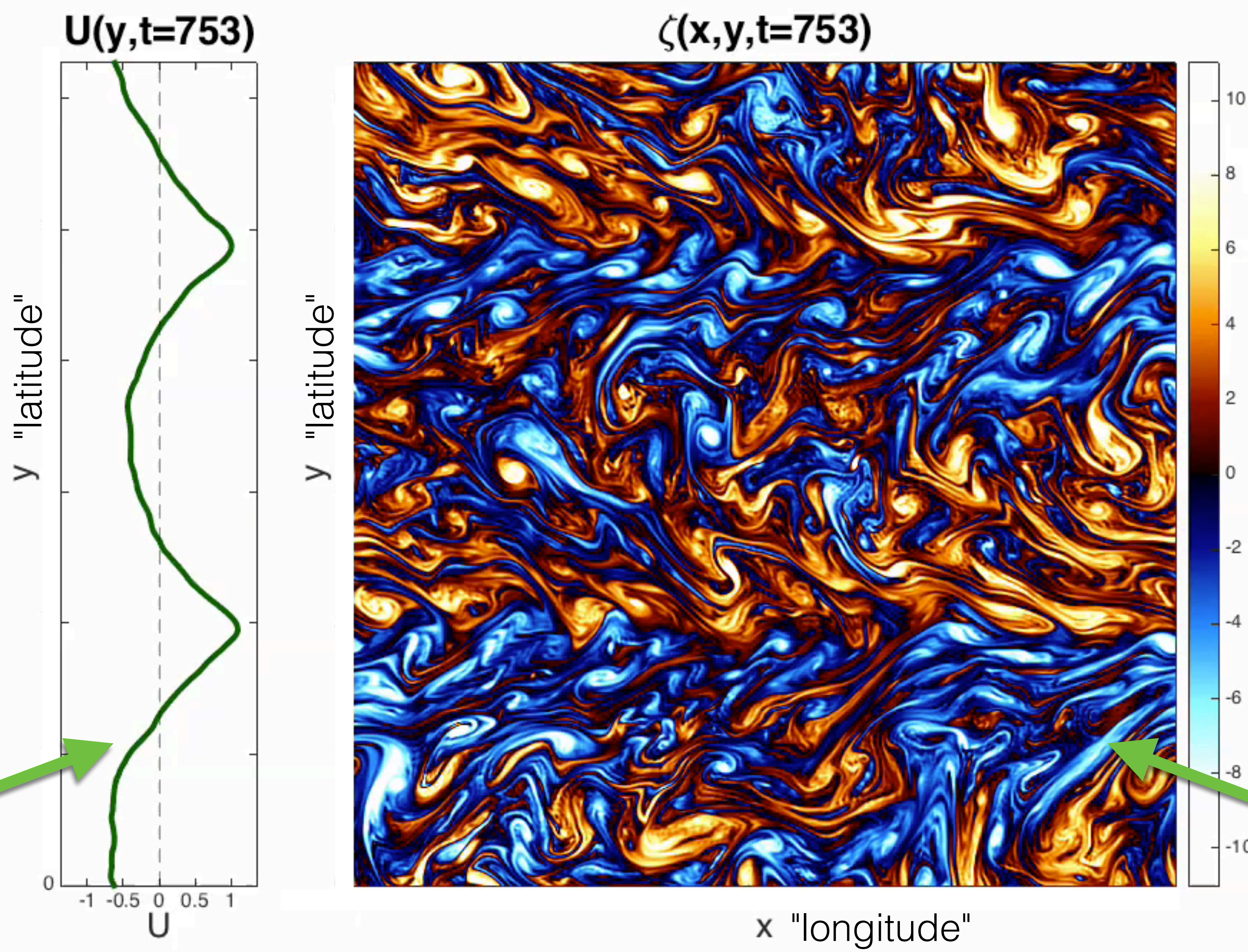
Magnetic suppression of zonal flows on a beta plane

Navid Constantinou



zonal velocity averaged over x (longitude)

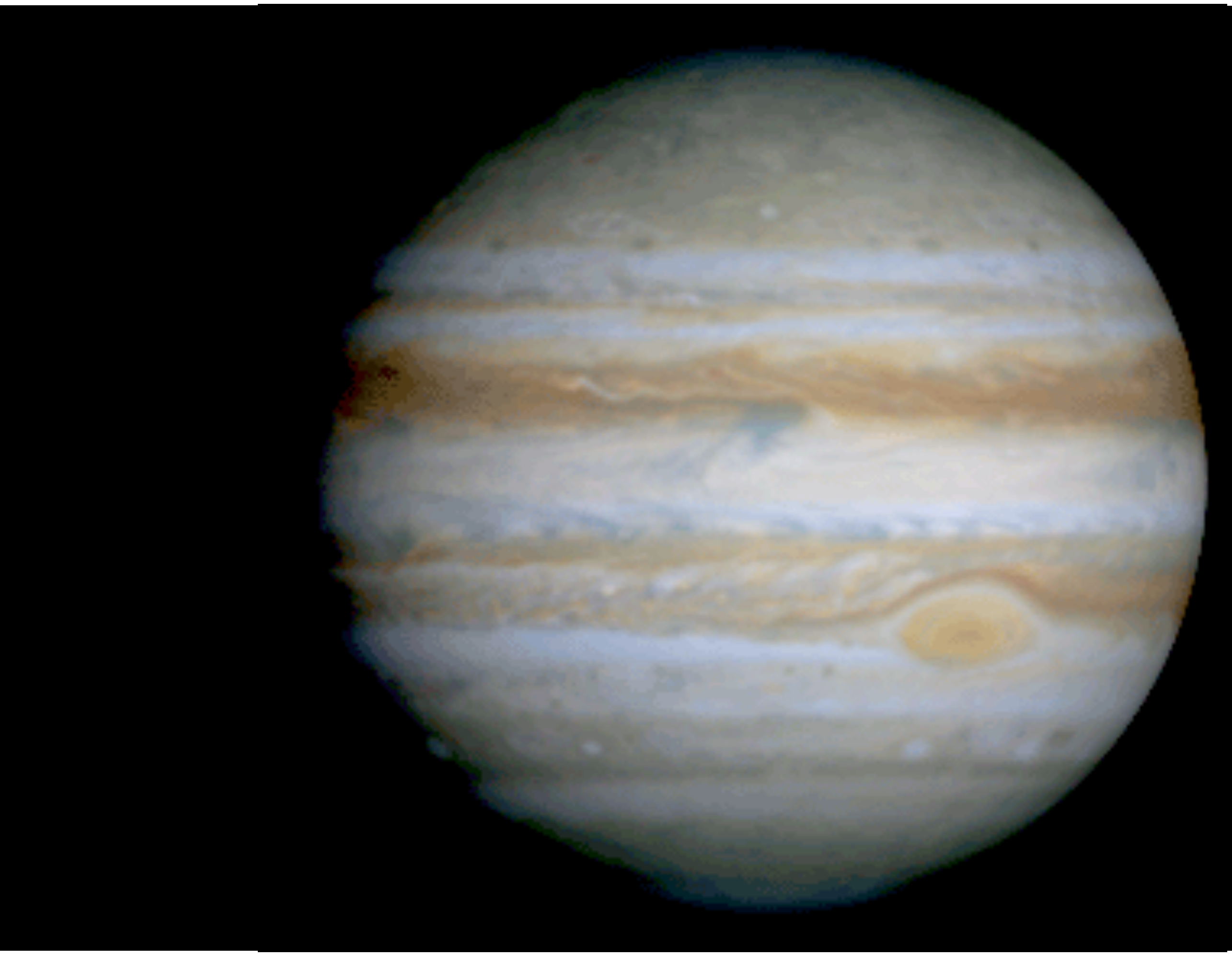
relative vorticity

zonal jet formation on a beta plane

Planetary turbulence

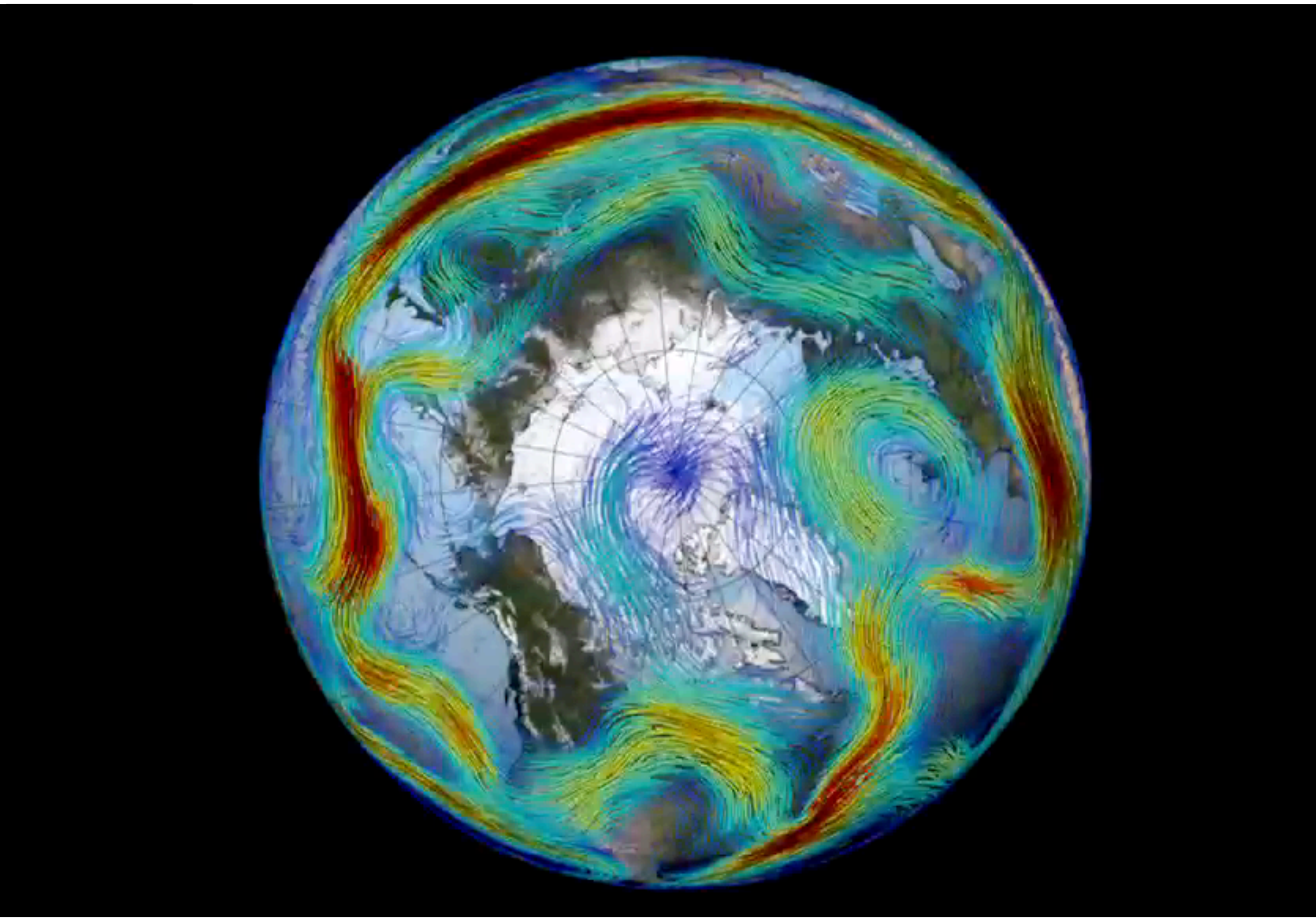
most of the energy of the flow is in large-scale coherent jets and vortices of specific form

not at the largest allowed scale (as inverse cascade might imply)
"arrest" of the cascade by jets



