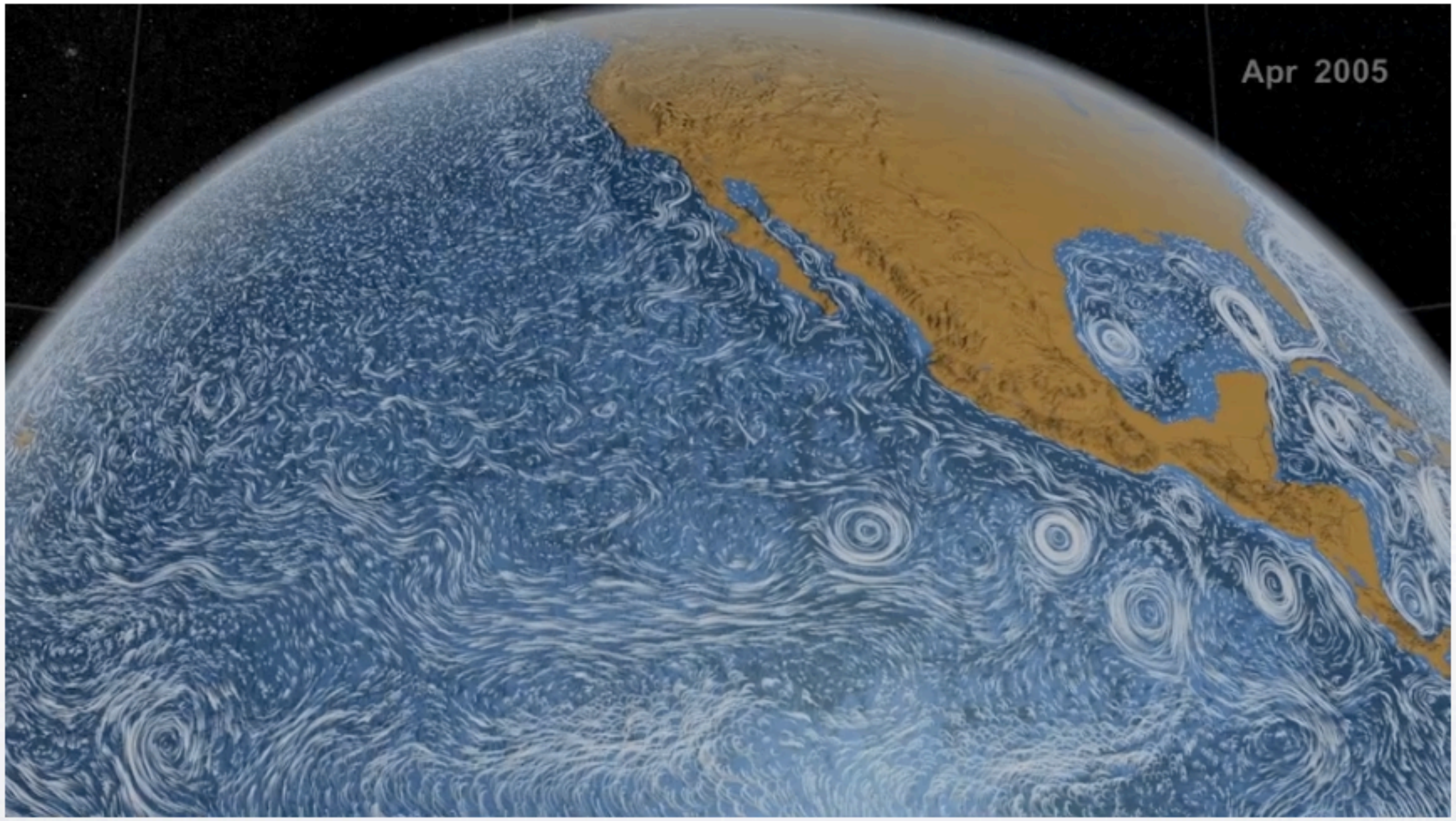
Σχηματισμός και εξισορρόπηση ζωνικών ανέμων σε πλανητικές τυρβώδεις ατμόσφαιρες

Ναβίτ Κωνσταντίνου και Πέτρος Ιωάννου
Τμήμα Φυσικής
Εθνικό και Καποδιστριακό Παν/μιο Αθηνών

