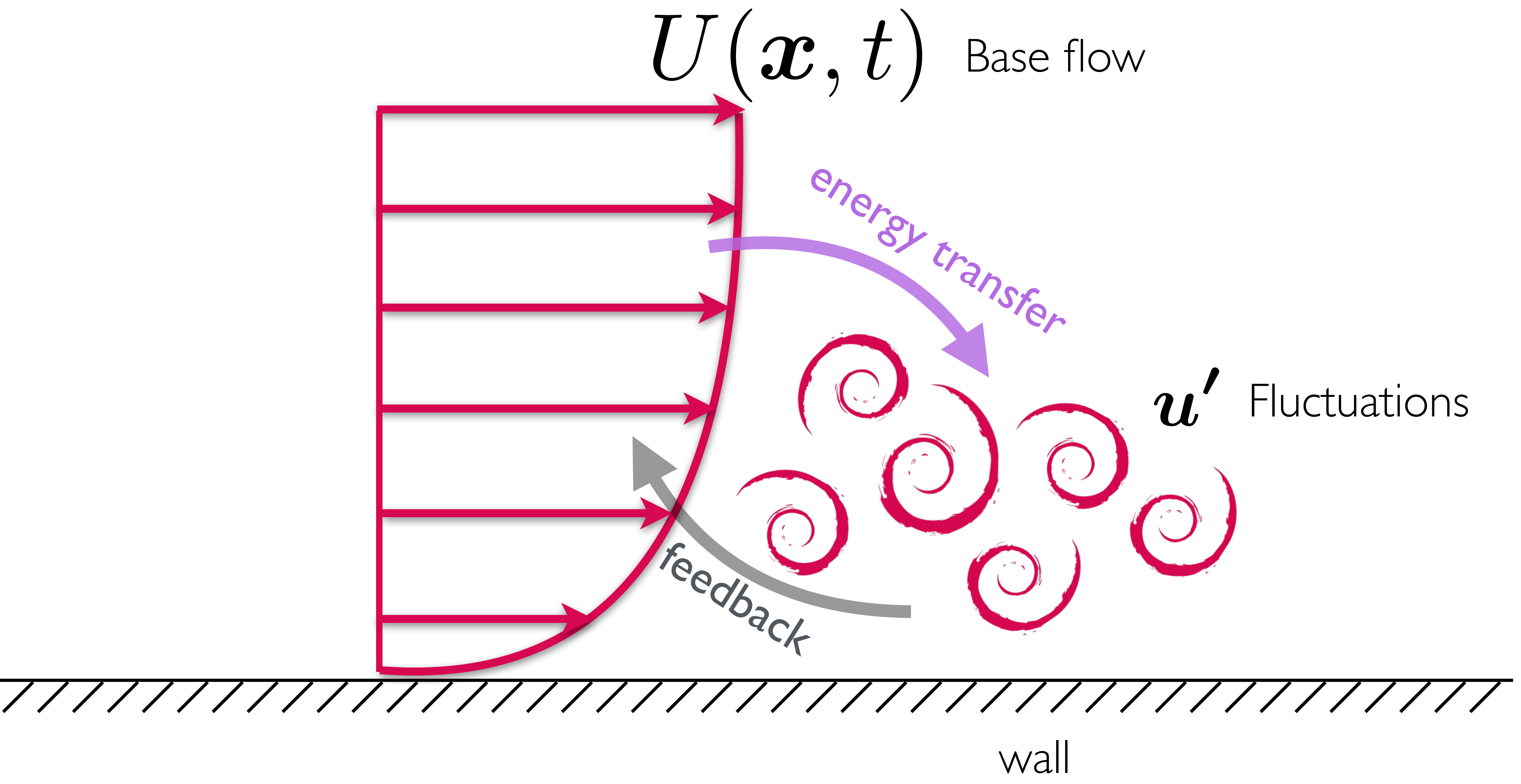
Poiseuille flow at Re_τ = 950

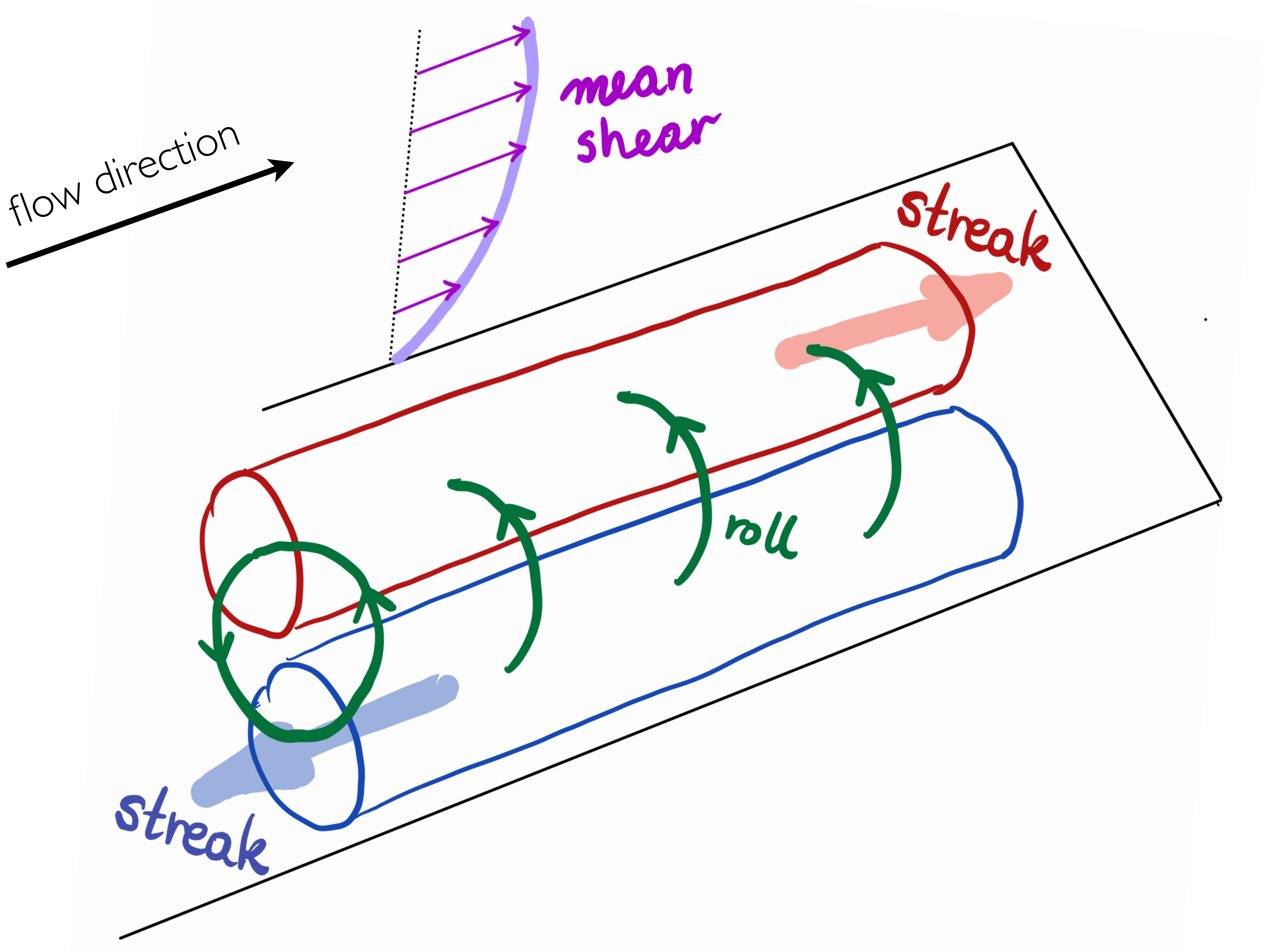
isocontours of streamwise velocity without k_x=k_z=0 mode

Coherent structures in wall-turbulence

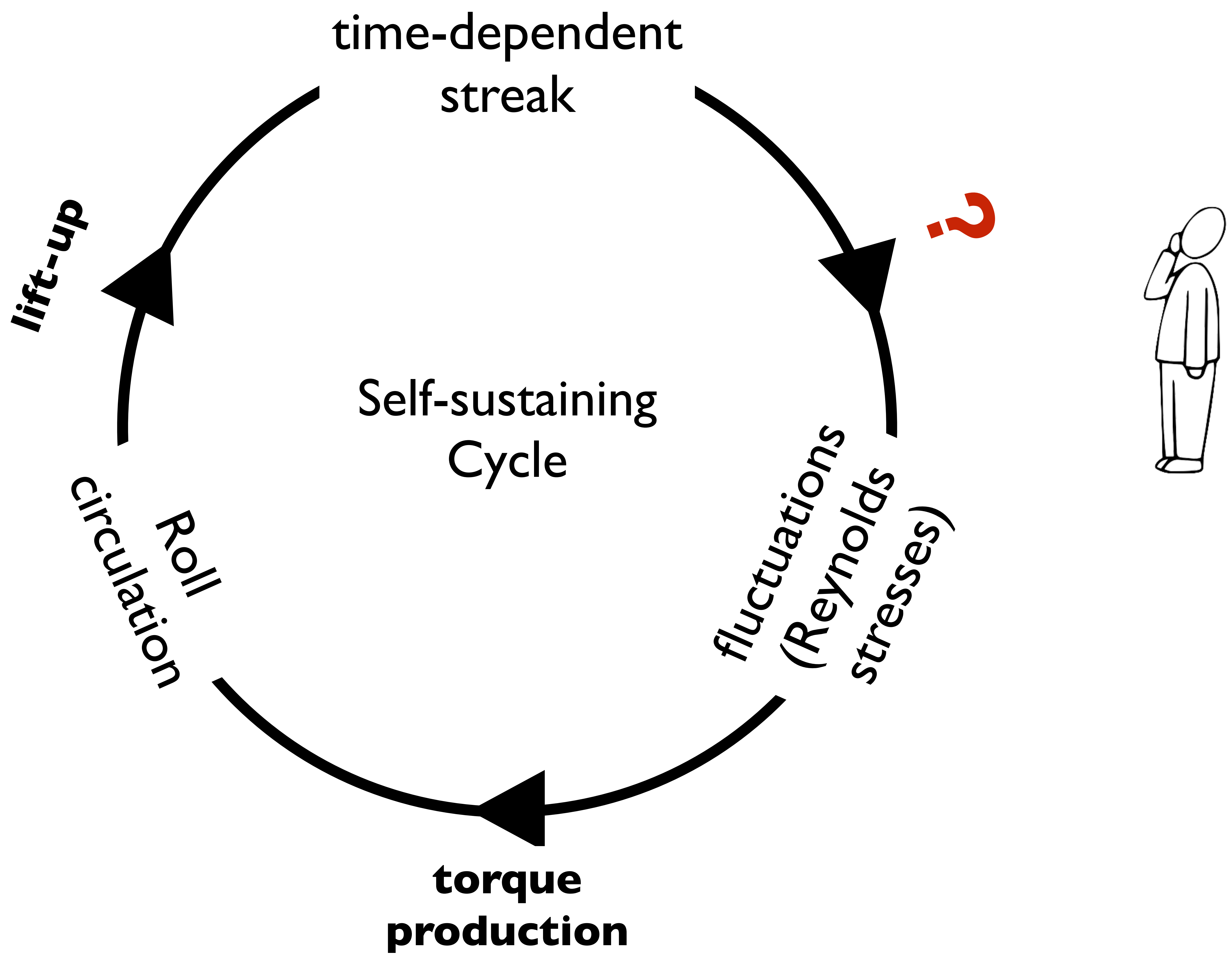
Mean shear profile — Rolls — Streaks — Fluctuations

Coherent **roll**−**streak** structure and turbulent fluctuations actively participate in a self-sustaining cycle

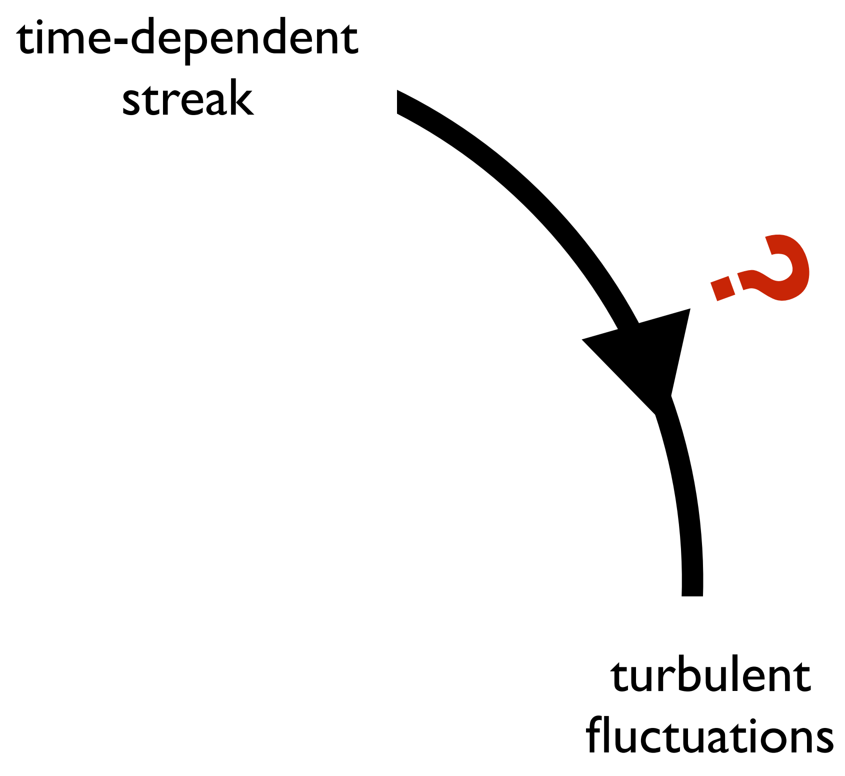




How is the loop closed?