banded Jovian jets

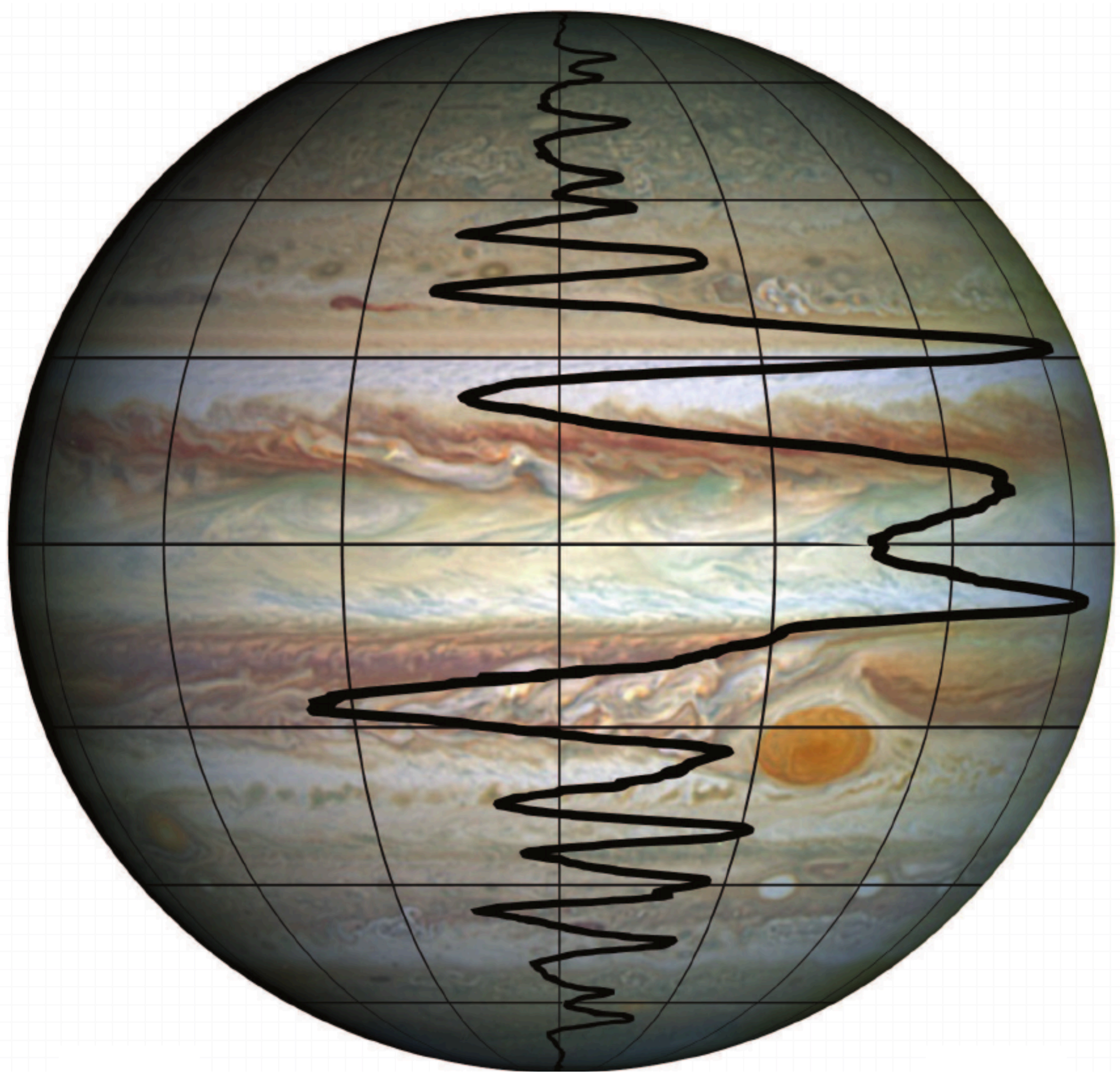
NASA/Cassini Jupiter Images

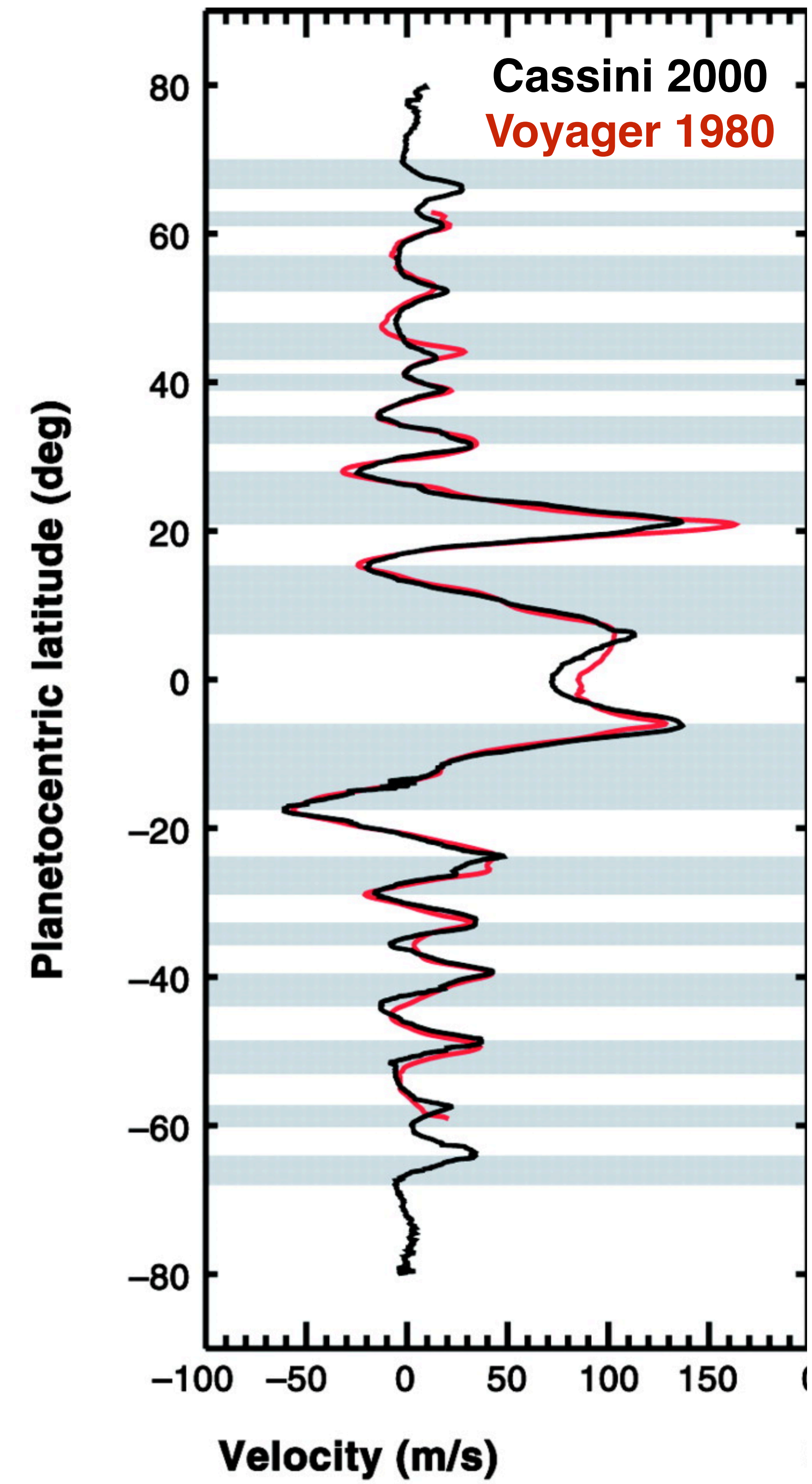


polar front jet

NASA/Goddard Space Flight Center

Jets appear to be “steady”



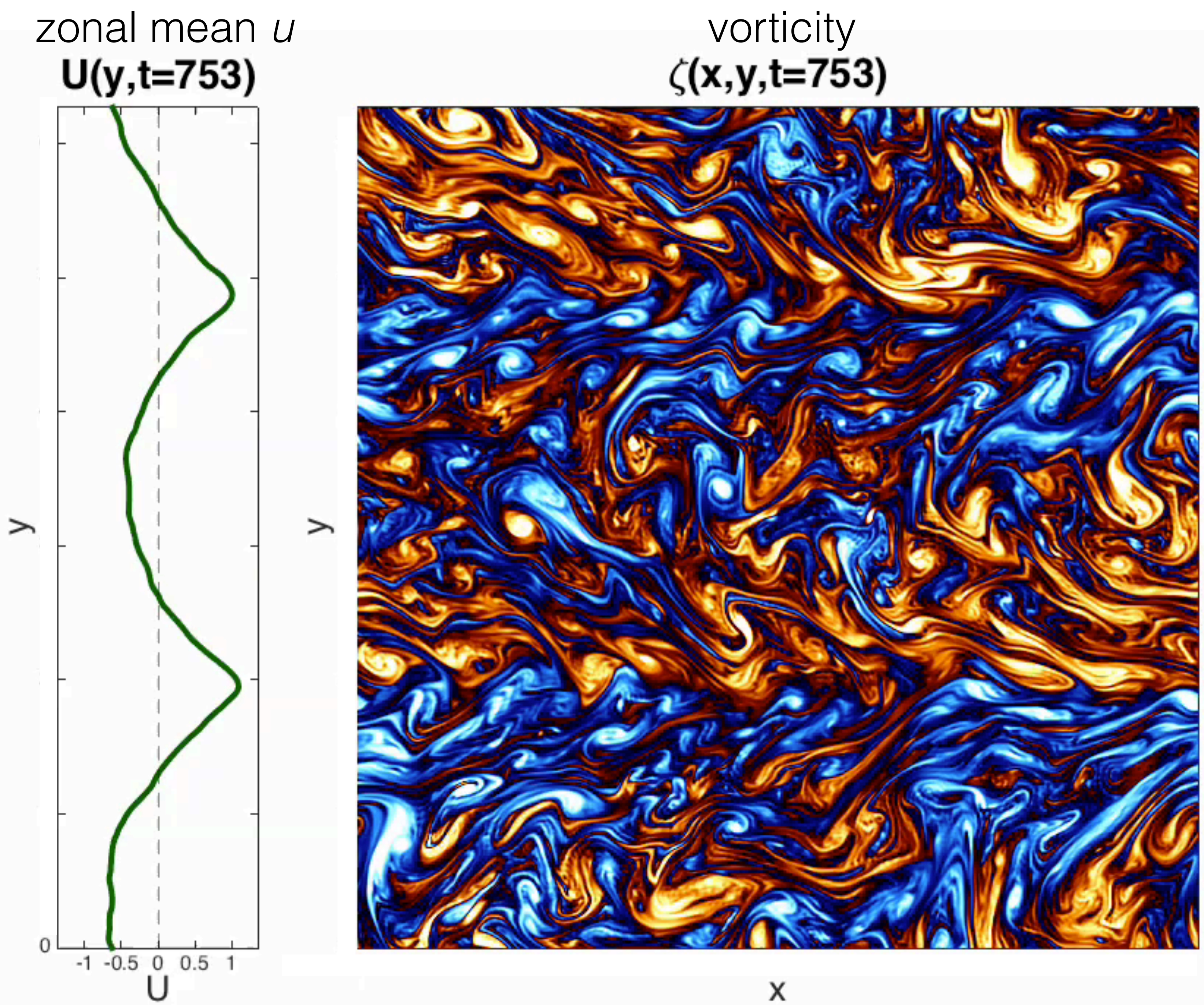


The problem to be addressed:

Understand how these *specific* structures arise
and how are they maintained

What's a *minimal model* for studying zonal jets?

zonal jet formation in forced-dissipative barotropic β plane



statistically homogeneous small-scale forcing

(forcing **does not** impose any inhomogeneity)

random flow inhomogeneities organize the turbulence so that they are reinforced

we observe:

- jet emerge
- jets appear to change *much slower* compared to the eddies
- jets may merge

non-dimensional parameters

$\varepsilon k_f^2 / \mu^3 = 10^6$ (≈"amplitude of forcing")

$\beta / (k_f \mu) = 67$ (≈"rotation of the planet")

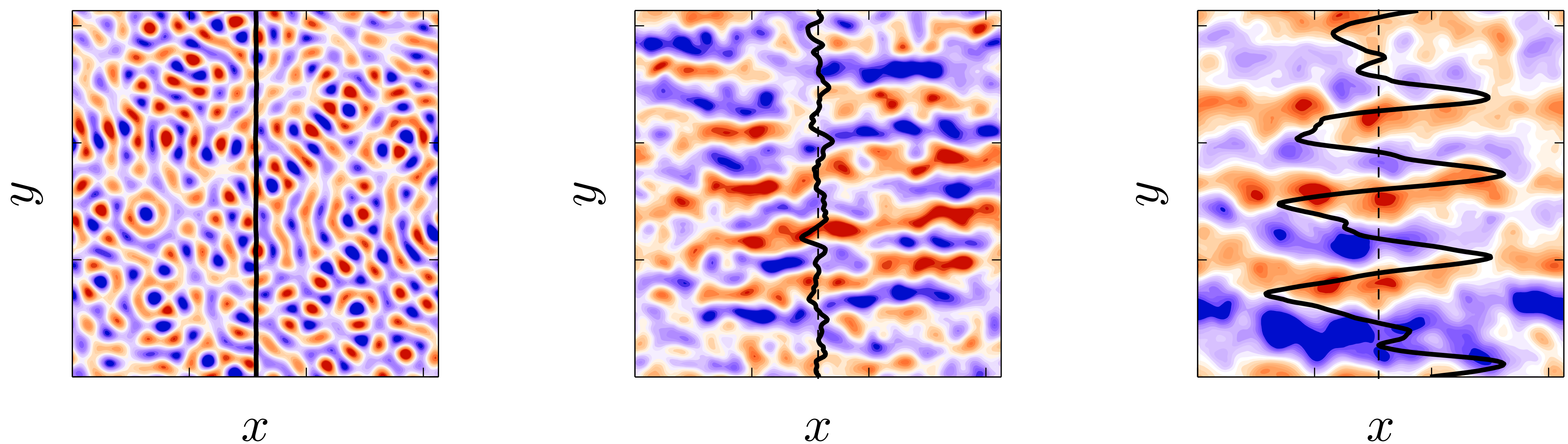
β gradient of Coriolis parameter, μ linear drag, ε energy injection rate by the forcing; k_f characteristic wavenumber of forcing

various β-plane flow regimes flows
at statistically steady state:

homogeneous — traveling waves — zonal jets

$\beta/(k_f r) = 67$

$\varepsilon k_f^2/\mu^3 \ = 10^2$ $\qquad 5\times10^3 \qquad 5\times10^4$



this suggests that there is some kind of transition as ε is increased

[snapshots of the streamfunction $\psi(\mathbf{x},t)$ with instantaneous zonal mean flow $U(y,t)$]

Are these transitions a result of the arrest of the inverse energy cascade by Rossby waves?