Πανεπιστήμιο Κύπρου
7 Ιανουαρίου 2014

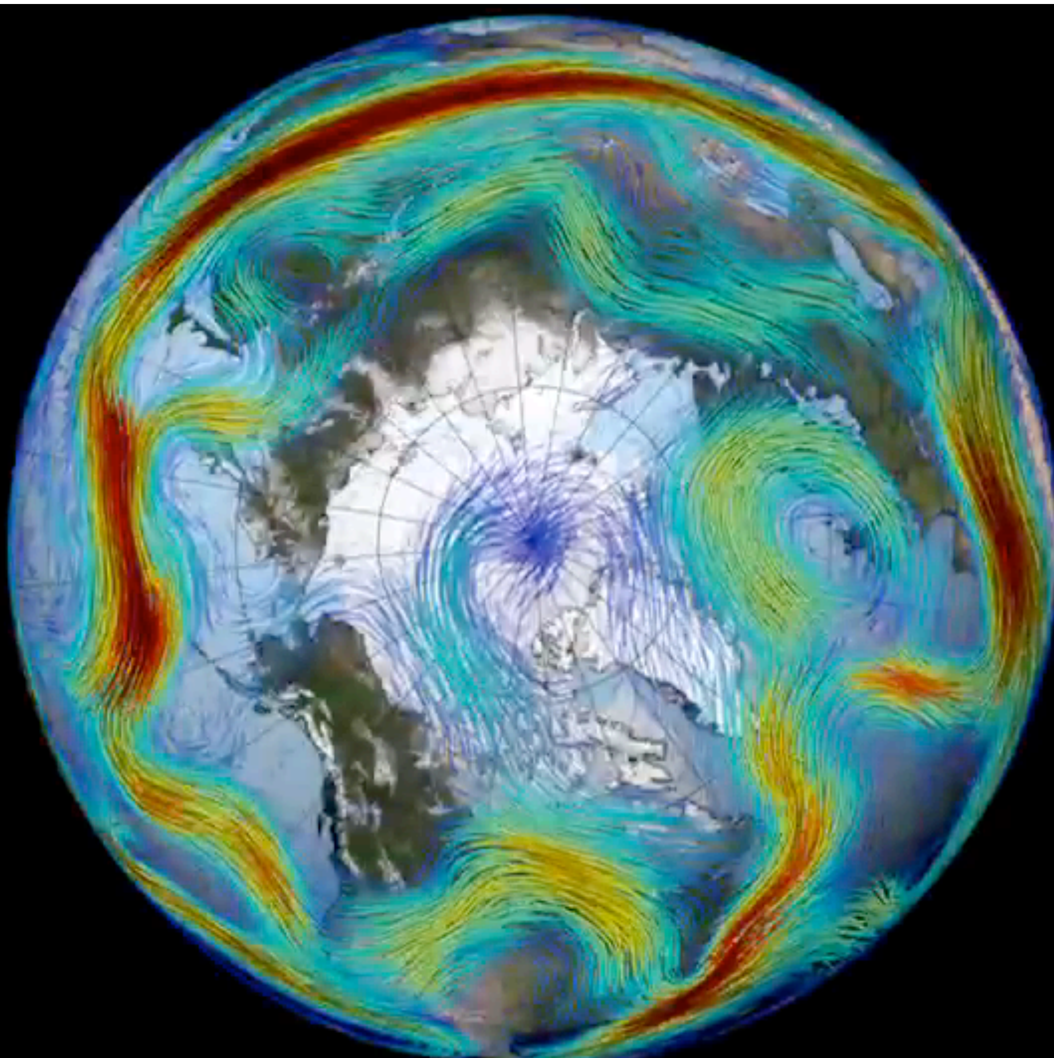
Coherent structures in turbulent flows