Proposed mechanism for energy transfer to turbulent fluctuations



Modal instabilities of the streak

[Waleffe 1997, Kawahara 2003, Hack & Moin 2018, …]

Transient growth due to non-normality of linear operator $\mathscr{L}$

[Schoppa & Hussain (2002), Farrell & Ioannou (2012), Giovanetti et al. (2017),…]

Neutral modes — vortex-wave interactions

[Hall & Smith (1988), Hall & Sherwin (2010),…]

Parametric instability (enhanced energy transfer due to time-varying $U(y, z, t)$)

[Farrell & Ioannou (2012), Farrell et al. (2016),…]

We will assess the role of each proposed mechanisms

for energy transfer from streak to the fluctuations.

Linear and nonlinear processes

$$\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u} \cdot \nabla \boldsymbol{u} = -\frac{1}{\rho} \nabla p + \nu \nabla^2 \boldsymbol{u} \qquad \nabla \cdot \boldsymbol{u} = 0$$

Linear and nonlinear processes

$$\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u} \cdot \nabla \boldsymbol{u} = -\frac{1}{\rho} \nabla p + \nu \nabla^2 \boldsymbol{u} \qquad \nabla \cdot \boldsymbol{u} = 0$$

decompose the flow as $\boldsymbol{u} = \boldsymbol{U} + \boldsymbol{u}'$ $\quad (\boldsymbol{U} \equiv \langle \boldsymbol{u} \rangle$; some average)

$$\frac{\partial \boldsymbol{U}}{\partial t} + \boldsymbol{U} \cdot \nabla \boldsymbol{U} = -\frac{1}{\rho} \nabla \langle p \rangle + \nu \nabla^2 \boldsymbol{U} - \underbrace{\langle \boldsymbol{u}' \cdot \nabla \boldsymbol{u}' \rangle}_{\text{Reynolds stresses}} \qquad \nabla \cdot \boldsymbol{U} = 0$$

$$\frac{\partial \boldsymbol{u}'}{\partial t} = \underbrace{\mathscr{L}(\boldsymbol{U})\, \boldsymbol{u}'}_{\substack{\text{linear} \\ \text{processes}}} + \underbrace{\mathscr{N}(\boldsymbol{u}')}_{\substack{\text{nonlinear} \\ \text{processes}}}$$

Linear and nonlinear processes

$$\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u}\cdot\nabla\boldsymbol{u} = -\frac{1}{\rho}\nabla p + \nu\nabla^2\boldsymbol{u} \qquad \nabla\cdot\boldsymbol{u} = 0$$

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$$\frac{\partial \boldsymbol{u}'}{\partial t} = \underbrace{\mathscr{L}(\boldsymbol{U})\,\boldsymbol{u}'}_{\substack{\text{linear}\\\text{processes}}} + \underbrace{\mathscr{N}(\boldsymbol{u}')}_{\substack{\text{nonlinear}\\\text{processes}}}$$

We *didn't linearise* about a solution $\boldsymbol{U}$!

We *decomposed* the flow and call "linear" anything included in $\mathscr{L}(\boldsymbol{U})\boldsymbol{u}'$.

A different choice for U can make a process included in $\mathscr{L}(\boldsymbol{U})\boldsymbol{u}'$ to become part of $\mathscr{N}(\boldsymbol{u}')$.

Linear processes energise the fluctuations

$$\boldsymbol{u} = \boldsymbol{U} + \boldsymbol{u}'$$

$$\text{flow} = \begin{matrix}\text{base}\\\text{flow}\end{matrix} + \text{fluctuations}$$

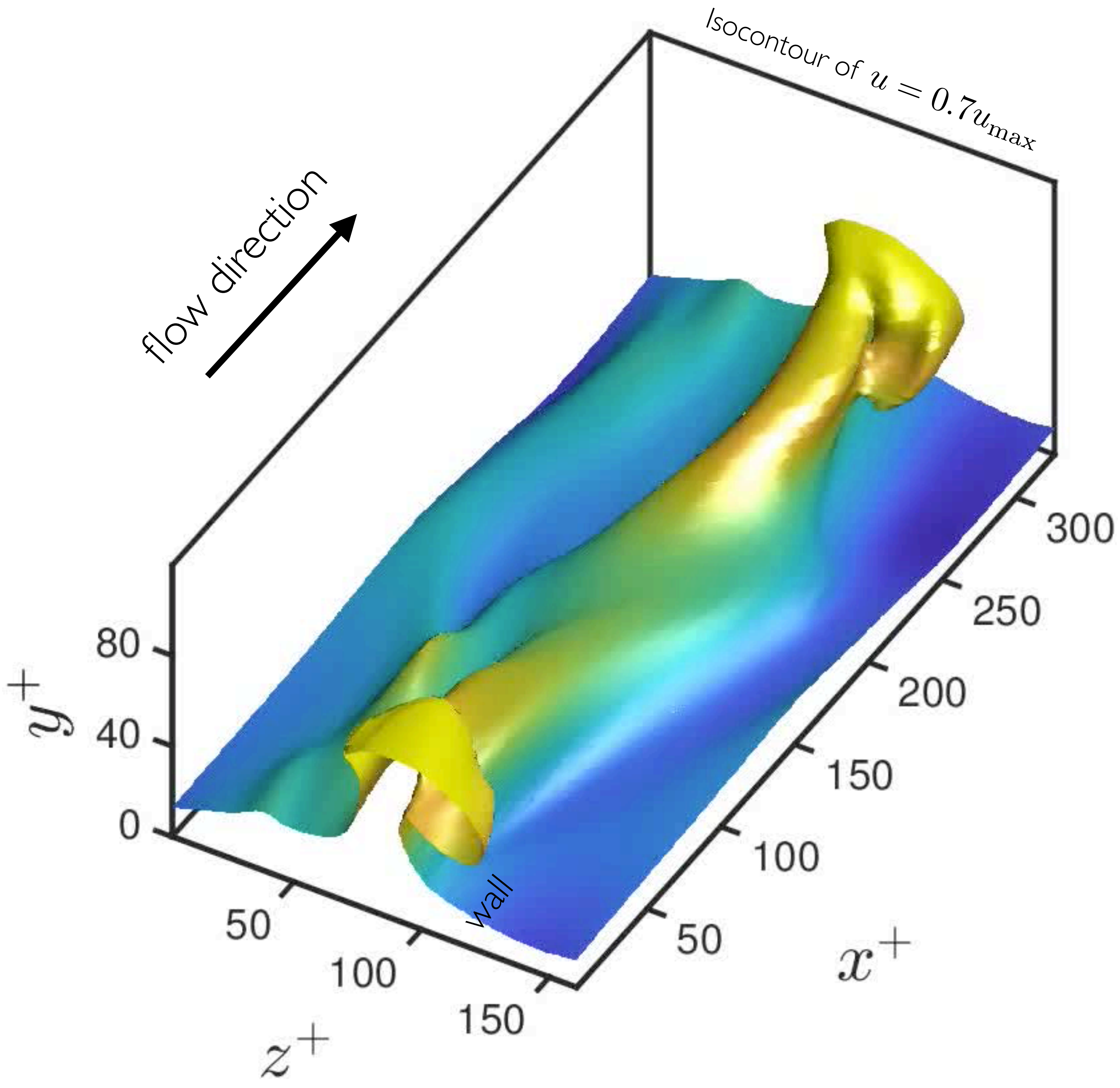
fluctuation dynamics

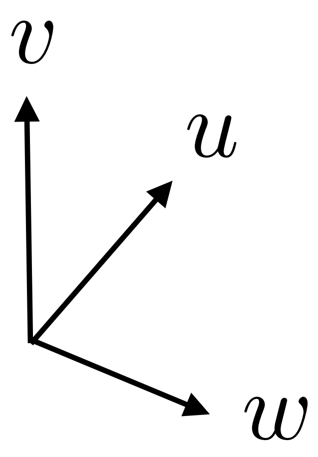
$$\frac{\partial \boldsymbol{u}'}{\partial t} = \underbrace{\mathscr{L}(\boldsymbol{U})\,\boldsymbol{u}'}_{\substack{\text{linear}\\\text{processes}}} + \underbrace{\mathscr{N}(\boldsymbol{u}')}_{\substack{\text{nonlinear}\\\text{processes}}}$$

If $\displaystyle\int \boldsymbol{u}'\cdot\mathscr{N}(\boldsymbol{u}')\,\mathrm{d}V = 0$ then

$$\frac{\mathrm{d}}{\mathrm{d}t}\underbrace{\int \tfrac{1}{2}|\boldsymbol{u}'|^2\,\mathrm{d}V}_{\substack{\text{turbulent}\\\text{kinetic energy}}} = \int \boldsymbol{u}'\cdot\left[\mathscr{L}(\boldsymbol{U})\boldsymbol{u}'\right]\mathrm{d}V$$