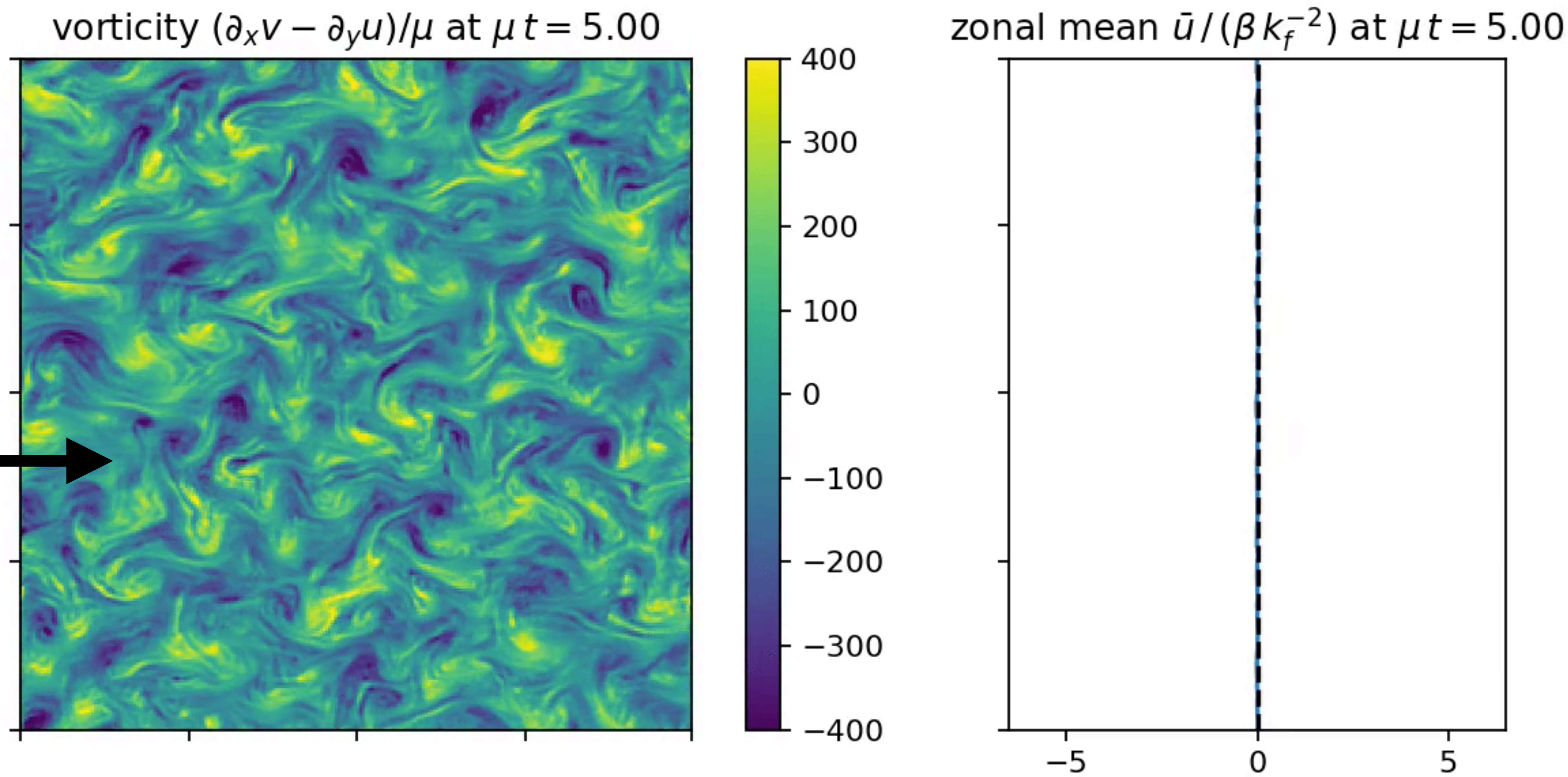
(This is what all textbooks say at least...)

Or do these transitions are similar to phase transitions, i.e., occur at critical threshold parameter values?

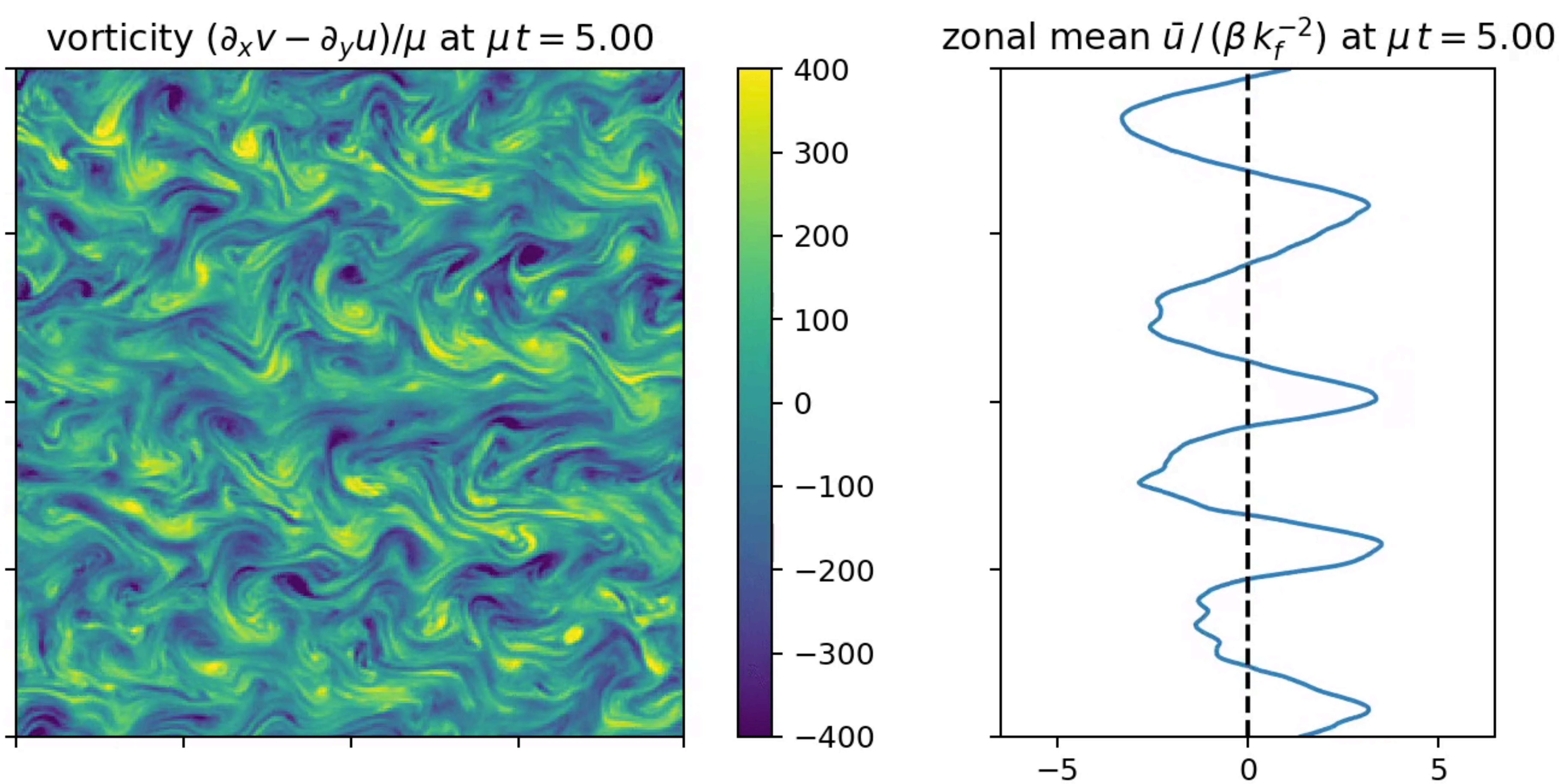
That would be the case if transitions occur due to instabilities...
But how to we study stability of ***turbulent*** flows?

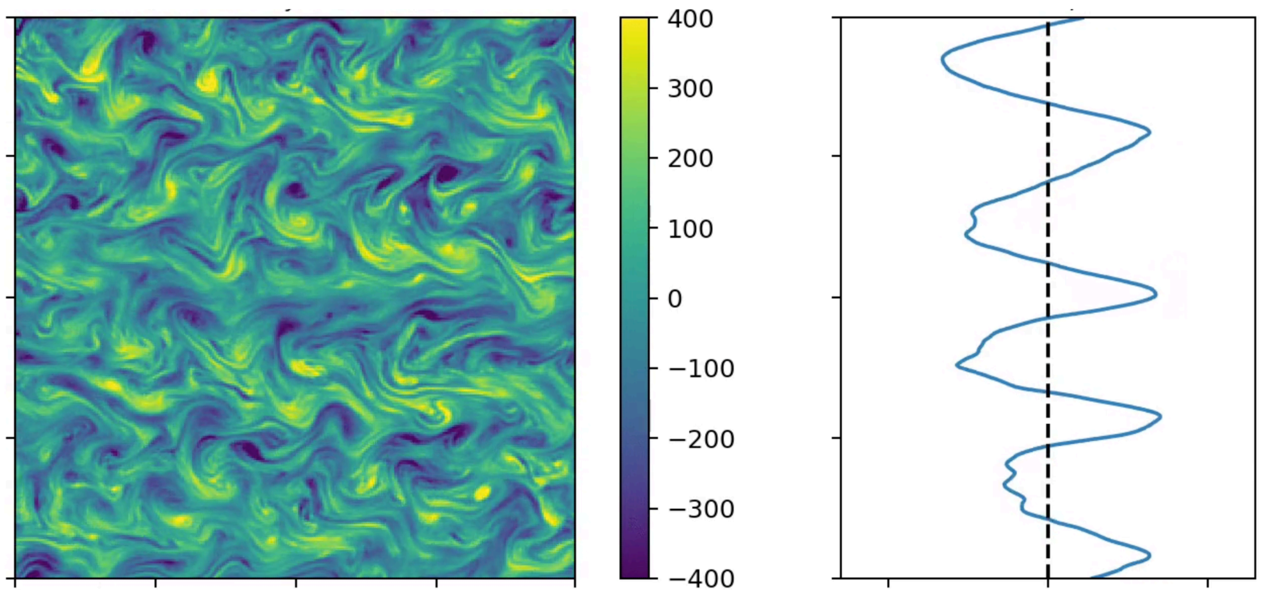
how do we show that
a flow like this ...

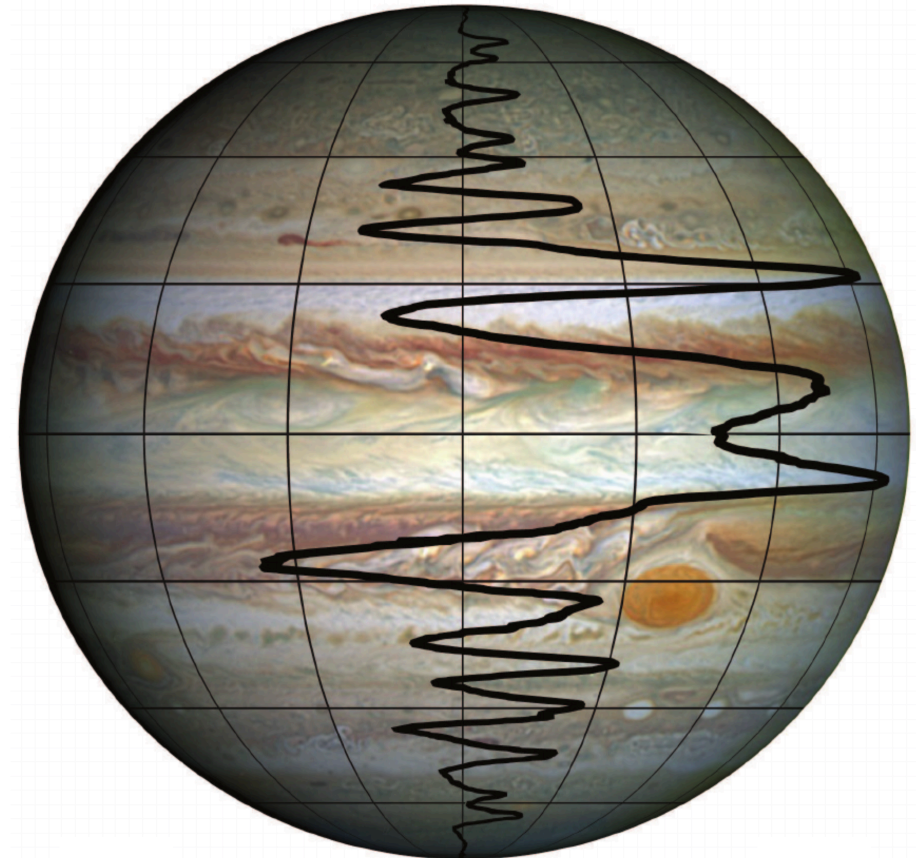
[simulation in which we kill the k_x=0 component at each time step]



... is ***unstable*** leading
to forming four jets?