ocean currents

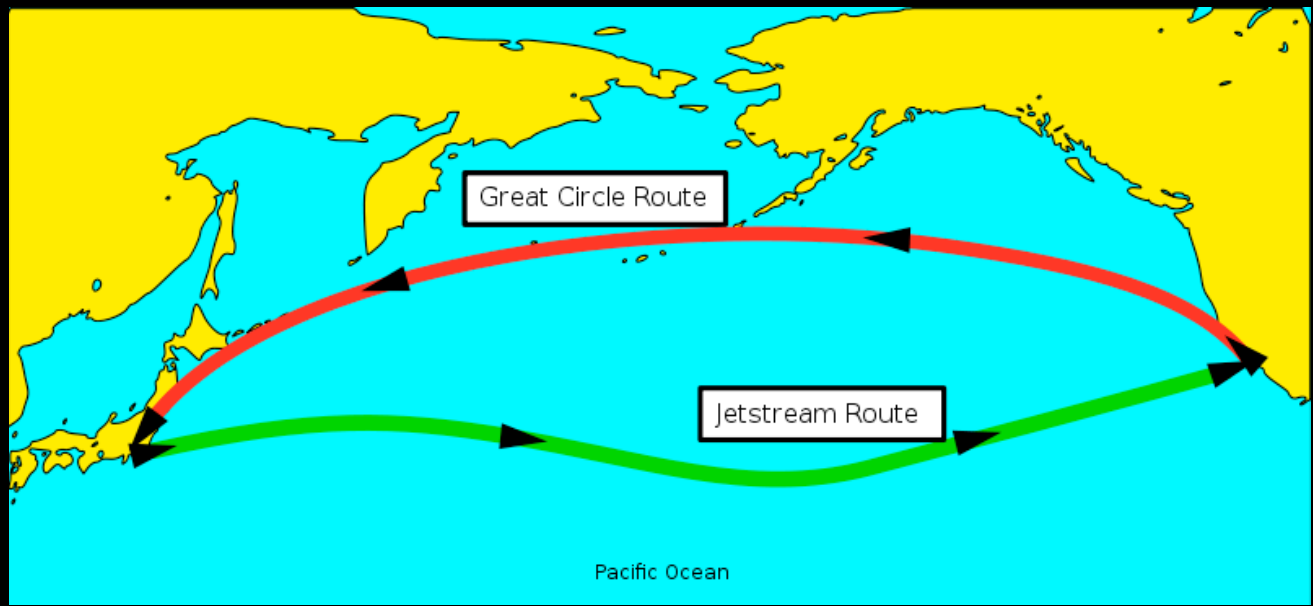
NASA/Goddard Space Flight Center

Earth's atmospheric polar jet stream