Problem set-up: minimal turbulent channel





Half channel flow

Constant pressure gradient

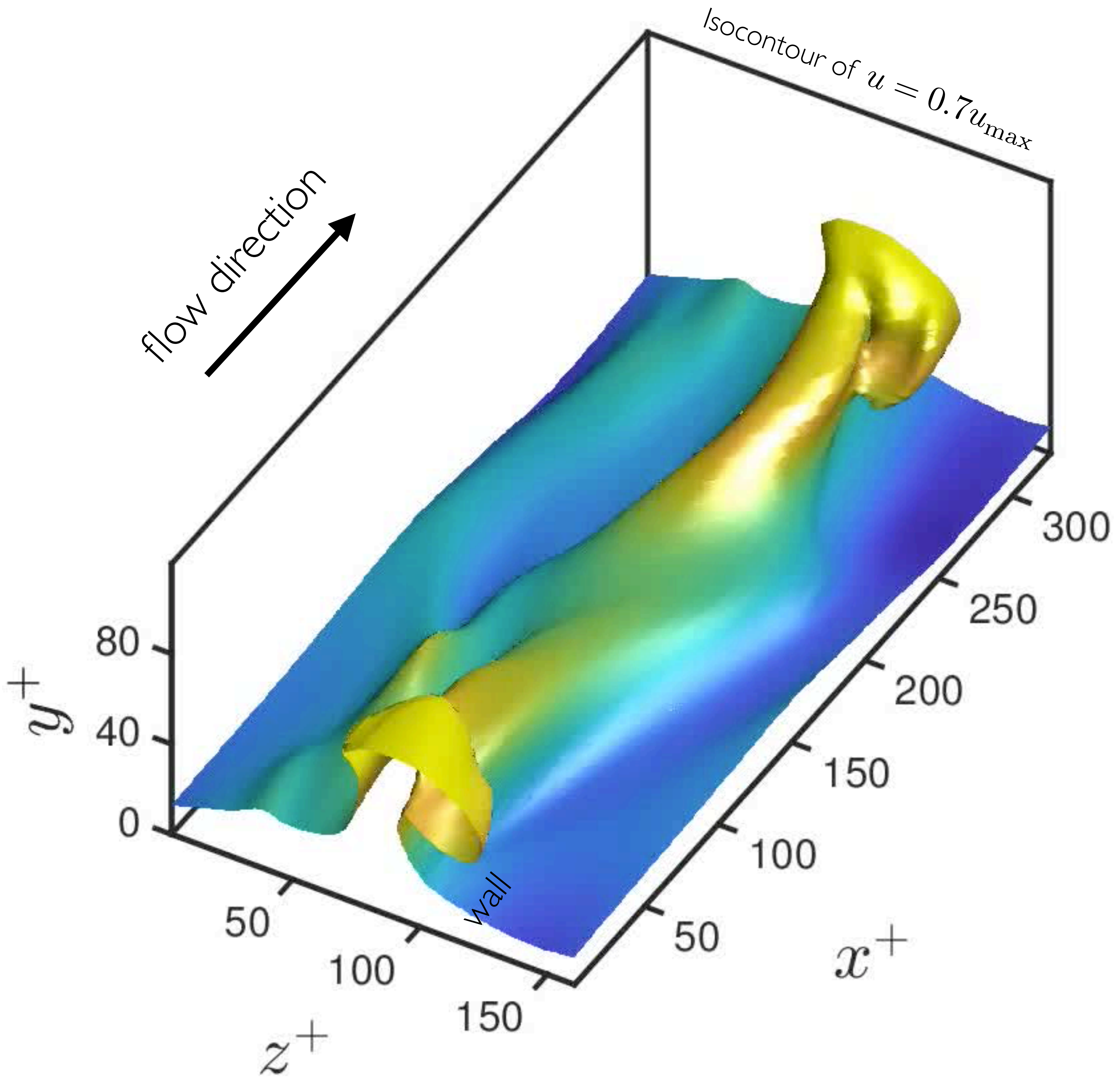
Solution by
Direct Numerical Simulation

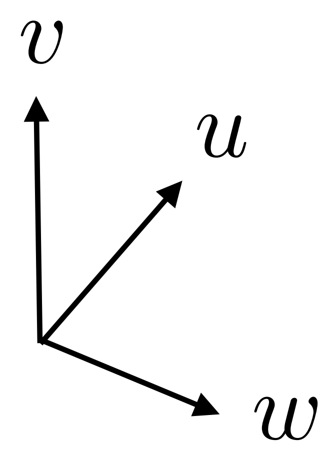
$Re_\tau = 184$

h wall-normal height

u_τ friction velocity

Problem set-up: minimal turbulent channel

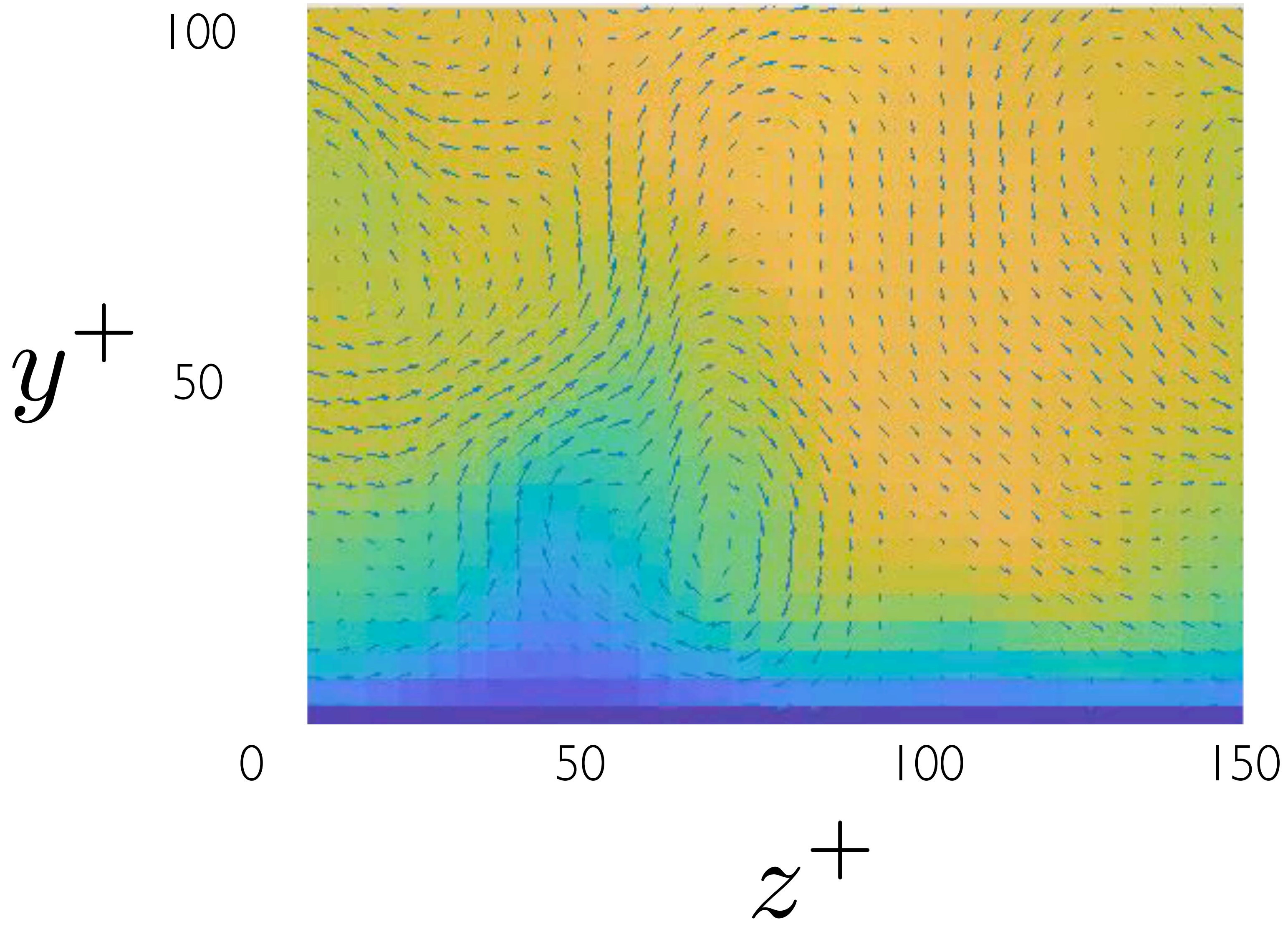




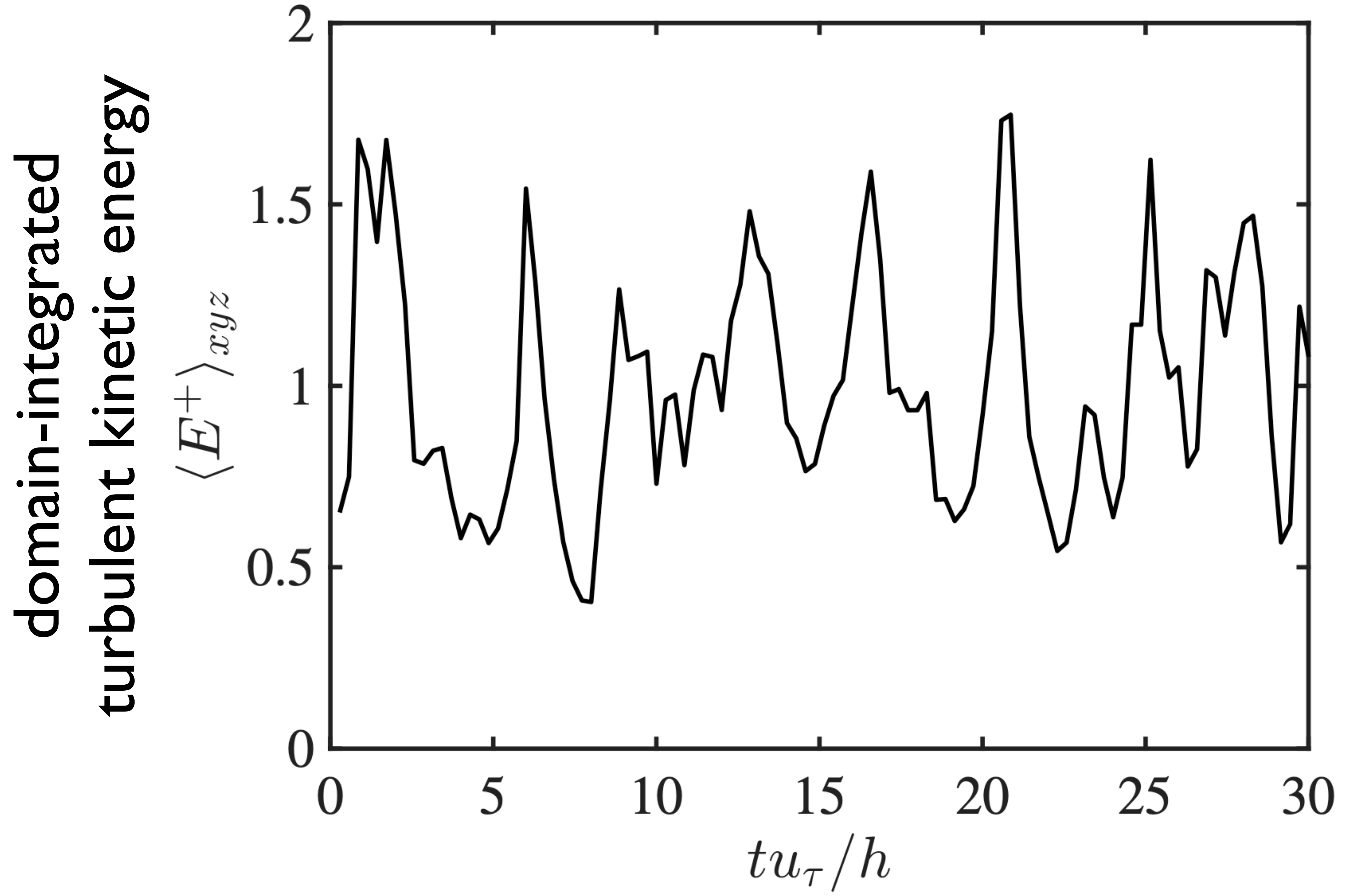
Streaky base flow

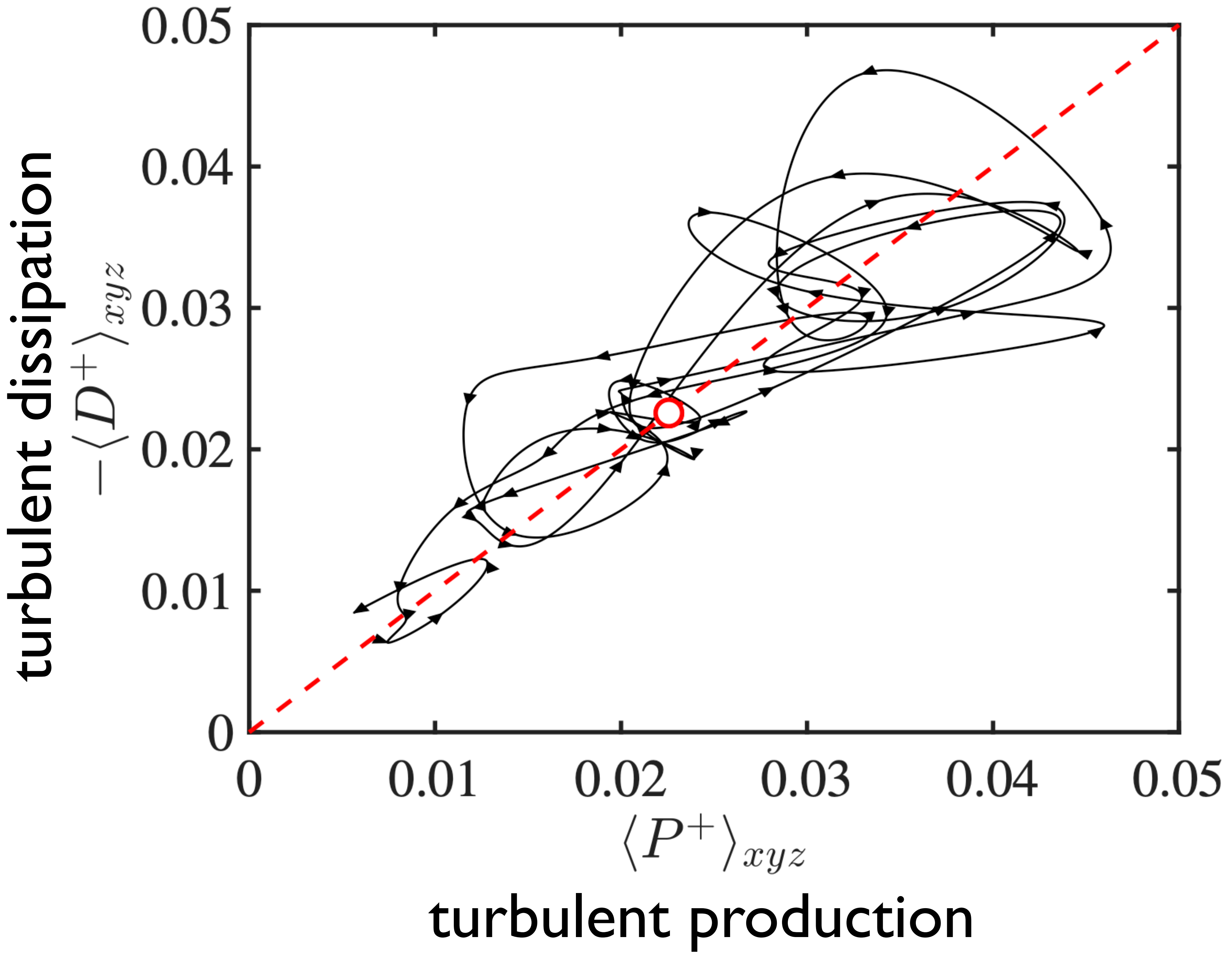
$$\boldsymbol{U} = U(y, z, t)\,\hat{\boldsymbol{x}} \qquad U(y, z, t) \equiv \int u(x, y, z, t)\,\mathrm{d}x / L_x$$

(only x-component)



Problem set-up: minimal turbulent channel





We run DNS for >600h/u_τ and keep *all* snapshots of base flow $U(y, z, t)$

Two ways to assess various mechanisms

Interrogate DNS output

non-intrusive

Sensibly modify equations of motion to *preclude* some mechanisms

allows infer casual relationships

Modal instabilities of the streaky base flow

$$\mathscr{L}(U(y,z,t)) = \underbrace{\mathscr{U} \begin{pmatrix} \lambda_1 + \mathrm{i}\omega_1 & & & \\ & \lambda_2 + \mathrm{i}\omega_2 & & \\ & & \lambda_3 + \mathrm{i}\omega_3 & \\ & & & \ddots \end{pmatrix} \mathscr{U}^{-1}}_{\text{Eigen-decomposition of } \mathscr{L}}$$