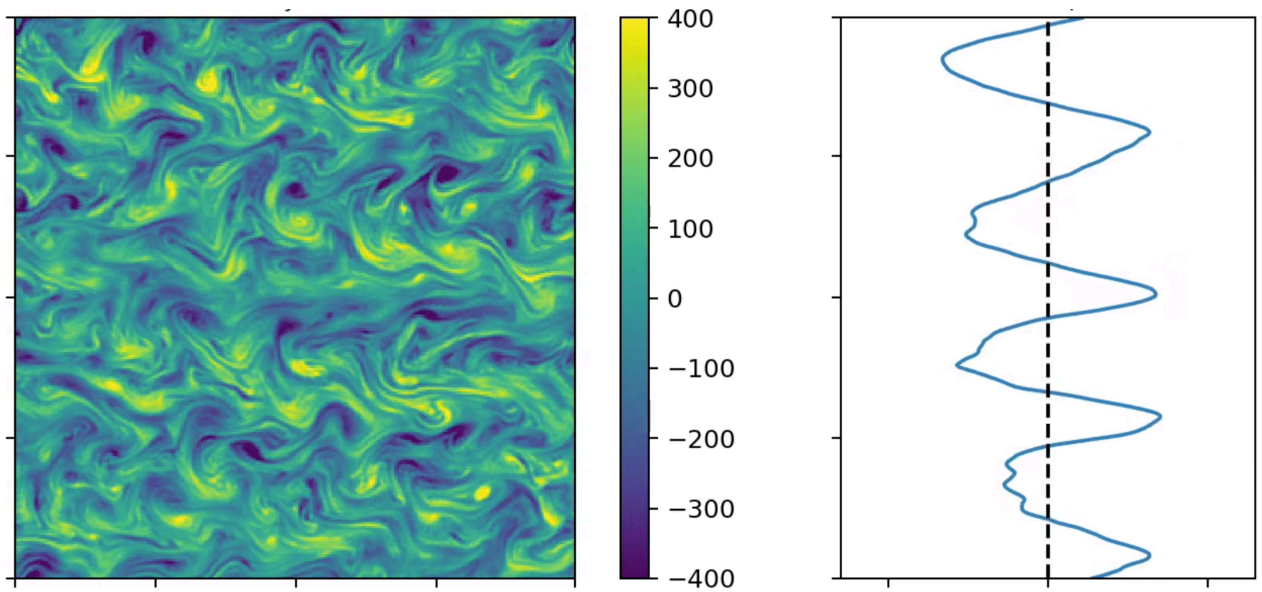


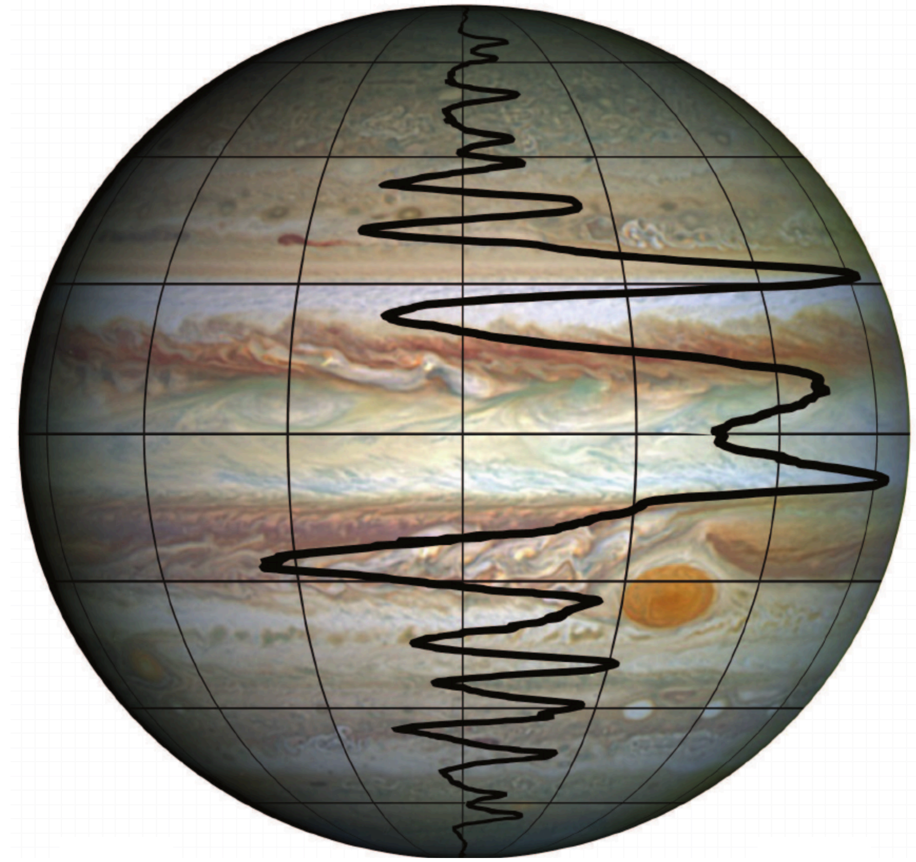
at statistical equilibrium:

jets + turbulent eddies

$\approx$ steady

strongly time-dependent





at statistical equilibrium:

jets **+** eddy **statistics**

≈steady ≈stationary

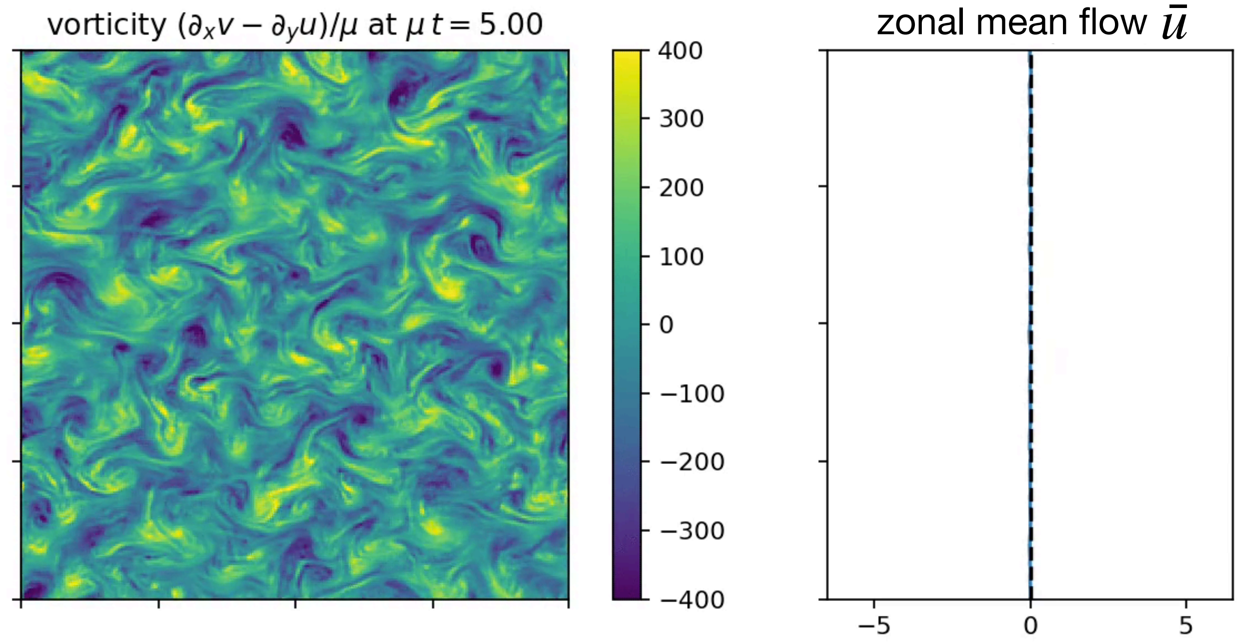
Lorenz's vision

Ed Lorenz

"More than any other theoretical procedure, numerical integration is also subject to the criticism that it yields little insight into the problem. [...] *An alternative procedure which does not suffer this disadvantage consists of deriving a new system of equations whose unknowns are the statistics themselves.*"

The Nature and Theory of the General Circulation of the Atmosphere, by E. N. Lorenz, **1967**

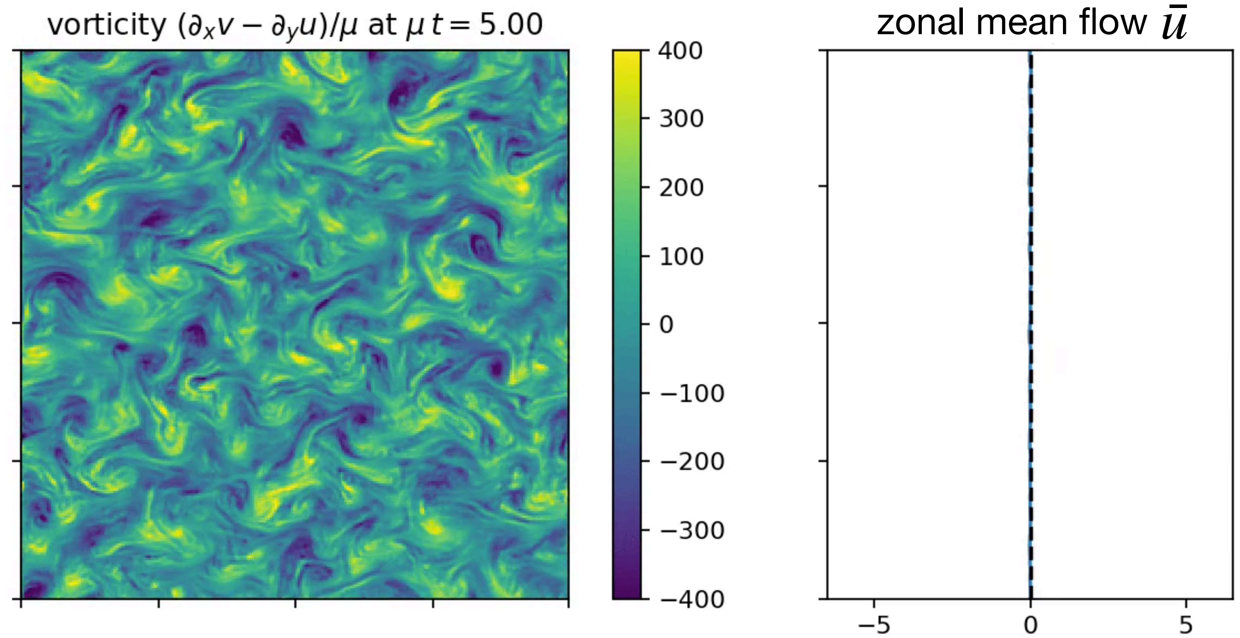
Statistical State Dynamics (SSD):
the dynamics that govern the statistics of the flow rather than those governing single flow realizations



homogeneous
stationary
second-order
eddy statistics

+ no mean flow

fixed point of the second-order closure of the SSD



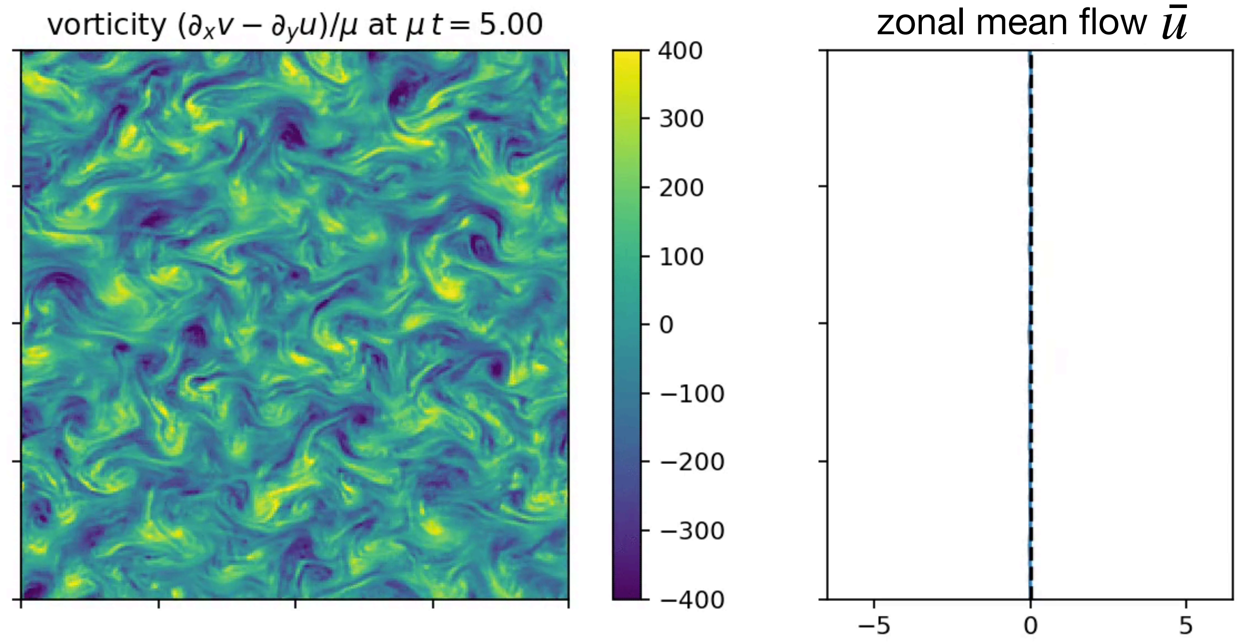
homogeneous
stationary
second-order
eddy statistics

+ no mean flow

fixed point of the second-order closure of the SSD

let's perturb it and study its stability...