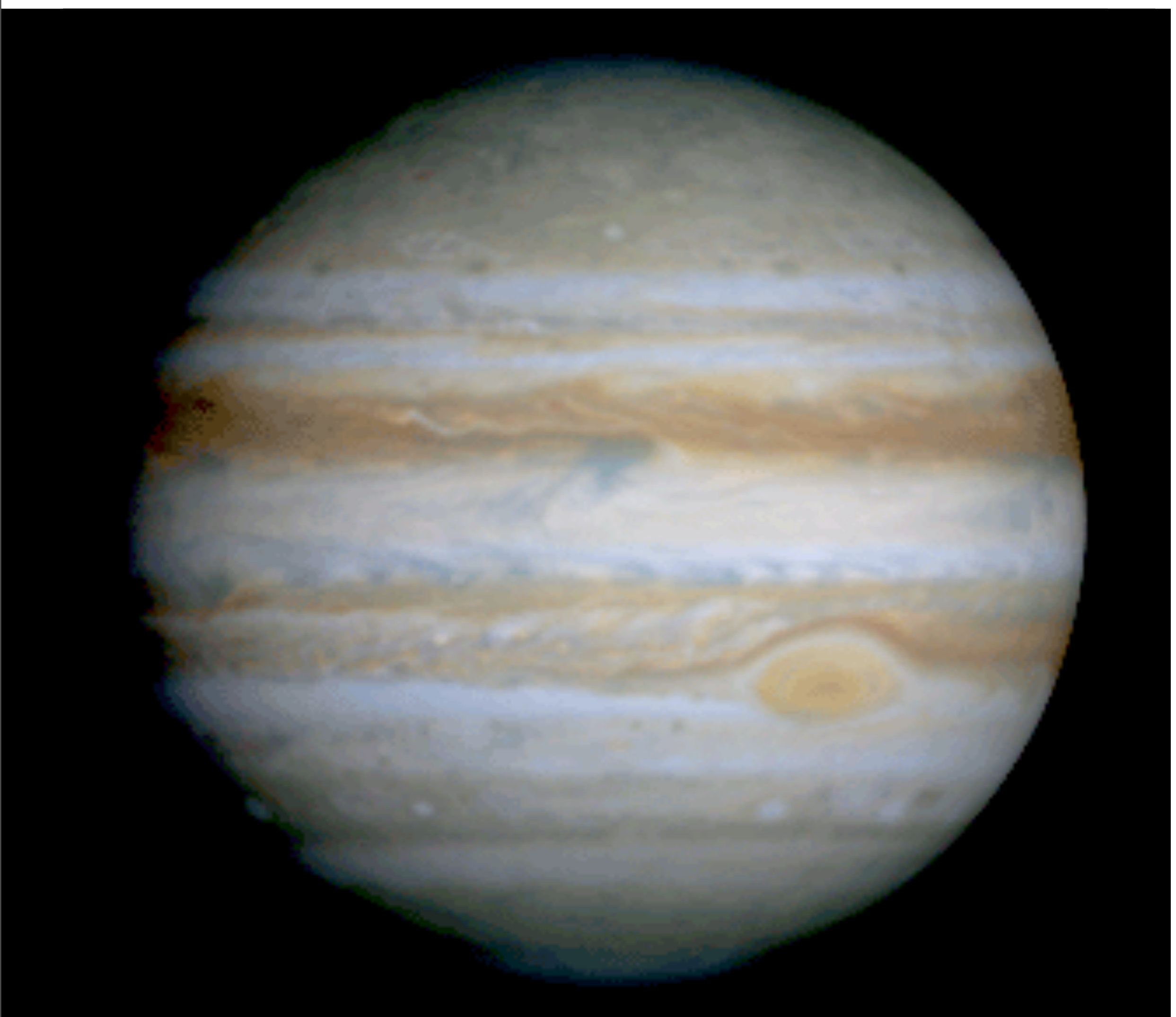
polar front jet

NASA/Goddard Space Flight Center



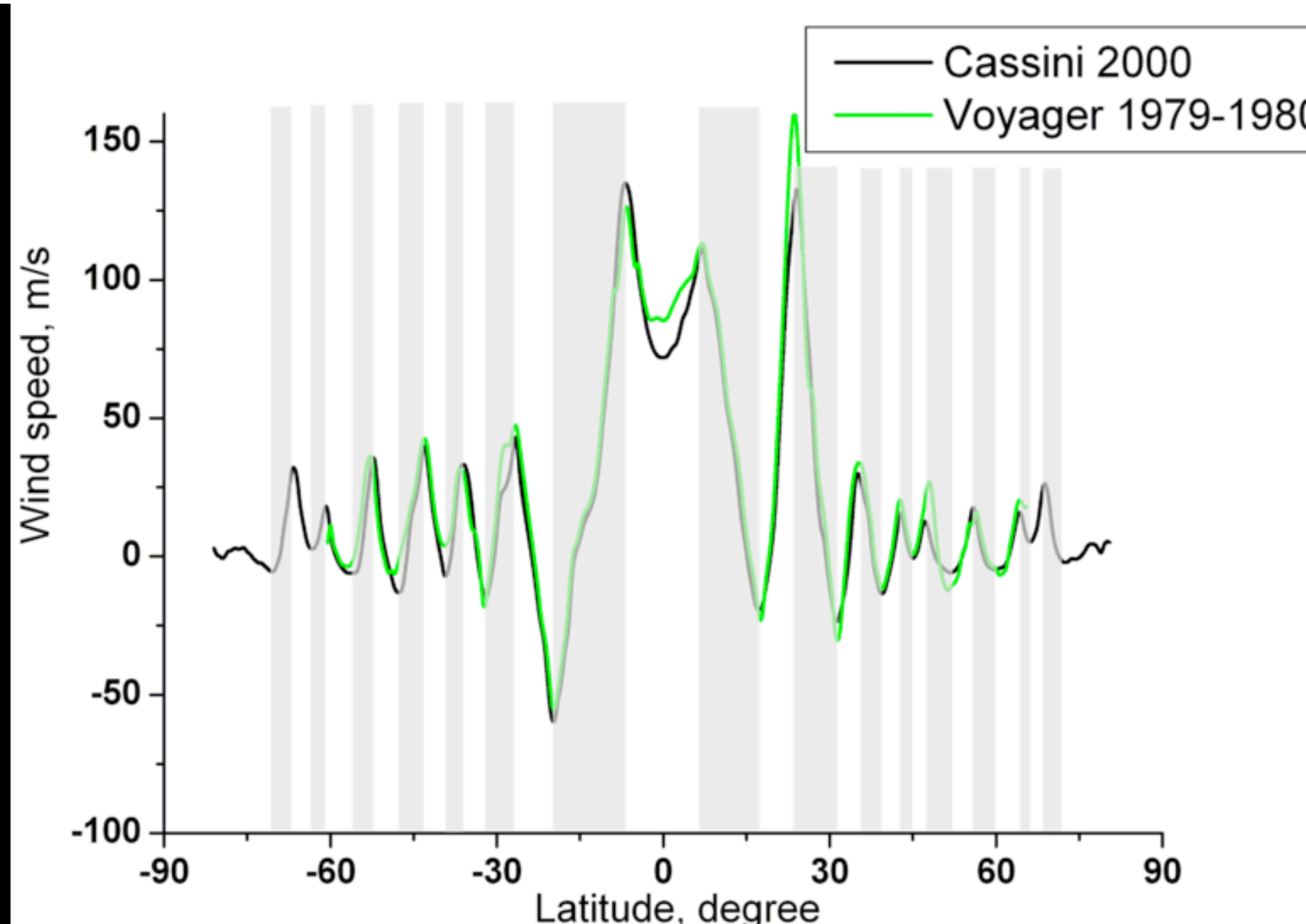