Eigenvalues

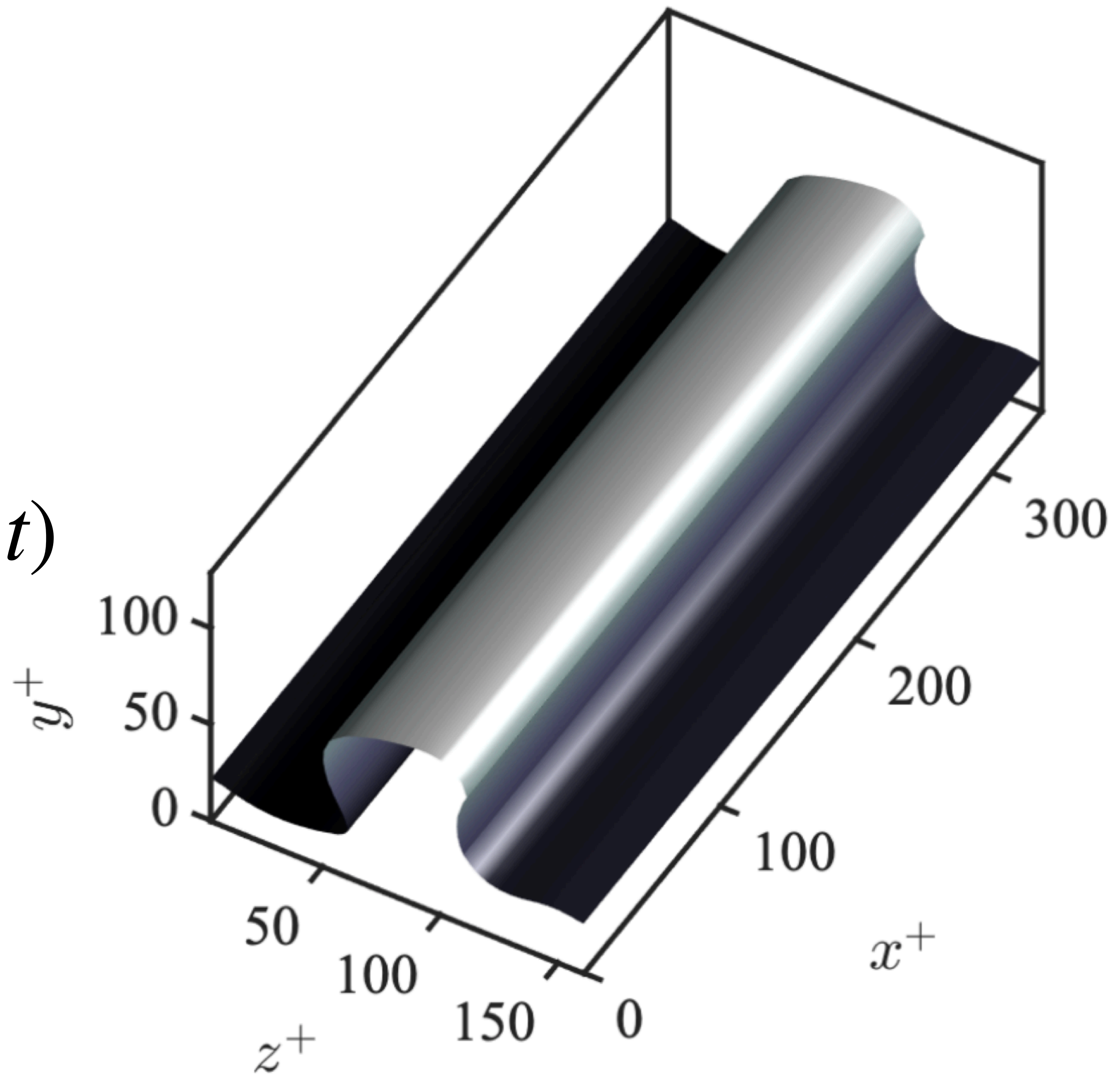
growth rates
$\lambda_1 \geq \lambda_2 \geq \cdots$

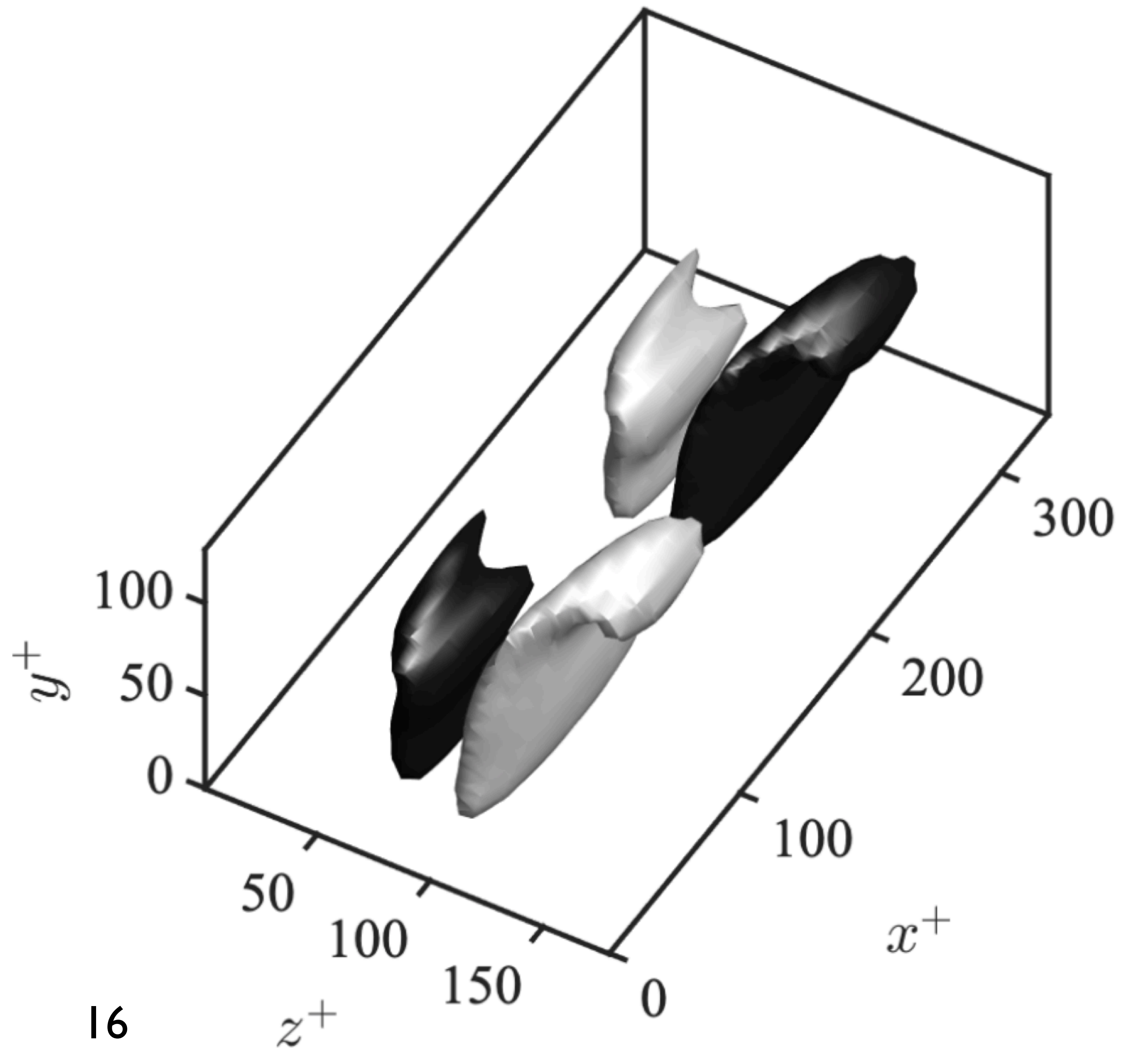
Modal instabilities of the streaky base flow

$$\mathscr{L}(U(y,z,t)) = \mathscr{U} \begin{pmatrix} \lambda_1 + i\omega_1 & & & \\ & \lambda_2 + i\omega_2 & & \\ & & \lambda_3 + i\omega_3 & \\ & & & \ddots \end{pmatrix} \mathscr{U}^{-1}$$