(doable, but we have to solve an eigenvalue problem of dimension $n^4 \times n^4$)



homogeneous
stationary
second-order
eddy statistics

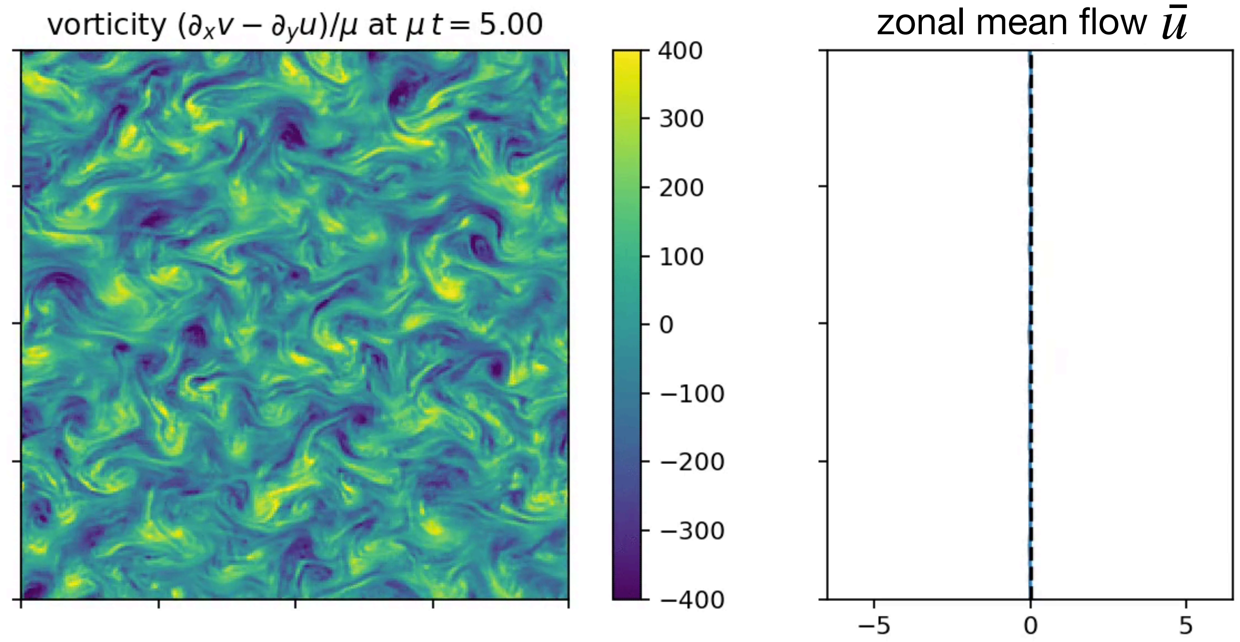
+ no mean flow

fixed point of the second-order closure of the SSD

let's perturb it and study its stability...

(doable, but we have to solve an eigenvalue problem of dimension $n^4 \times n^4$)

note: we've linearized about a turbulent state!

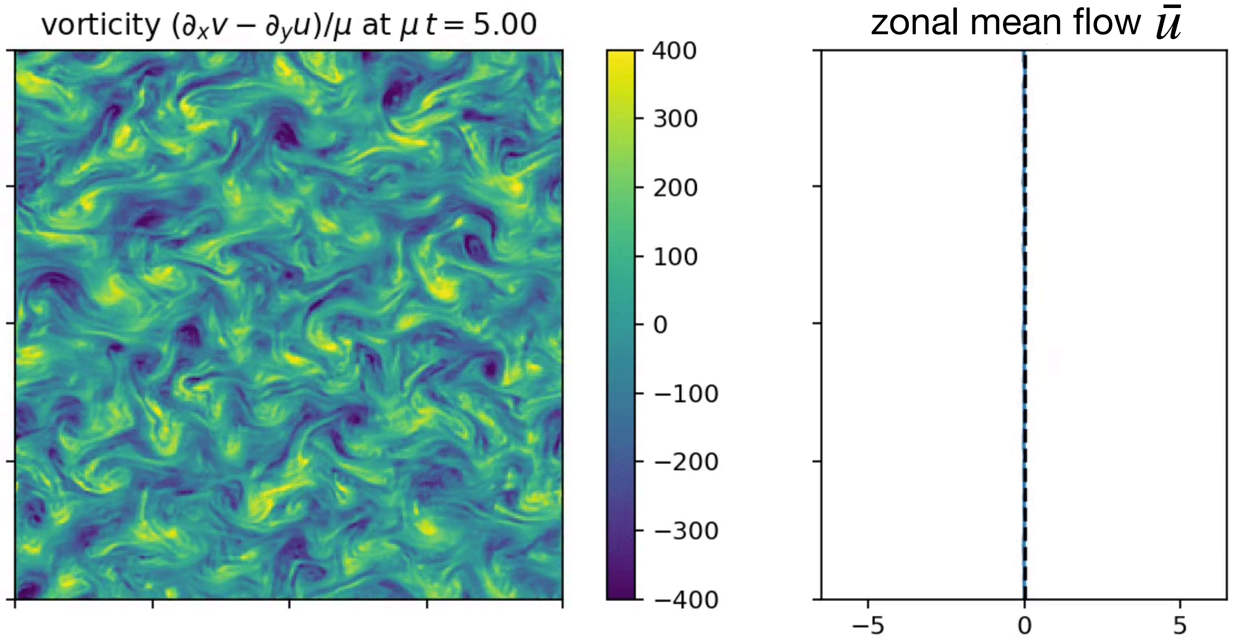


homogeneous
stationary
second-order
eddy statistics

+ no mean flow

as we cross a threshold value of $\varepsilon k_f^2 / \mu^3$

the homogeneous turbulent state ***becomes unstable*** to infinitesimal zonal jet mean flow perturbations



homogeneous
stationary
second-order
eddy statistics

+ no mean flow

as we cross a threshold value of $\varepsilon k_f^2 / \mu^3$

the homogeneous turbulent state ***becomes unstable*** to infinitesimal zonal jet mean flow perturbations

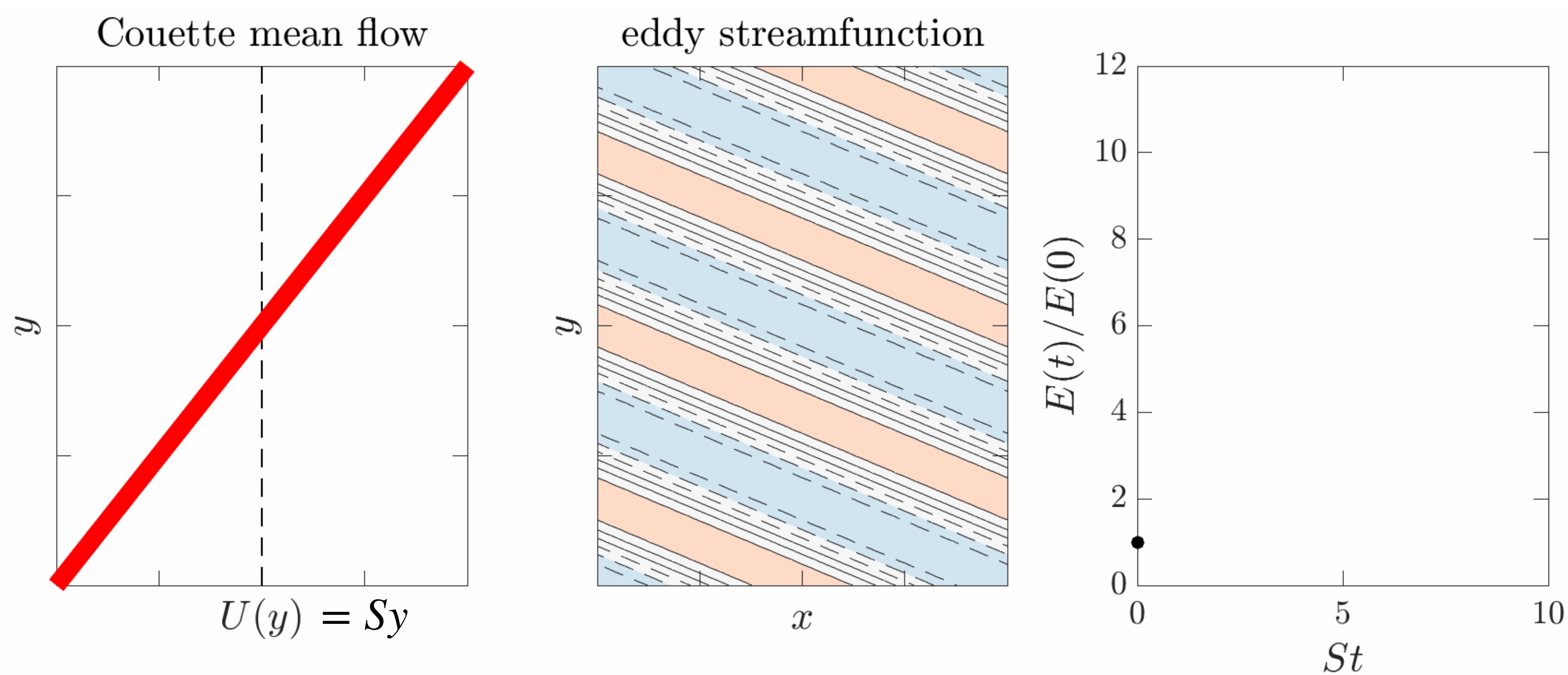
Kelvin–Orr wave solution

linearized barotropic vorticity equation

$$(\partial_t + Sy\partial_x)\,\nabla^2\psi = 0$$

$$\nabla^2\psi(t=0) = Z_0 \mathrm{e}^{\mathrm{i}[k_x x + k_y y]} \Rightarrow \nabla^2\psi = Z_0 \mathrm{e}^{\mathrm{i}[k_x x + (k_y - Sk_x t)y]}$$

$$\Rightarrow \psi = -\frac{Z_0 \mathrm{e}^{\mathrm{i}[k_x x + (k_y - Sk_x t)y]}}{k_x^2 + (k_y - Sk_x t)^2}$$