airplane trip from L.A. to Tokyo

‘striped’ Jupiter



banded Jovian jets

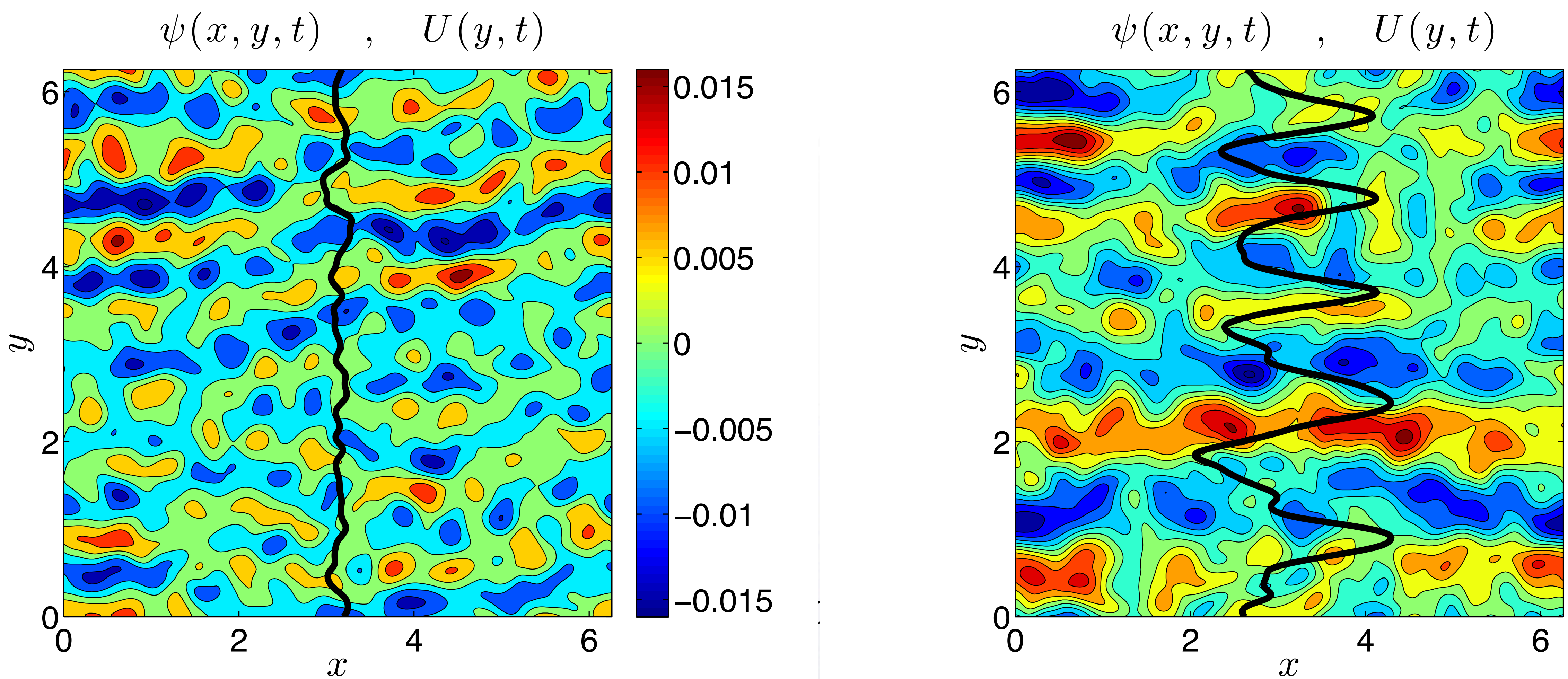