growth rates
$\lambda_1 \geq \lambda_2 \geq \cdots$

base flow $U(y, z, t)$



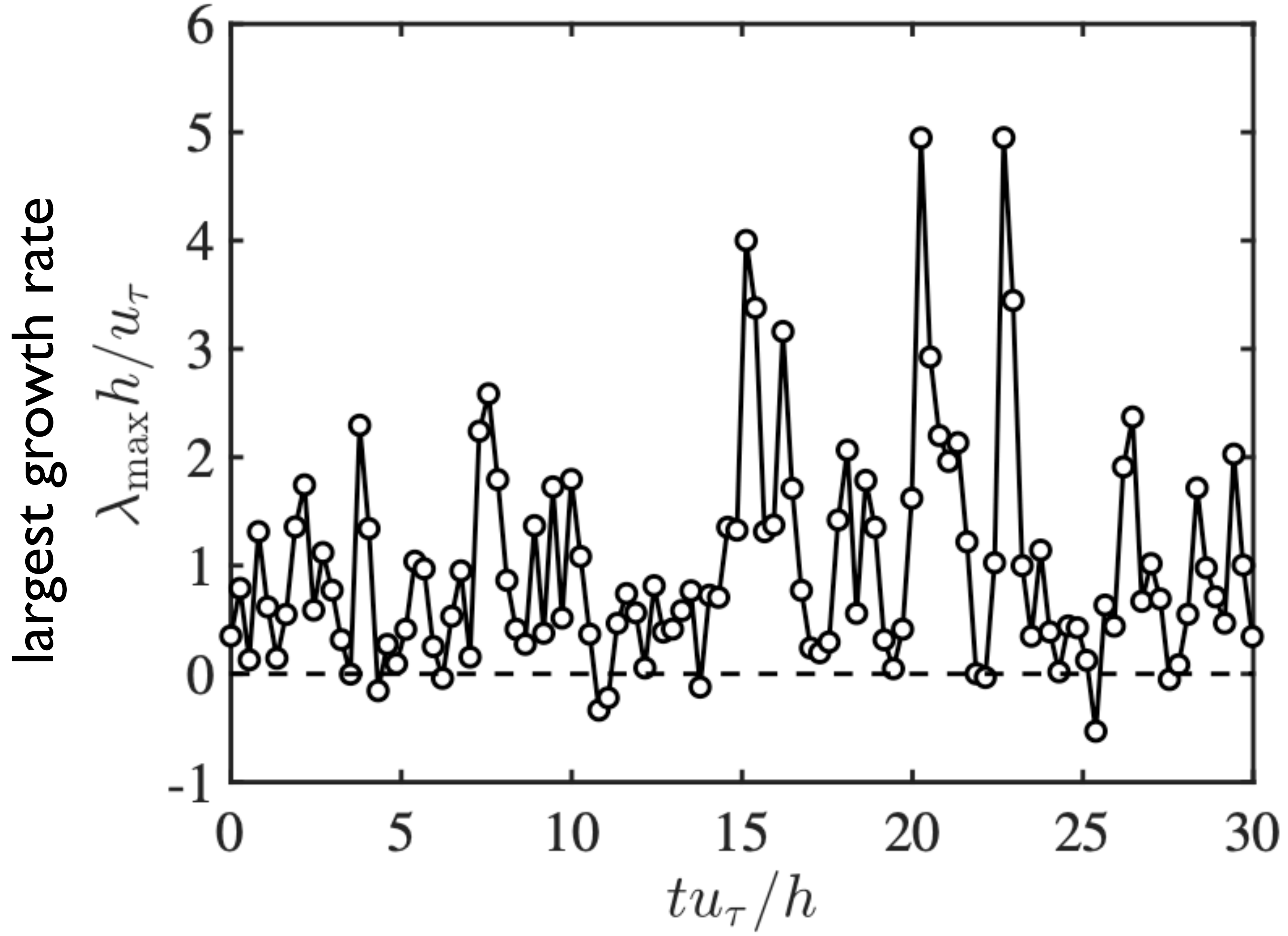


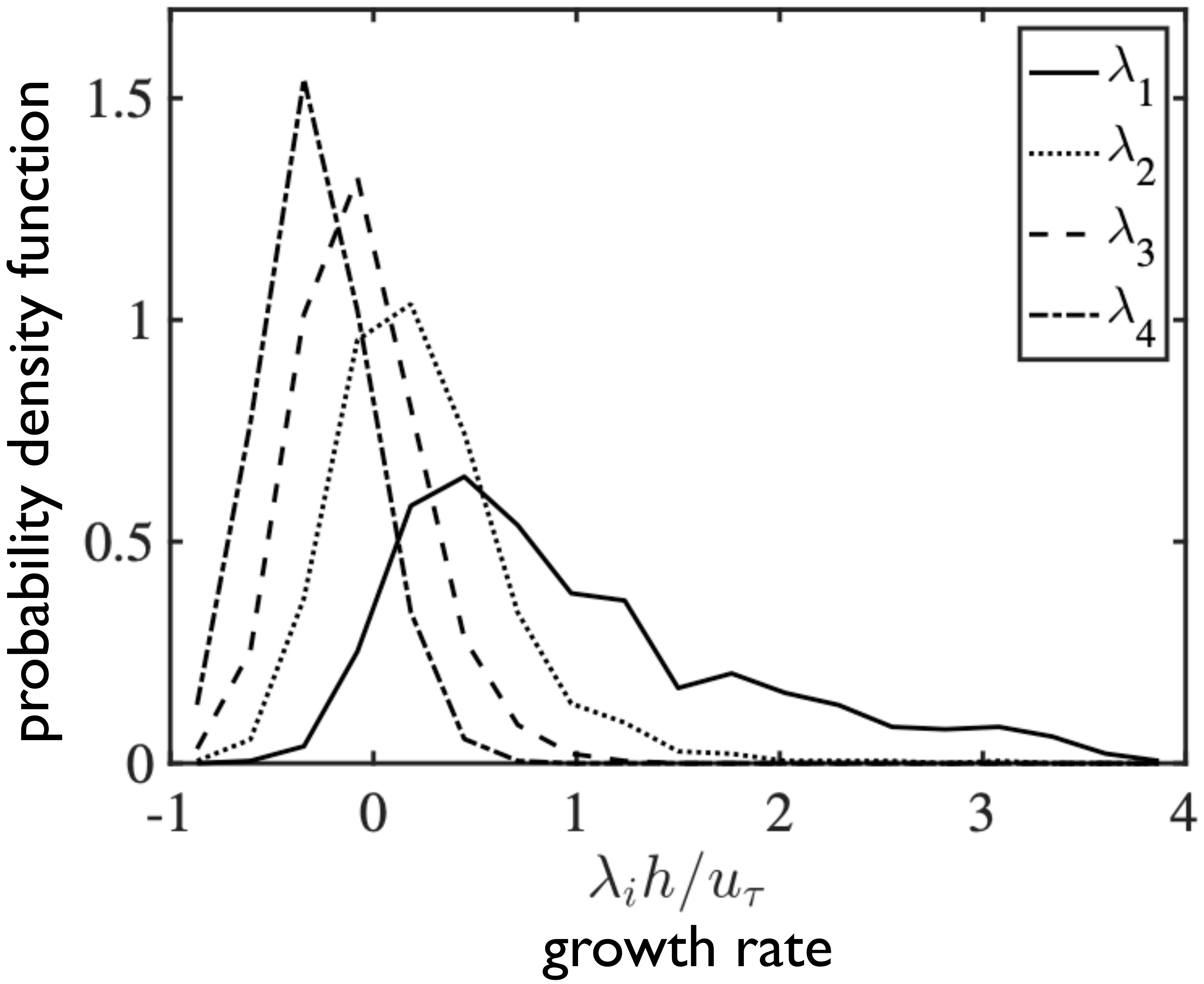
sinuous unstable eigenmode
$\lambda h / u_\tau \approx 3$

Modal instabilities of the streaky base flow

U is unstable $\gtrapprox$90% of the time

~2-3 unstable modes



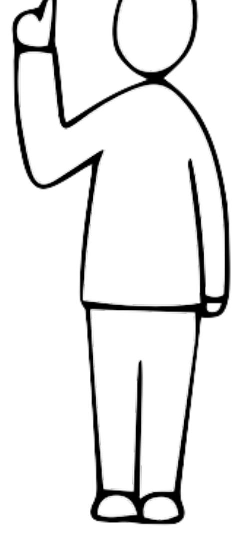


Autocorrelation of $U \Rightarrow$ base flow changes (at least) ~3 x slower than e-folding $1/\lambda$

$\Rightarrow$ modal instabilities *do* have time to grow

If modal instabilities are *crucial* for the self-sustaining cycle

flow should laminarise without them…



Suppressing modal instabilities of the streaky base flow

$$\mathscr{L}(U(y,z,t)) = \mathscr{U}\begin{pmatrix} \lambda_1 + \mathrm{i}\omega_1 & & & \\ & \lambda_2 + \mathrm{i}\omega_2 & & \\ & & \lambda_3 + \mathrm{i}\omega_3 & \\ & & & \ddots \end{pmatrix}\mathscr{U}^{-1} \qquad \lambda_1 \geq \lambda_2 \geq \cdots$$

@ *every* instance we stabilise $\mathscr{L} \implies$ if $\lambda_j > 0$, replace with $-\lambda_j$

E.g., for 2 unstable modes:

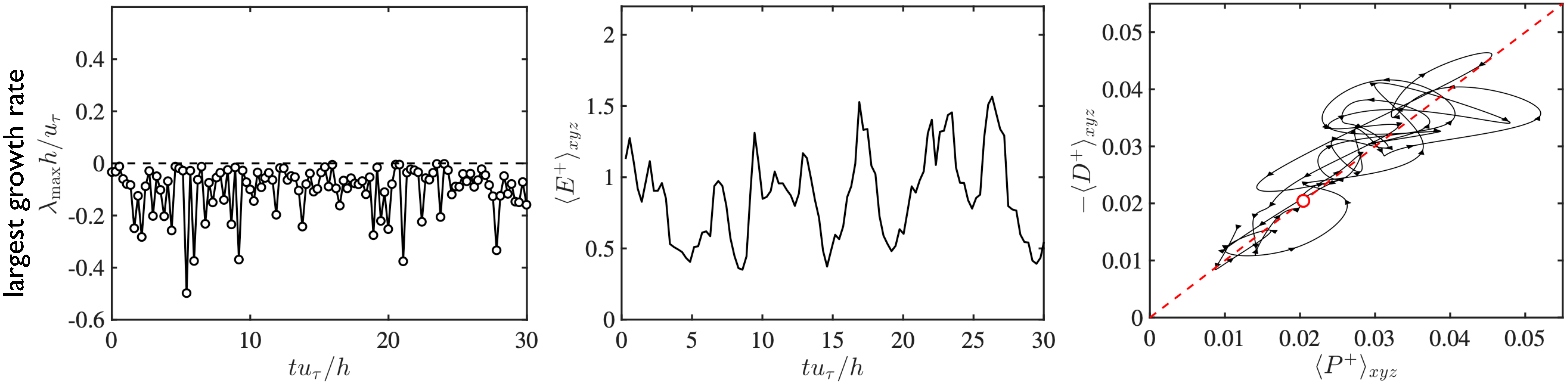
$$\widetilde{\mathscr{L}}(U(y,z,t)) = \mathscr{U}\begin{pmatrix} -\lambda_1 + \mathrm{i}\omega_1 & & & \\ & -\lambda_2 + \mathrm{i}\omega_2 & & \\ & & \lambda_3 + \mathrm{i}\omega_3 & \\ & & & \ddots \end{pmatrix}\mathscr{U}^{-1}$$

Modally stable wall-turbulence

$$\frac{\partial \boldsymbol{U}}{\partial t} + \boldsymbol{U}\cdot\nabla\boldsymbol{U} = -\frac{1}{\rho}\nabla P + \nu\,\nabla^2\boldsymbol{U} - \langle \boldsymbol{u}'\cdot\nabla\boldsymbol{u}'\rangle \qquad \nabla\cdot\boldsymbol{U} = 0$$

$$\frac{\partial \boldsymbol{u}'}{\partial t} = \widetilde{\mathscr{L}}(U)\,\boldsymbol{u}' + \mathscr{N}(\boldsymbol{u}')$$

fully coupled



turbulence persists…

[Turbulence also persist if $\mathscr{N}$ is set to 0!]

Farrell & Ioannou (2012)

Modally stable wall-turbulence

$$\frac{\partial \boldsymbol{U}}{\partial t} + \boldsymbol{U}\cdot\nabla\boldsymbol{U} = -\frac{1}{\rho}\nabla P + \nu\nabla^2\boldsymbol{U} - \langle \boldsymbol{u}'\cdot\nabla\boldsymbol{u}'\rangle \qquad \nabla\cdot\boldsymbol{U}=0$$

$$\frac{\partial \boldsymbol{u}'}{\partial t} = \widetilde{\mathscr{L}}(U)\,\boldsymbol{u}' + \mathscr{N}(\boldsymbol{u}')$$

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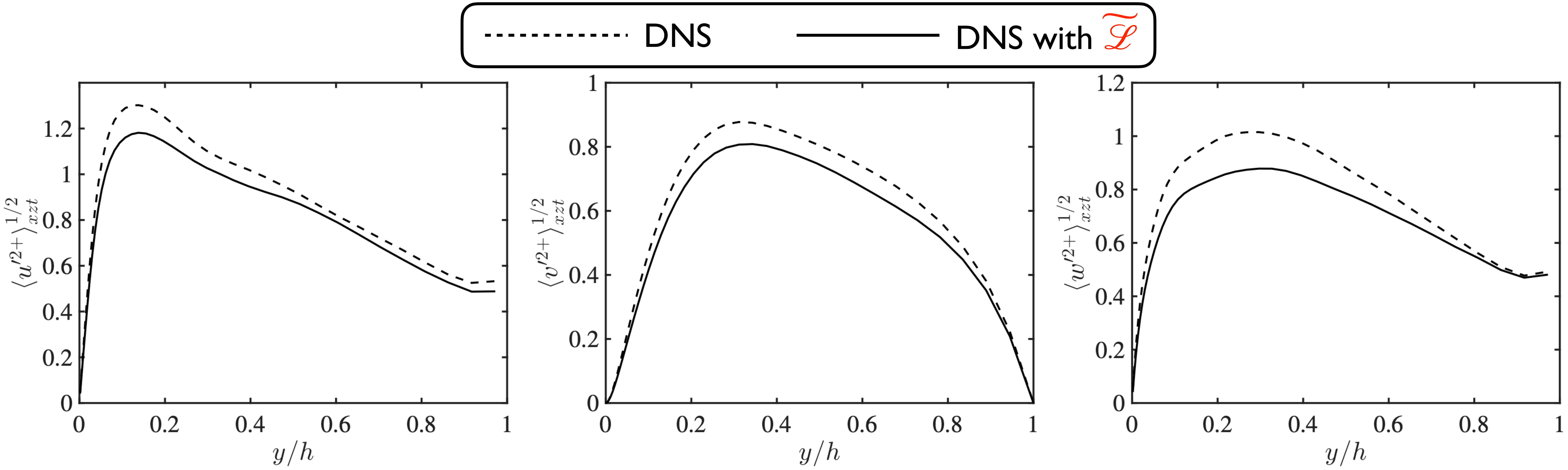


… and it's not that different from the DNS — turbulent intensities only drop by ~10%

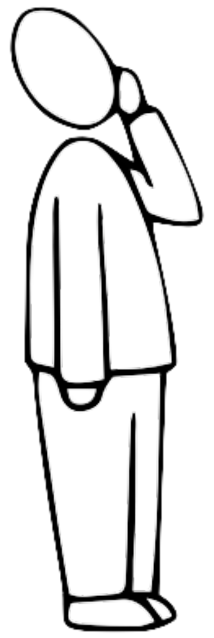
Non-modal transient growth

Since $\int \boldsymbol{u}' \cdot \mathcal{N}(\boldsymbol{u}')\,\mathrm{d}V = 0$, turbulent energy is governed by linear processes

$$\left.\begin{aligned} \frac{\partial \boldsymbol{u}'}{\partial t} &= \mathscr{L}(t)\,\boldsymbol{u}' \\ \boldsymbol{u}'(t = t_0) &= \boldsymbol{u}'_0 \end{aligned}\right\} \Longrightarrow \boldsymbol{u}'(t) = \underbrace{\Phi_{t,t_0}}_{\text{linear map from } t_0 \text{ to } t} \boldsymbol{u}'_0$$

$$\underbrace{G_{\max}(t_0, T)}_{\text{maximum energy gain}} = \sup_{\boldsymbol{u}'_0} \frac{\int |\boldsymbol{u}'(t_0+T)|^2\,\mathrm{d}V}{\int |\boldsymbol{u}'_0|^2\,\mathrm{d}V} = \sup_{\boldsymbol{u}'_0} \frac{\int |\Phi_{t_0,t_0+T}\,\boldsymbol{u}'_0|^2\,\mathrm{d}V}{\int |\boldsymbol{u}'_0|^2\,\mathrm{d}V} = \max\left[\mathrm{svd}\left(\Phi_{\mathrm{t_0,t_0+T}}\right)^2\right]$$

[Farrell & Ioannou (1996), Schmid (2007)]



How we can disentangle transient growth

from exponential instabilities?