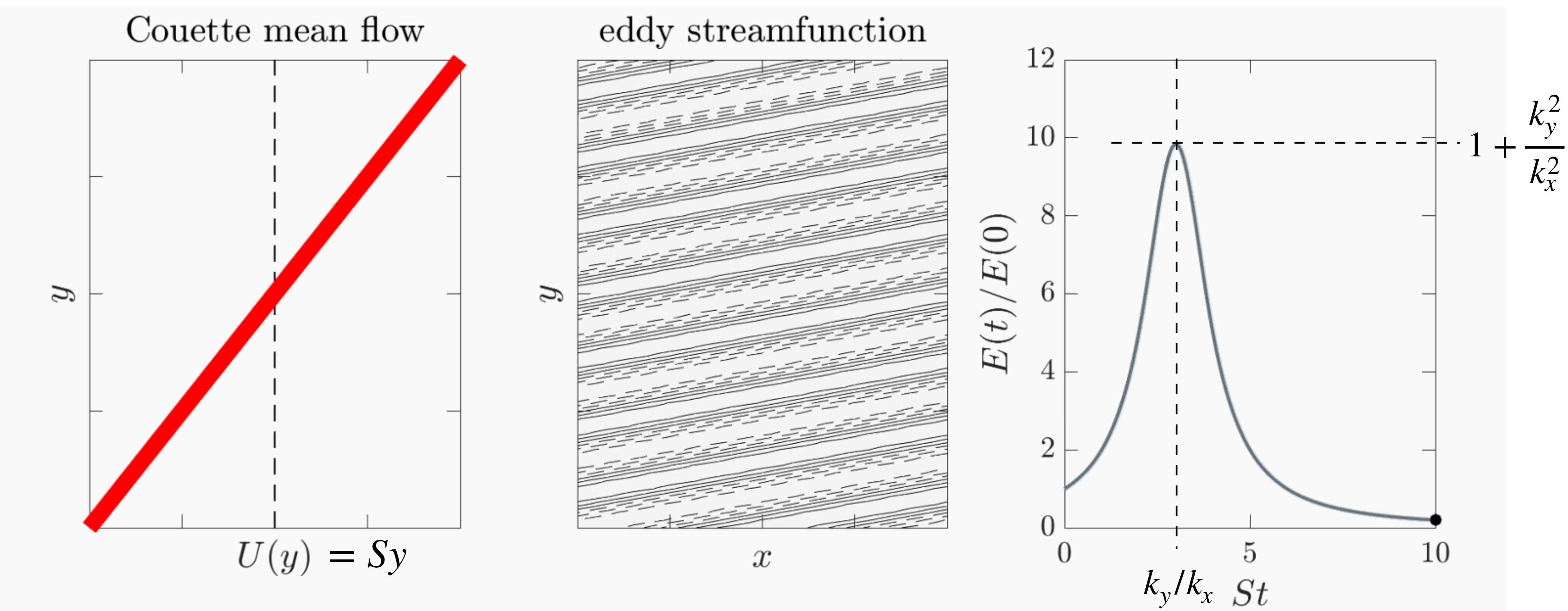
Kelvin–Orr wave solution

linearized barotropic vorticity equation

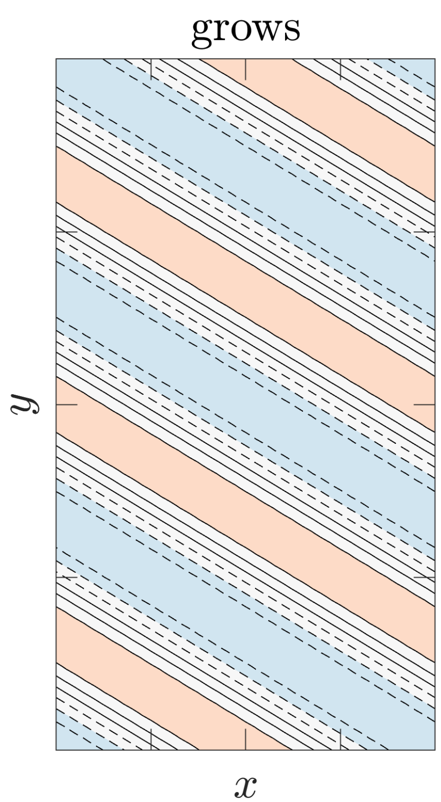
$$(\partial_t + Sy\partial_x)\nabla^2\psi = 0$$

$$\nabla^2\psi(t=0) = Z_0 e^{i[k_x x + k_y y]} \Rightarrow \nabla^2\psi = Z_0 e^{i[k_x x + (k_y - Sk_x t)y]}$$

$$\Rightarrow \psi = -\frac{Z_0 e^{i[k_x x + (k_y - Sk_x t)y]}}{k_x^2 + (k_y - Sk_x t)^2}$$

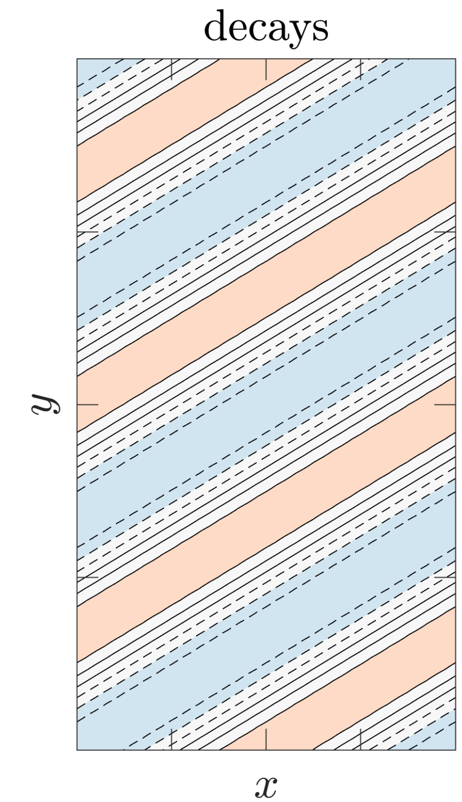


Kelvin–Orr wave solution
weak shear limit



takes energy from
zonal flow

gives energy to
zonal flow



the energy contribution to zonal flow
from a **pair** of vorticity waves initially with $(k_x, \pm k_y)$ is

$$\Delta E_{\text{zonal flow}} = \frac{S^2 k_x^2}{4\nu^2 k^4} \frac{k_x^2 - 5k_y^2}{k^6} |Z_0|^2$$

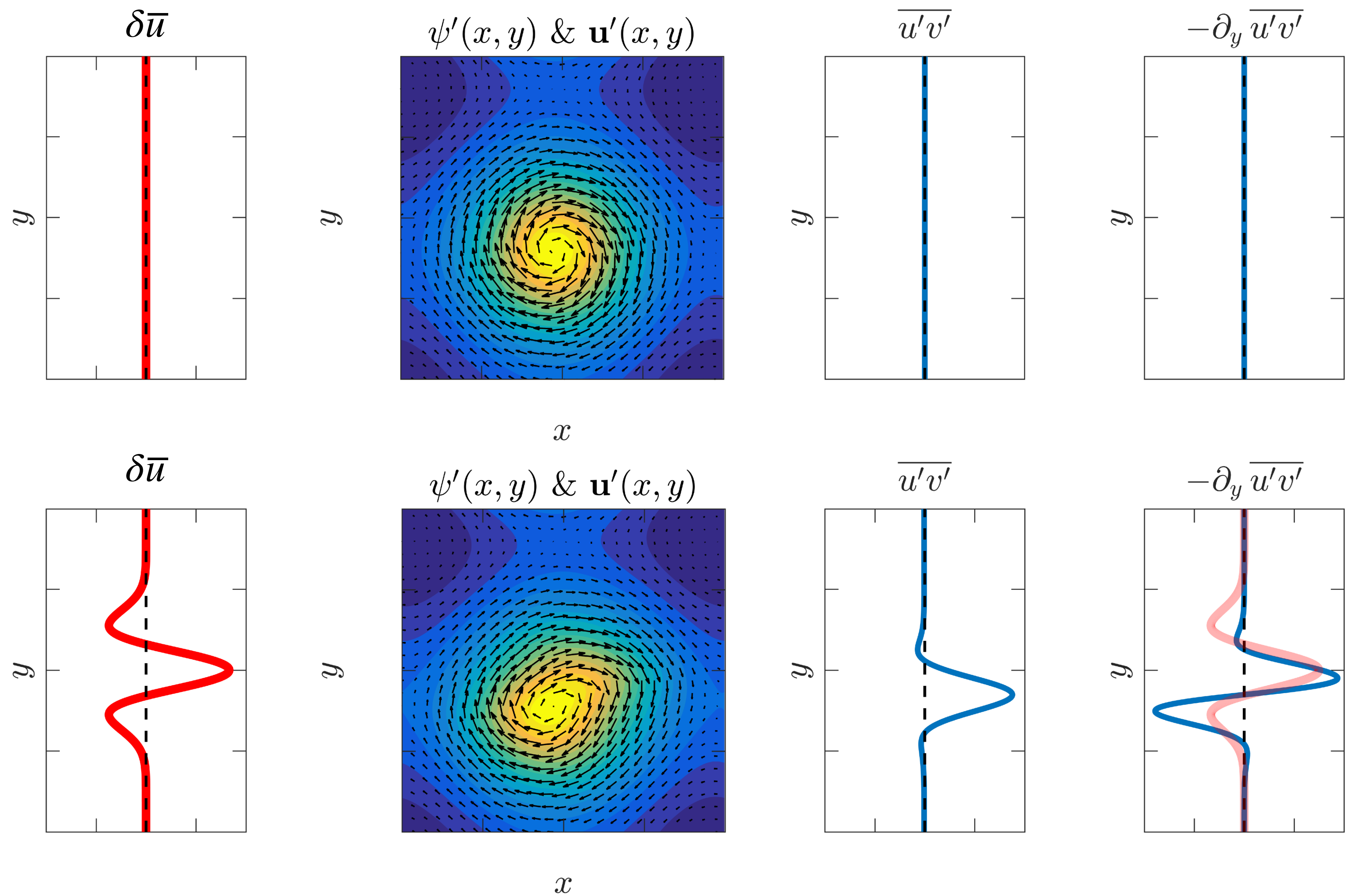
proof of concept

how does a zero jet state become unstable?

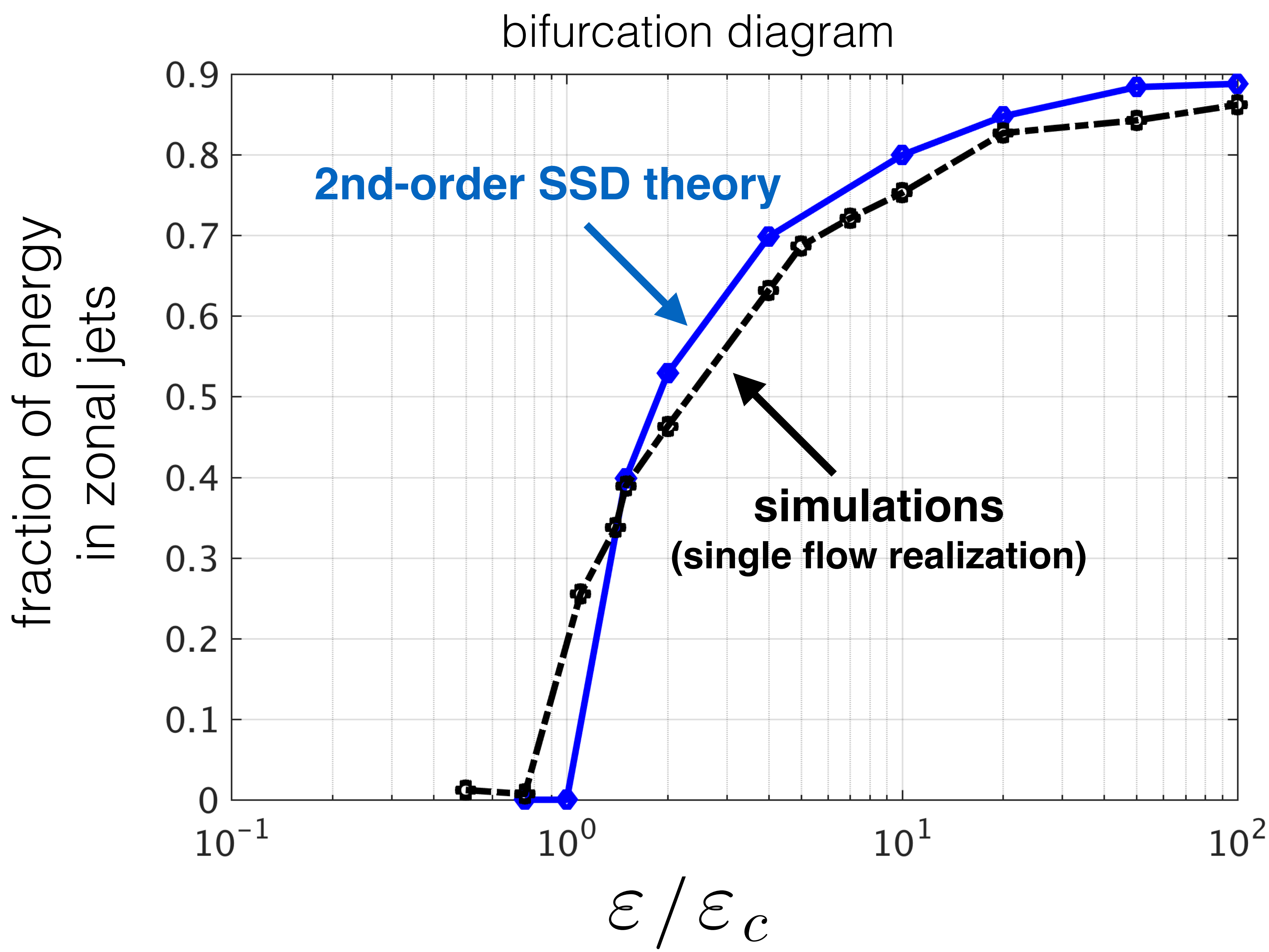
for certain parameters eddies have the tendency to reinforce
mean flow inhomogeneities (even if mean flow is infinitesimal!)