NASA/Cassini Jupiter Images



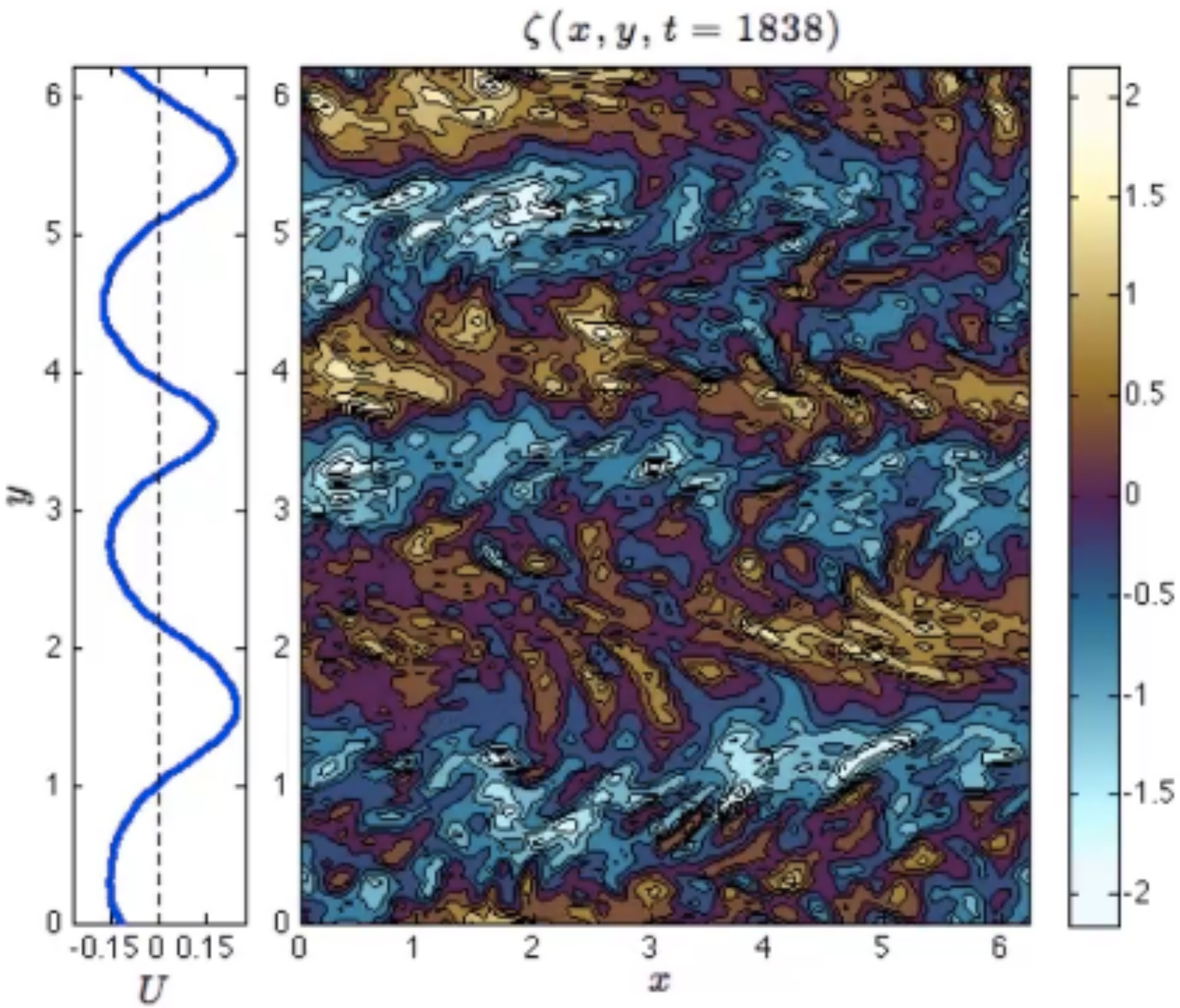
observed Jovian zonal winds at cloud level