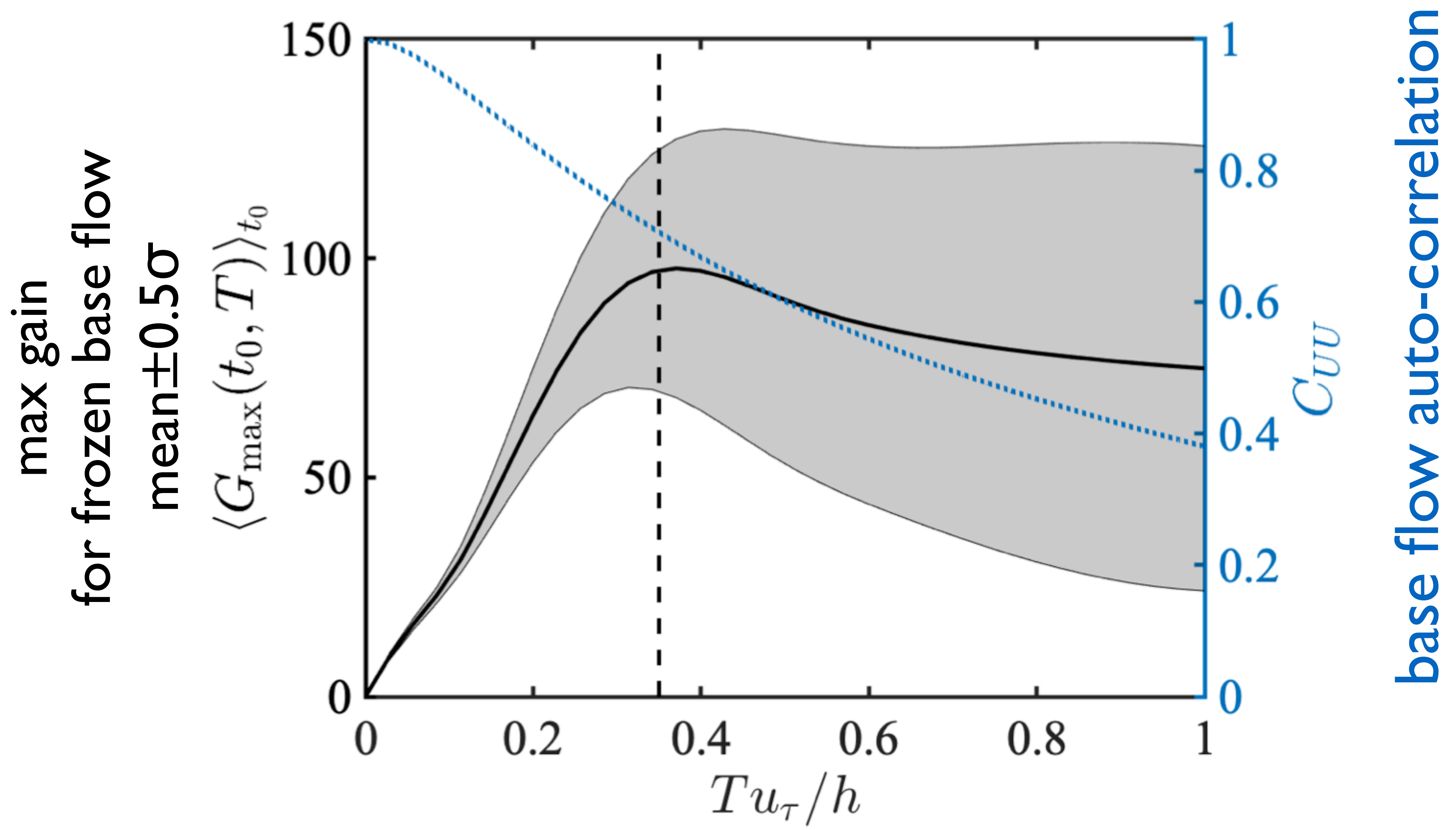
We can use the stabilised operator $\widetilde{\mathscr{L}}(U)$.

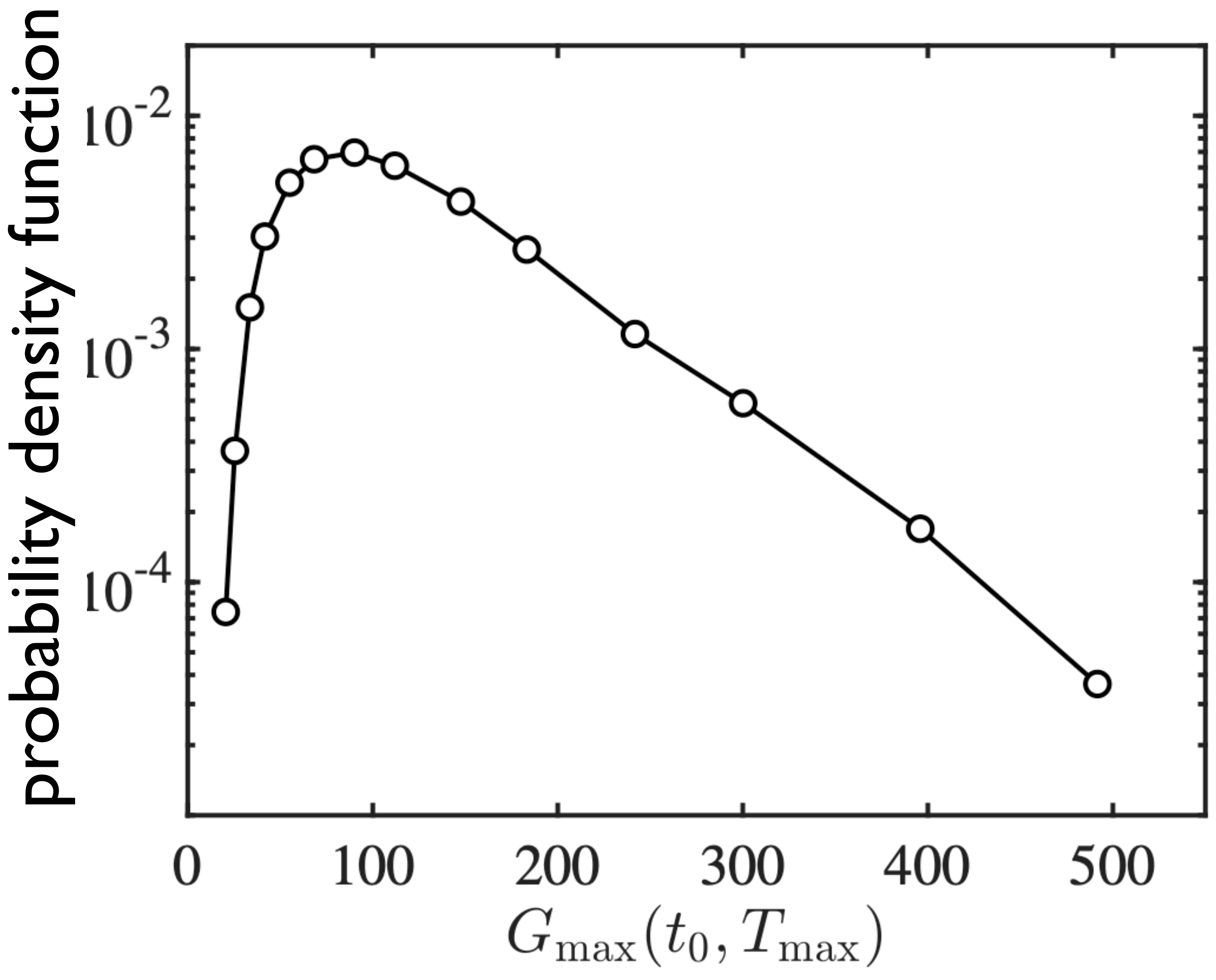
Non-modal transient growth
frozen base flow $U(y, z, t_0)$

$$G_{\max}(t_0, T) = \max\left[\mathrm{svd}\left(\widetilde{\Phi}_{t_0,t_0+T}\right)^2\right]$$

$G_{\max}(t_0, T)$: maximum energy gain

$\widetilde{\Phi}_{t_0,t_0+T}$: linear map for the stabilised $\widetilde{\mathscr{L}}$ 





[Note that streaky base flow $U(y, z, t_0)$ gives gains $O(100)$. Base flows $U(y)$ induce gain $O(10)$.]

Del Alamo & Jiménez 2006; Pujals et al. 2009; Cossu et al. 2009

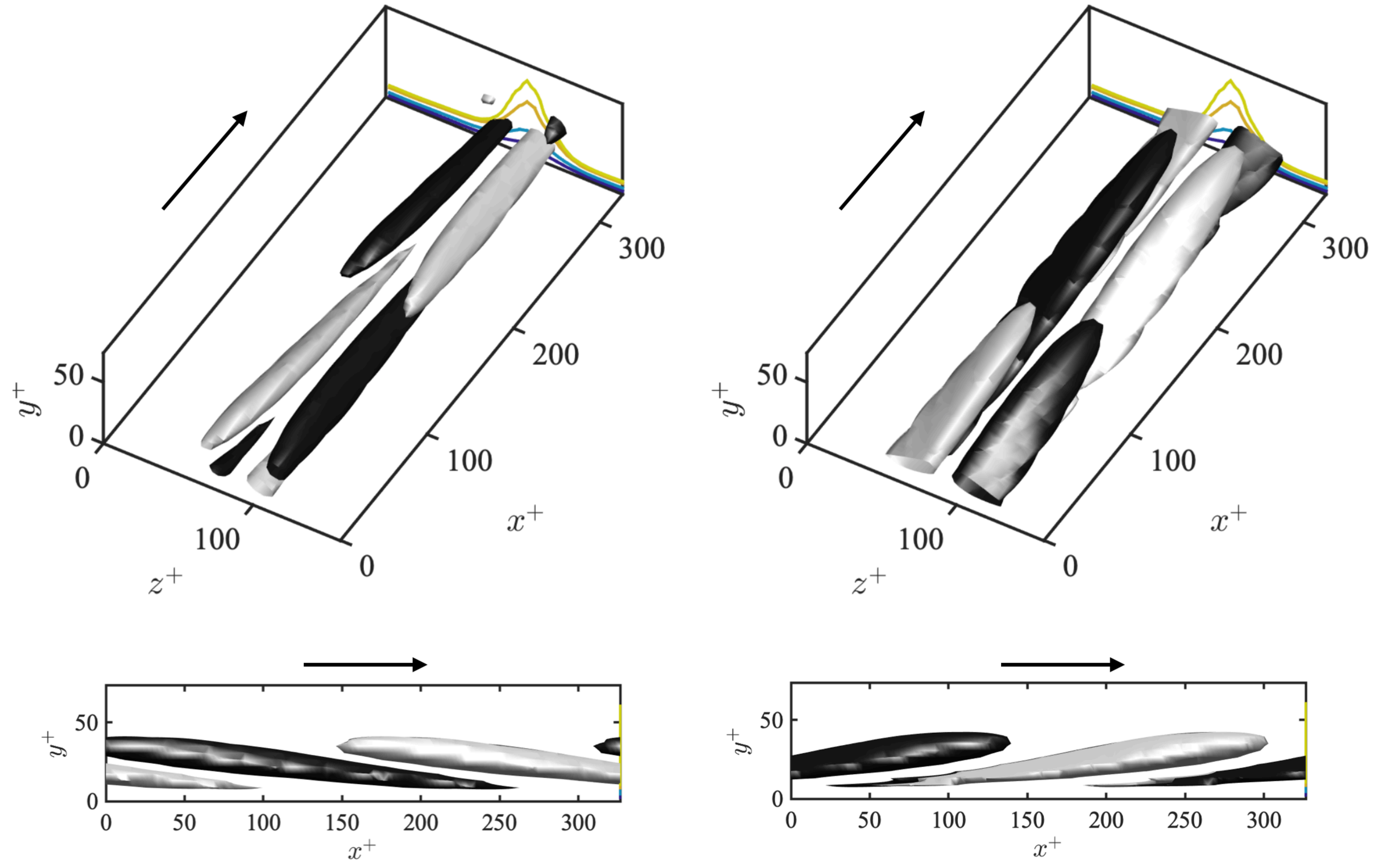
Non-modal transient growth
frozen base flow $U(y, z, t_0)$

$$\underbrace{G_{\max}\left(t_0, T\right)}_{\text{maximum energy gain}} = \max\left[\mathrm{svd}\left(\underbrace{\widetilde{\Phi}_{t_0,t_0+T}}_{\text{linear map for the stabilised } \widetilde{\mathscr{L}}}\right)^2\right]$$