$$\partial_t \overline{u} = -\,\partial_y \overline{u'v'} + \text{dissipation}$$

Reynolds stress

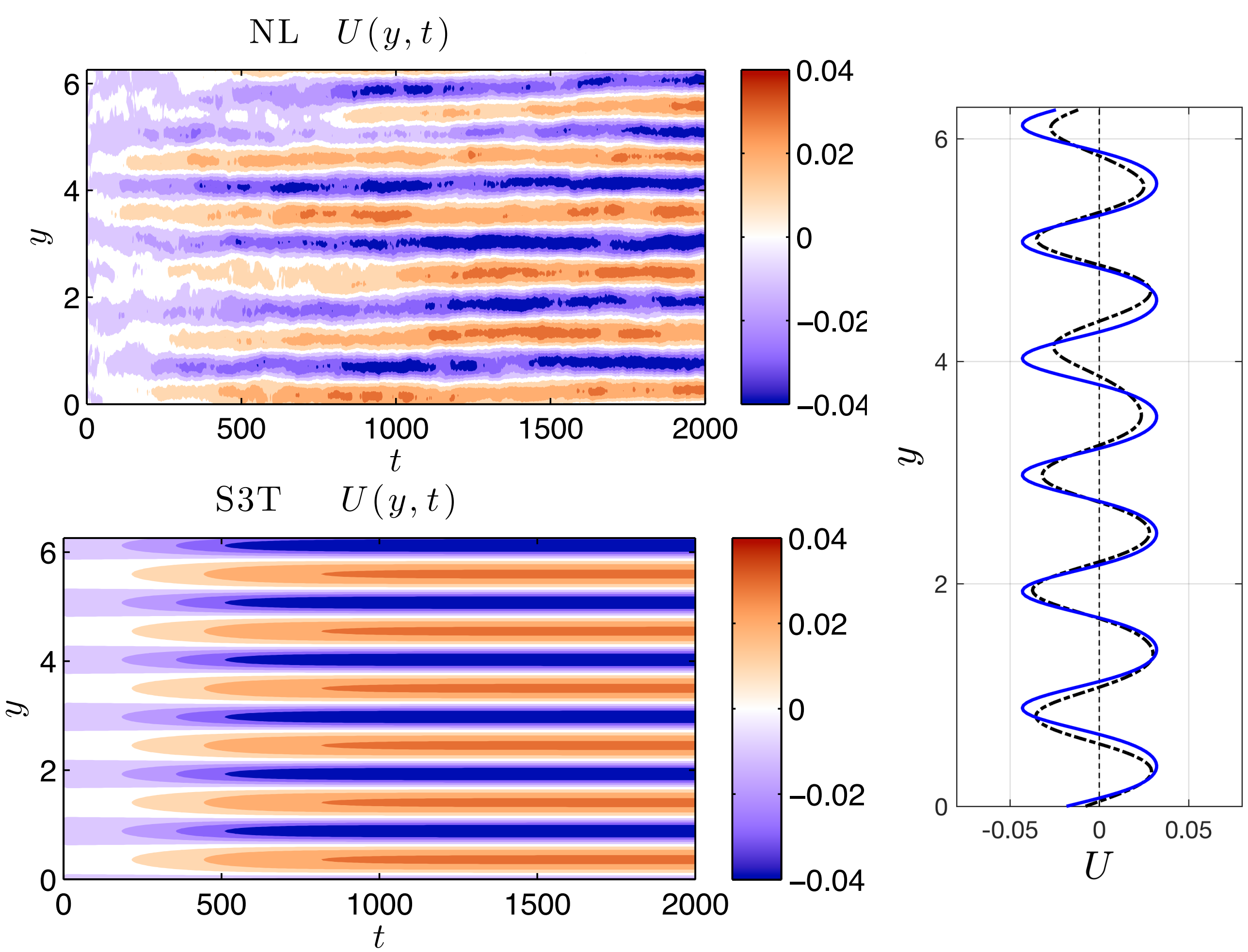


Verification of SSD stability predictions for the jet formation bifurcation

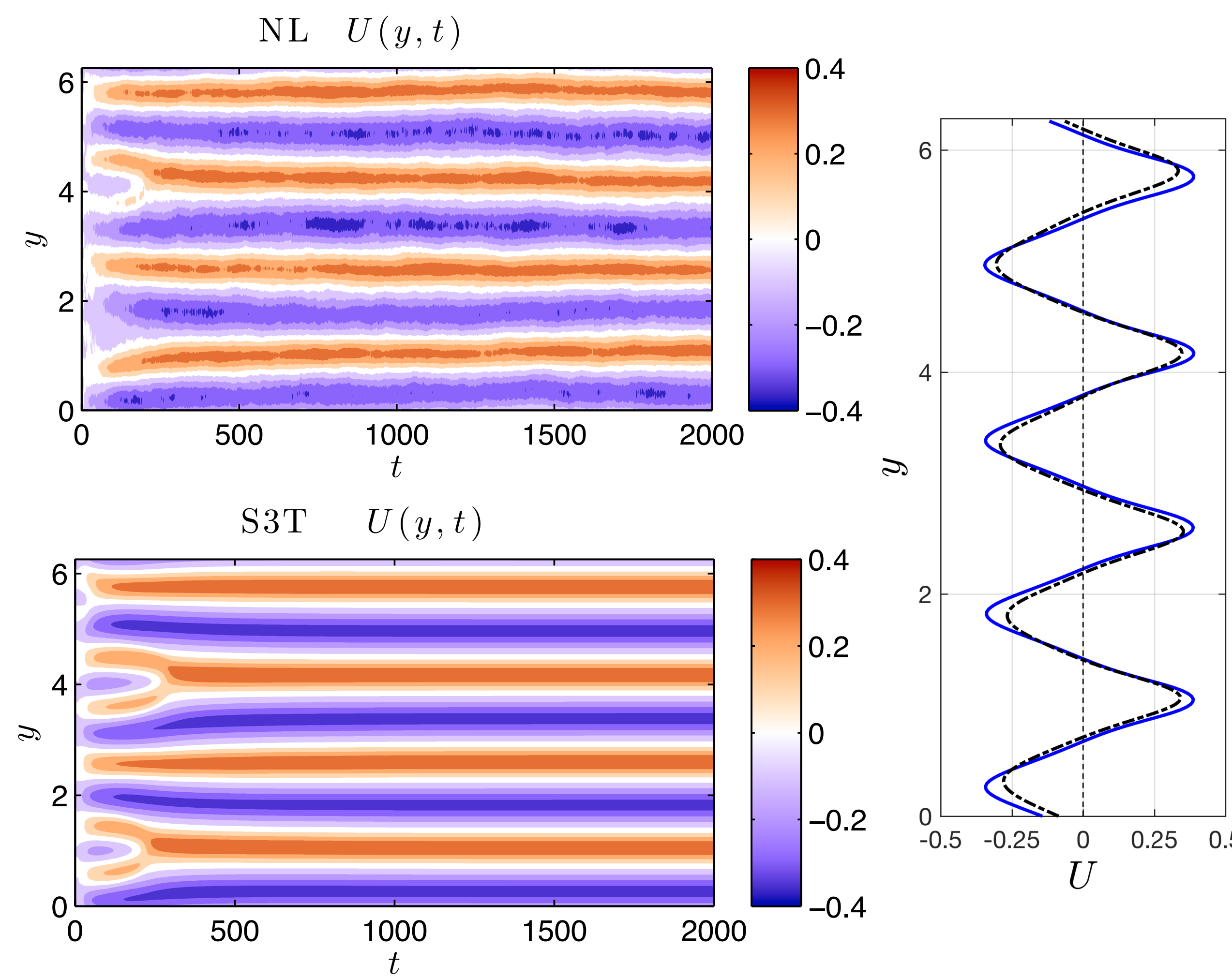


SSD predictions for jet formation and equilibration at finite amplitude

$\varepsilon/\varepsilon_c = 1.5$



$\varepsilon/\varepsilon_c = 20$



SSD instabilities grow and reach finite amplitude
to produce new inhomogeneous turbulent equilibria with jets

Zonal flows + Magnetic fields

MHD equations on a beta plane
imposed toroidal magnetic field B_0
stochastic forcing in the hydrodynamic equation

$$\partial_t \zeta + \mathsf{J}(\psi, \zeta + \beta y) = \underbrace{\mathsf{J}(A + B_0 y, \nabla^2 A)}_{\text{curl of Lorentz force}} + \nu \nabla^2 \zeta + \underbrace{\xi}_{\text{stoch. forcing}}$$

$$\partial_t A + \mathsf{J}(\psi, A + B_0 y) = \eta \nabla^2 A$$

$$(u, v) = (-\partial_y \psi, \partial_x \psi) \qquad (B_x, B_y) = (B_0 + \partial_y A, -\partial_x A)$$

β : latitudinal gradient of Coriolis parameter $\quad$ η : resistivity $\quad$ ν : viscosity

ρ (mass density) $= \mu_0$ (permeability) $= 1$

what's known?

Tobias et al 2007, ApJ
DNS show that as imposed B_0 increases zonal flows die

Tobias et al 2011, ApJ
Qualitatively similar results for DNS on surface of sphere

Tobias et al, (unpublished; personal communication)
Qualitatively similar results for DNS with imposed poloidal B_0

but what's the mechanism of zonal flow suppression?

Kelvin–Orr wave solution
weak shear limit

the energy contribution to zonal flow
from a **pair** of vorticity waves initially with $(k_x, \pm k_y)$ is

$$\Delta E_{\text{zonal flow}} = \frac{S^2 k_x^2}{4\nu^2 k^4} \left[\frac{k_x^2 - 5k_y^2}{k^6} \underbrace{|Z_0|^2}_{\substack{\text{vorticity}\\ \text{wave ampl.}}} - \frac{k_x^2 - k_y^2}{k^2} \underbrace{|A_0|^2}_{\substack{\text{magnetic}\\ \text{wave ampl.}}} \right]$$

Kelvin–Orr wave solution
weak shear limit

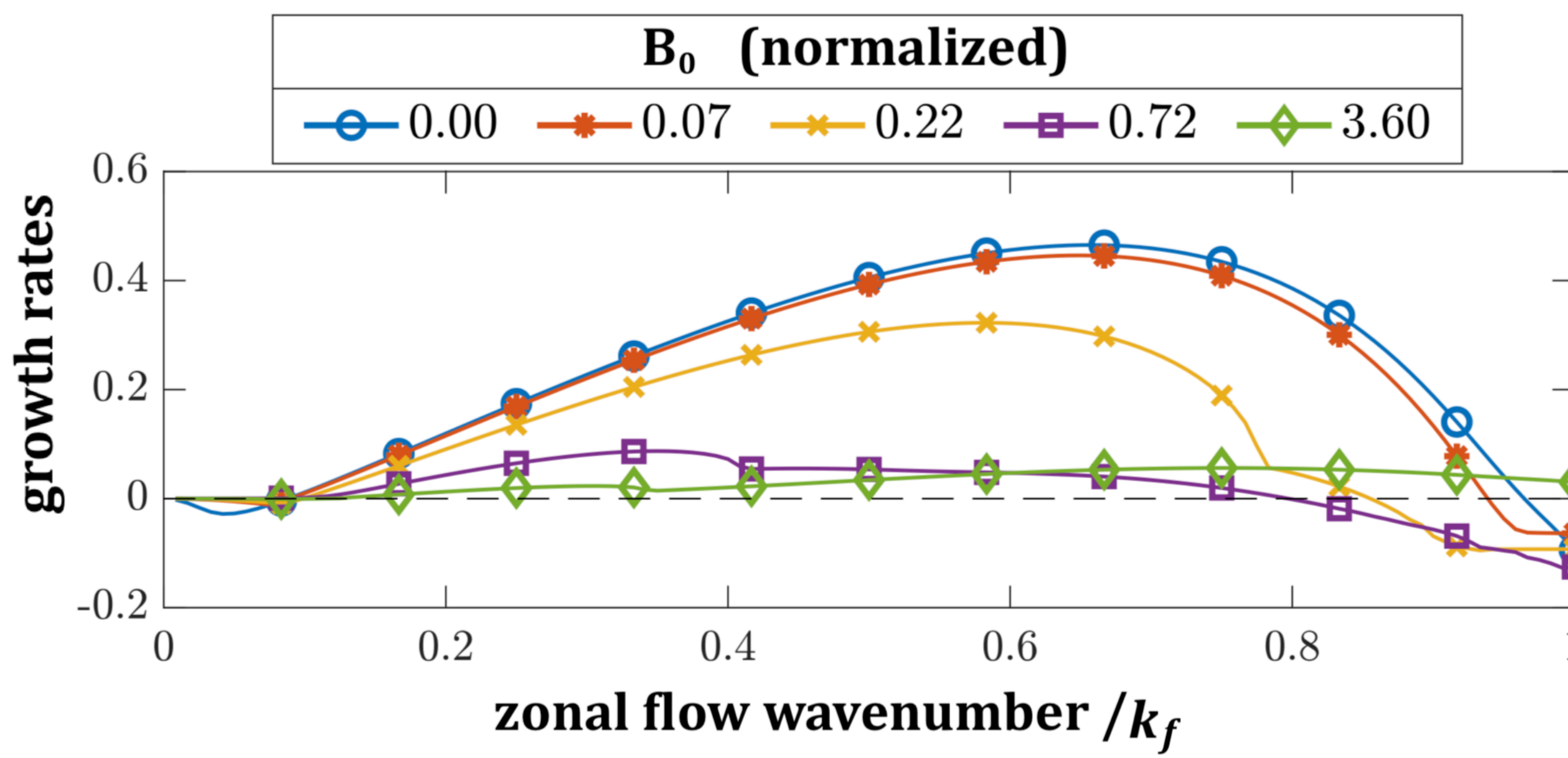
the energy contribution to zonal flow
from a **pair** of vorticity waves initially with $(k_x, \pm k_y)$ is