typical optimal of $\widetilde{\Phi}$
for $T = 0.35 h/u_\tau$
$G_{\max} = 136$

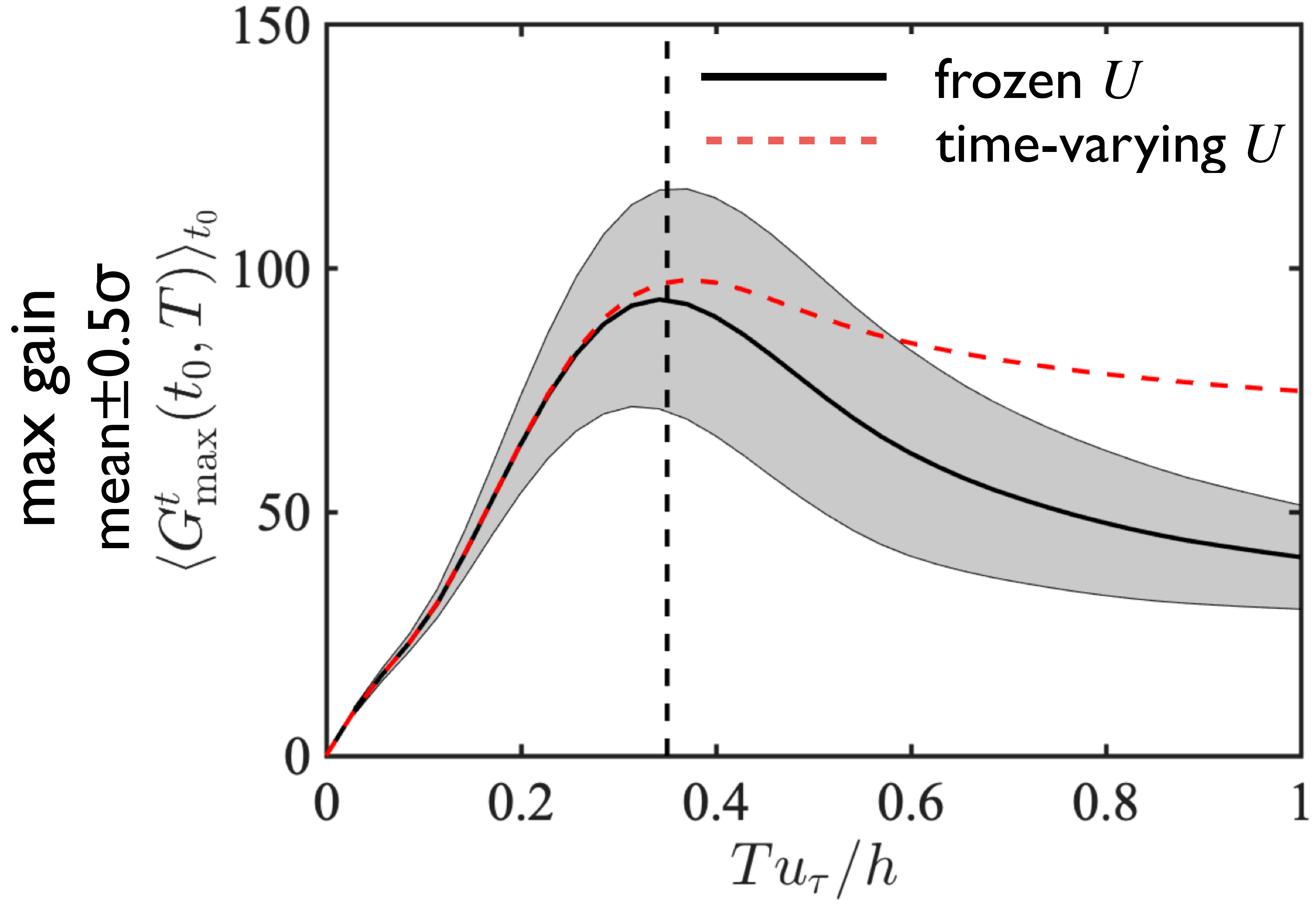
input mode/
right singular vector

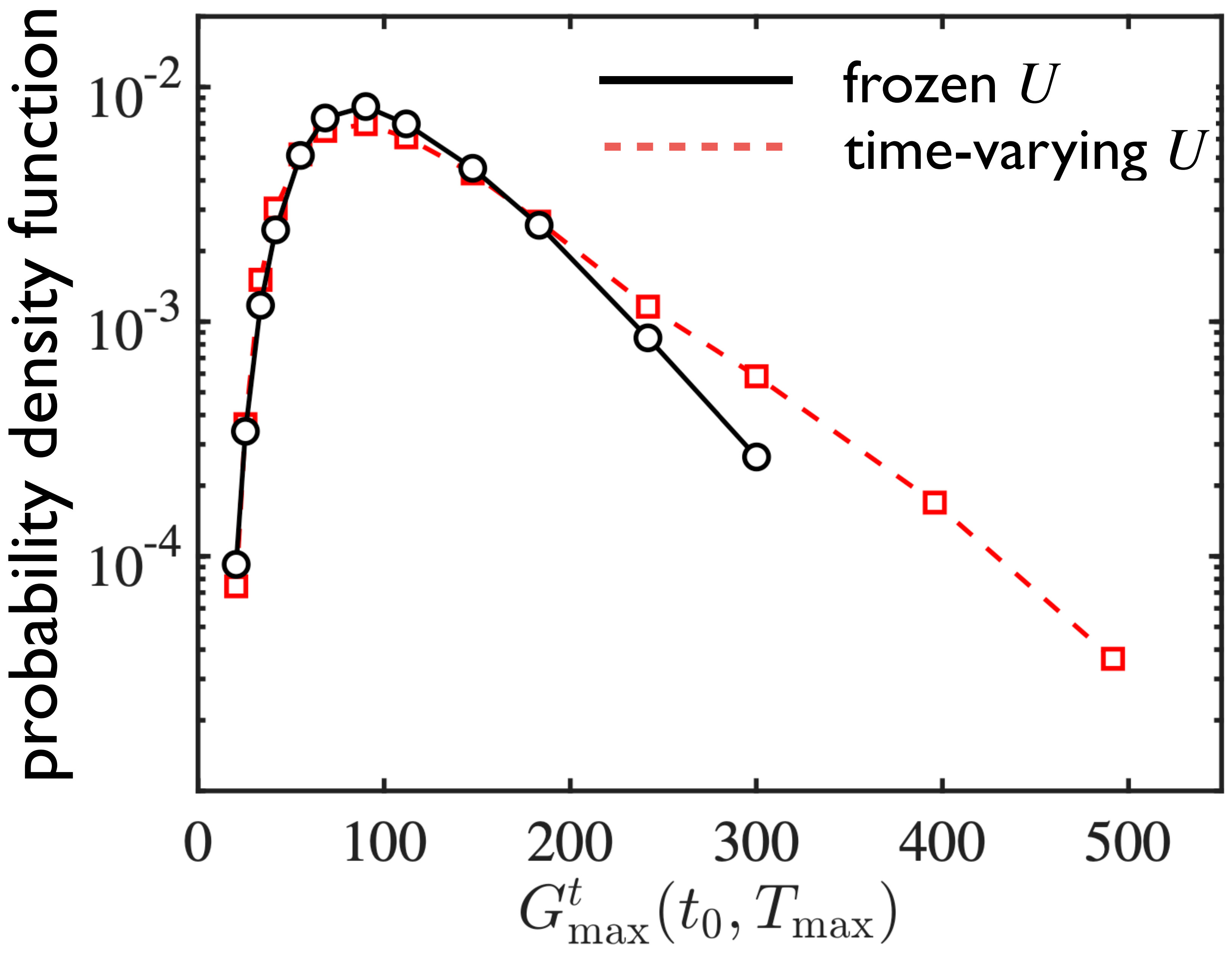
output mode/
left singular vector



Non-modal transient growth
time-varying base flow $U(y,z,t)$

$$\underbrace{G_{\max}(t_0, T)}_{\text{maximum energy gain}} = \max\left[\operatorname{svd}\Big(\underbrace{\widetilde{\Phi}_{t_0,t_0+T}}_{\text{linear map for the stabilised } \widetilde{\mathscr{L}}}\Big)^2\right]$$





Time-variability of the base flow $U(y,z,t)$ does not enhance energy transfer to fluctuations for short times.

Turbulence with *only* transient growth operable

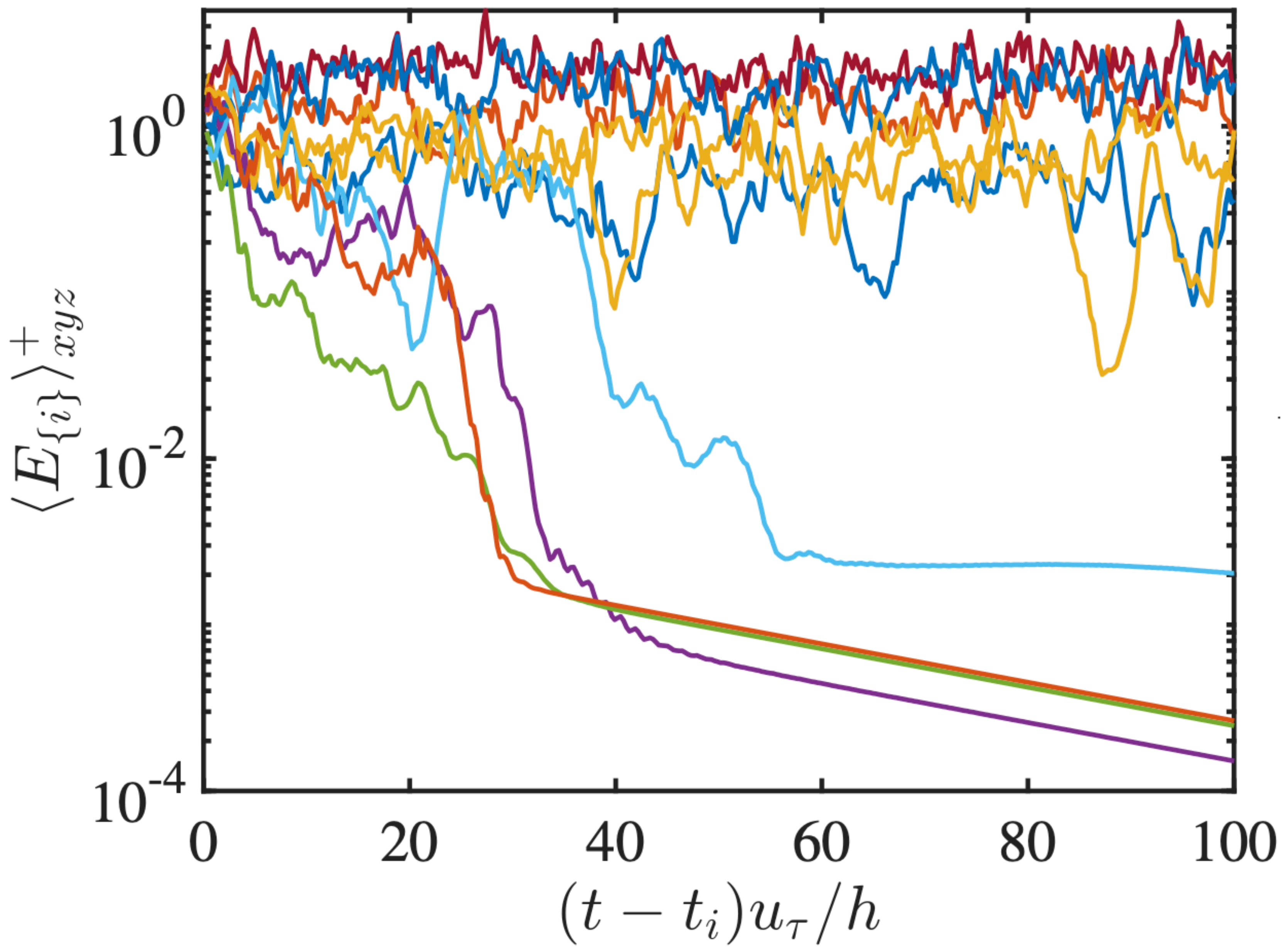
500 simulations

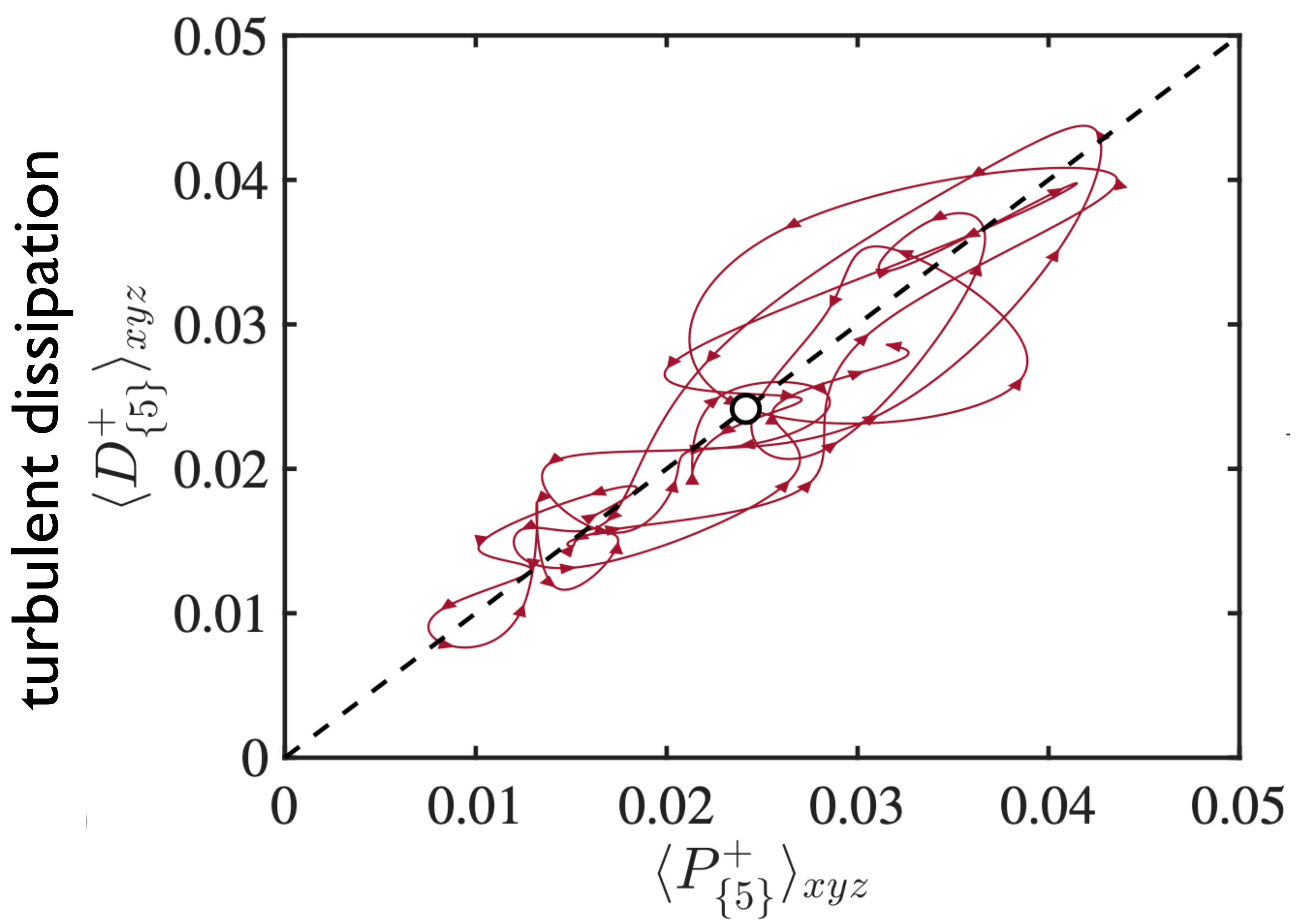
$$\frac{\partial \boldsymbol{u}'}{\partial t} = \widetilde{\mathscr{L}}(U(y,z,t_i))\,\boldsymbol{u}' + \mathscr{N}(\boldsymbol{u}') \qquad i = 1,2,\ldots,500$$

with a ***frozen*** snapshot $U(y,z,t_j)$ from DNS

Turbulence persist in ≈80% of the simulations.

TKE for 10 of the simulations





a case that sustains

Turbulence with *only* transient growth operable

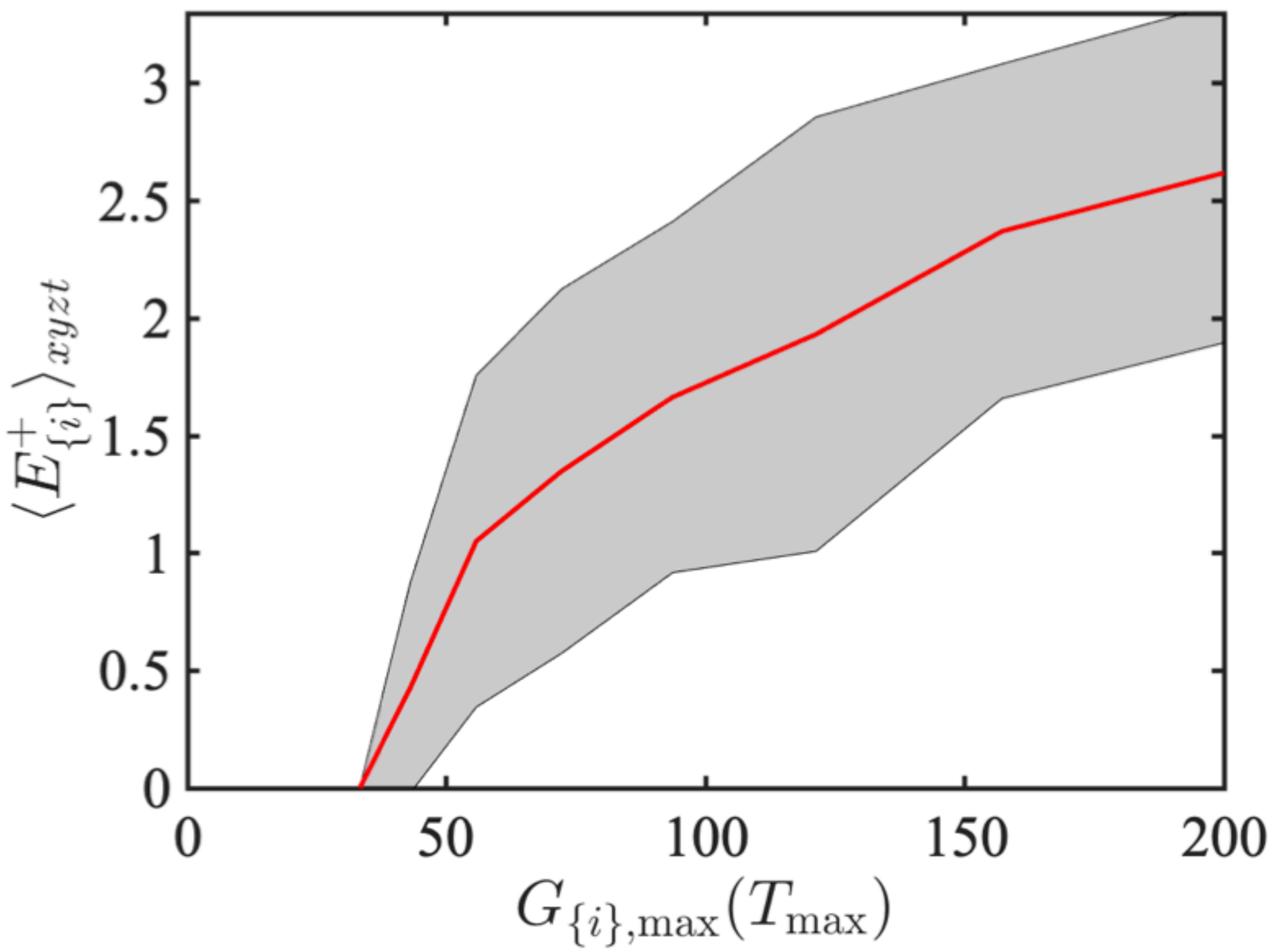
500 simulations

$$\frac{\partial \boldsymbol{u}'}{\partial t} = \widetilde{\mathscr{L}}(U(y,z,t_i))\,\boldsymbol{u}' + \mathscr{N}(\boldsymbol{u}')$$

$i = 1,2,\ldots,500$

with a ***frozen*** snapshot $U(y,z,t_j)$ from DNS

mean TKE $\pm 1\sigma$



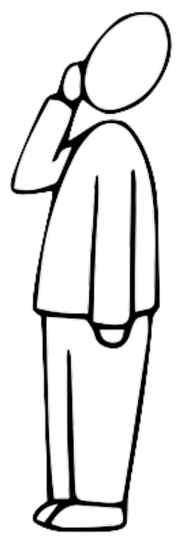
maximum gain

frozen base flows $U(y,z,t_i)$
with gain $\gtrapprox$40
sustain turbulence

(for $Re_\tau = 180$)

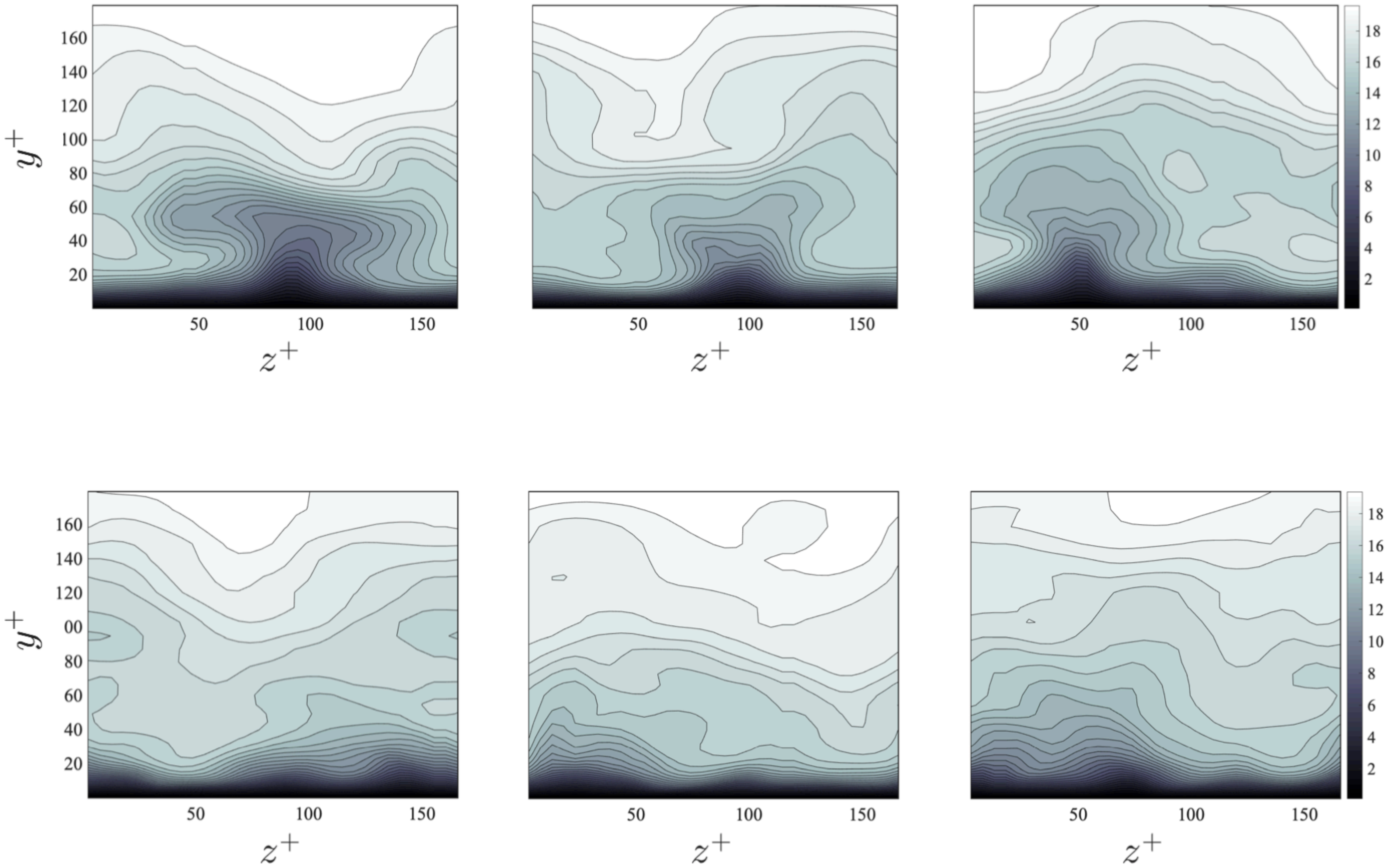
What differentiates the ***frozen*** base flows $U(y, z, t_i)$

that sustain turbulence from those which laminarise?



Spanwise streaky structure turns out *crucial* for $U(y, z, t_i)$ to sustain

these $U(y, z, t_i)$ sustain →



← these $U(y, z, t_i)$ laminarise

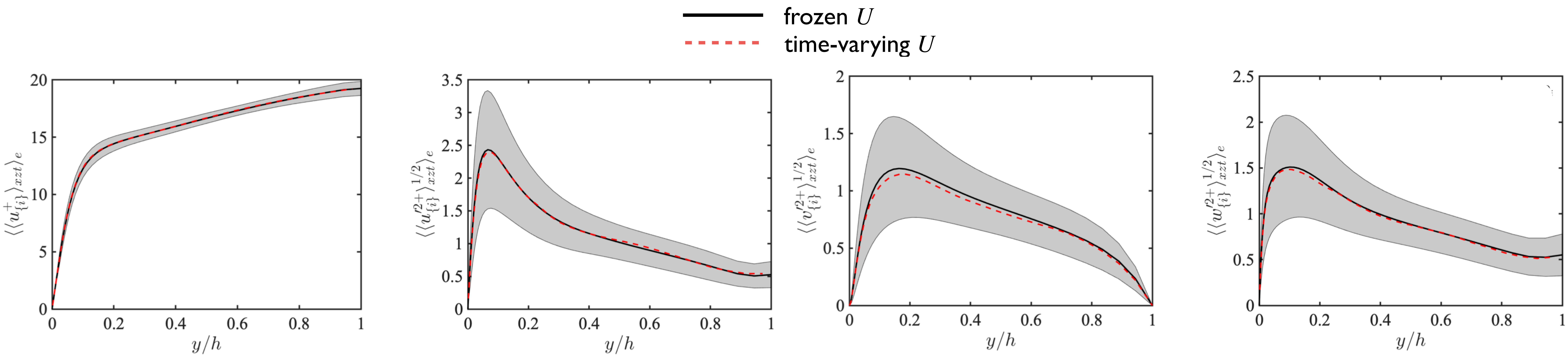
Precluding the 'push-over' mechanism due to spanwise base-flow shear leads to laminarisation.

[for detailed experiments demonstrating this see our paper: Lozano et al. *JFM* 2020]

Turbulence with *only* transient growth operable but time-varying U

$$\frac{\partial \boldsymbol{u}'}{\partial t} = \widetilde{\mathscr{L}}(U(y,z,t))\,\boldsymbol{u}' + \mathscr{N}(\boldsymbol{u}')$$

with a ***time-varying*** $U(y, z, t_j)$ from the DNS



ensemble of frozen snapshots $U(y, z, t_i)$ = time-varying $U(y, z, t)$

summary

modal instabilities of streaks are not crucial

how does energy go from the mean flow to the perturbations?

simple answer: transient growth

what produces this **transient growth?**

the ***spanwise*** shear of the streak & Orr mechanism

(not discussed here; see paper)

time-variability of the streak does not enhance energy transfer to fluctuations
but allows flow to “sample” independent transient-growth events resulting to the observed statistics

realistic wall-turbulence can be exclusively supported by transient growth

Lozano-Duran et al. (2020) Cause-and-effect of linear mechanisms sustaining wall turbulence, *J. Fluid Mech.* (In press; arXiv:2005.05303)