$$\Delta E_{\text{zonal flow}} = \frac{S^2 k_x^2}{4\nu^2 k^4} \left[\frac{k_x^2 - 5k_y^2}{k^6} \underbrace{|Z_0|^2}_{\text{vorticity wave ampl.}} - \frac{k_x^2 - k_y^2}{k^2} \underbrace{|A_0|^2}_{\text{magnetic wave ampl.}} \right]$$

the magnetic wave **always acts** to take energy away from zonal flow

SSD stability of the homogeneous turbulent state with magnetic fields

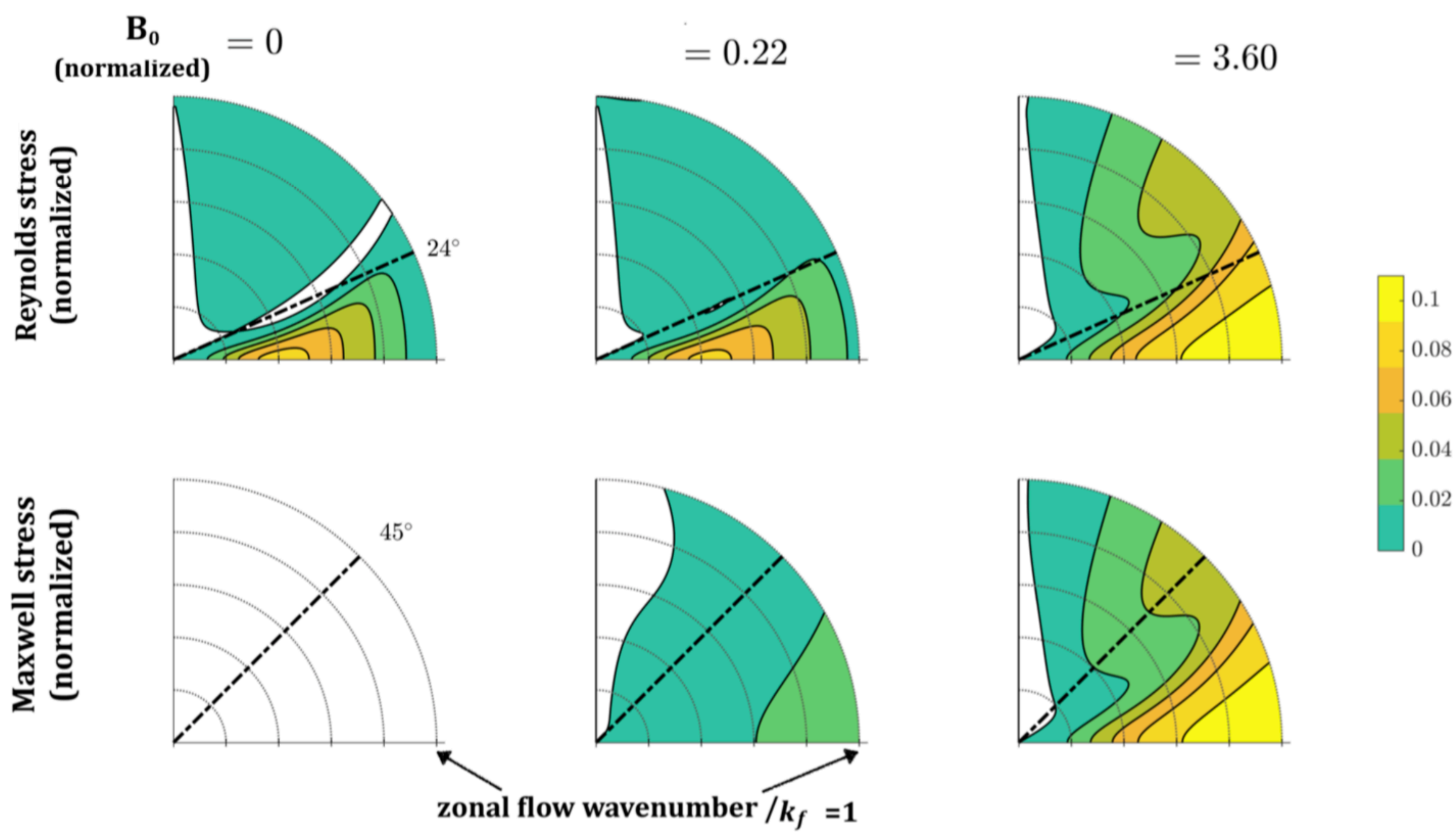
$$\partial_t \overline{u} = \underbrace{-\,\partial_y \overline{u'v'}}_{\text{Reynolds stress}} - \Big(\underbrace{-\partial_y \overline{B'_x B'_y}}_{\text{Maxwell stress}}\Big) + \text{dissipation}$$



As B_0 increases the jet-forming instability is suppressed.

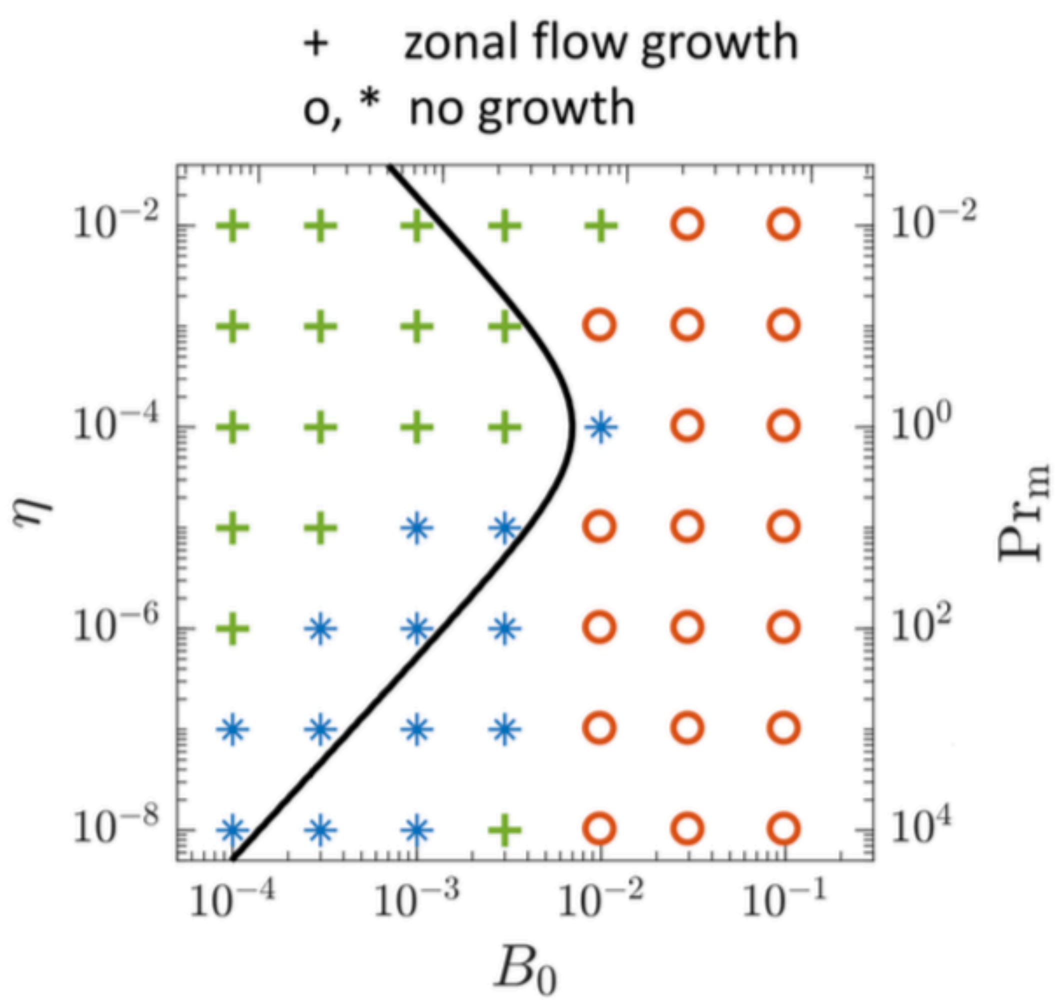
Maxwell stress competes with Reynolds stress

$$\partial_t \overline{u} = \underbrace{-\,\partial_y \overline{u'v'}}_{\text{Reynolds stress}} - \Big(\underbrace{-\partial_y \overline{B'_x B'_y}}_{\text{Maxwell stress}}\Big) + \text{dissipation}$$

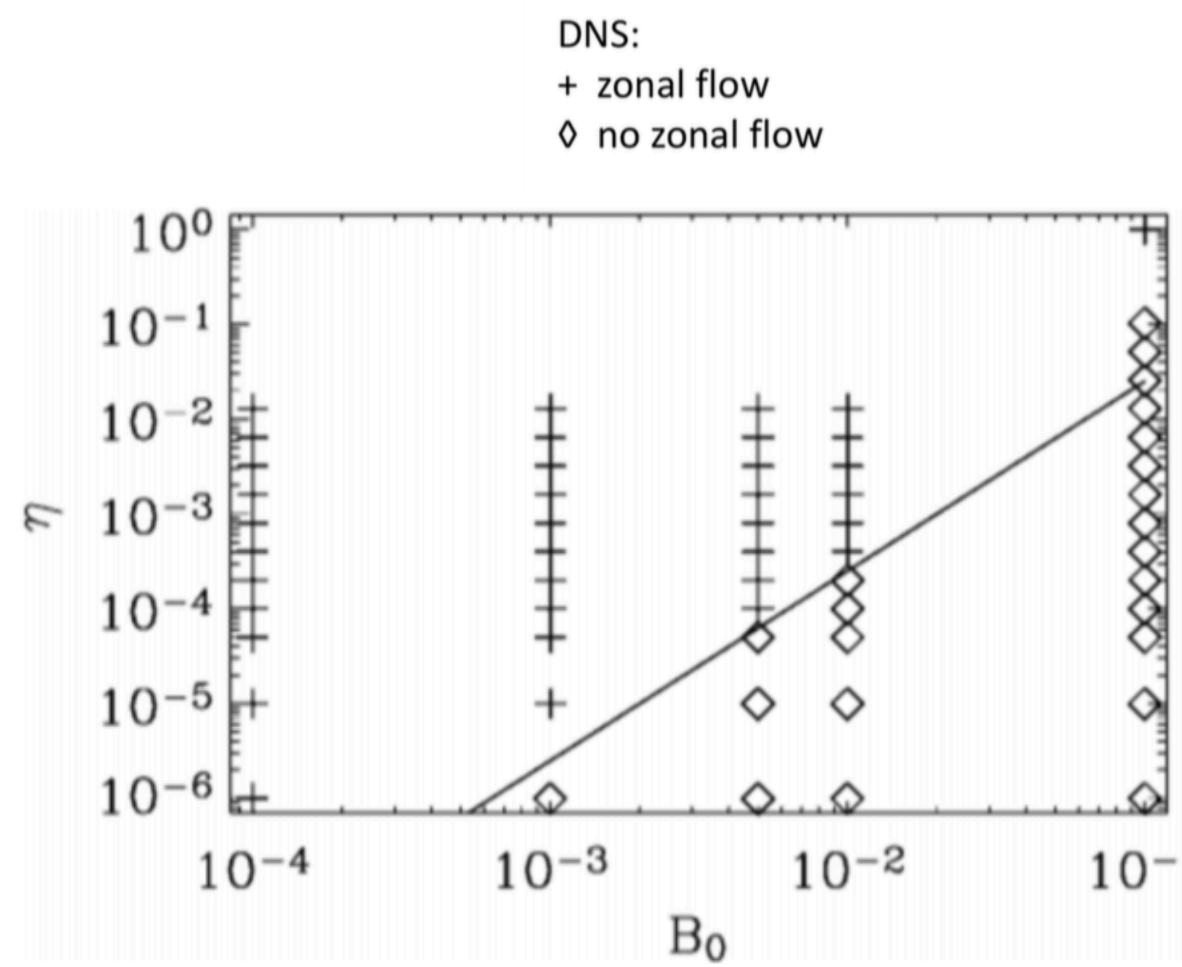


Stability calculations can predict DNS behavior

Stability of homogenous turbulent state



Direct Numerical Simulations (Tobias et al 2007)



Conclusions

- Magnetic fluctuations can suppress zonal flow through the Maxwell stress when the resistivity is sufficiently small
  - Consistent with earlier results of Tobias et al. (2007, 2011)
  - Here, we found that the suppression can be explained quasilinearly, and even occurs with weak zonal flows, without requiring nonlinear effects. The growth rate of zonal flow instability is suppressed.

- Results may explain the depth-extent of the zonal jets in Jupiter & Saturn.

Constantinou & Parker (2018) Magnetic suppression of zonal flows on a beta plane. *ApJ*, **863**, 46

thanks