Vasadava & Showman, 2005

Jet emergence on a barotropic beta-plane

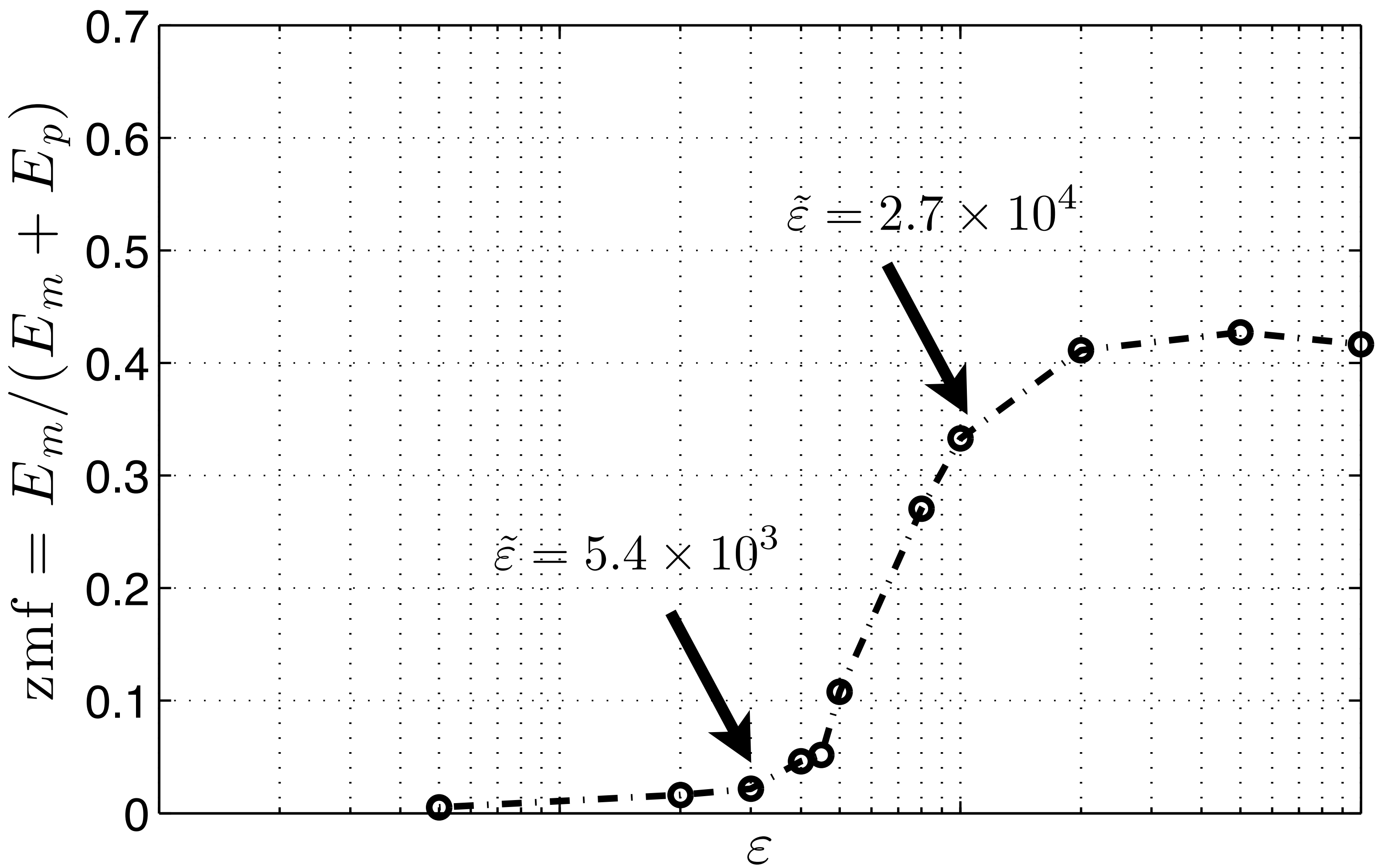


Turbulent flows organize into jets



Numerical simulation (barotropic beta-plane)

Jets seem to emerge as a bifurcation



E_m : zonal energy , E_p : eddy energy

Classical hydrodynamic stability

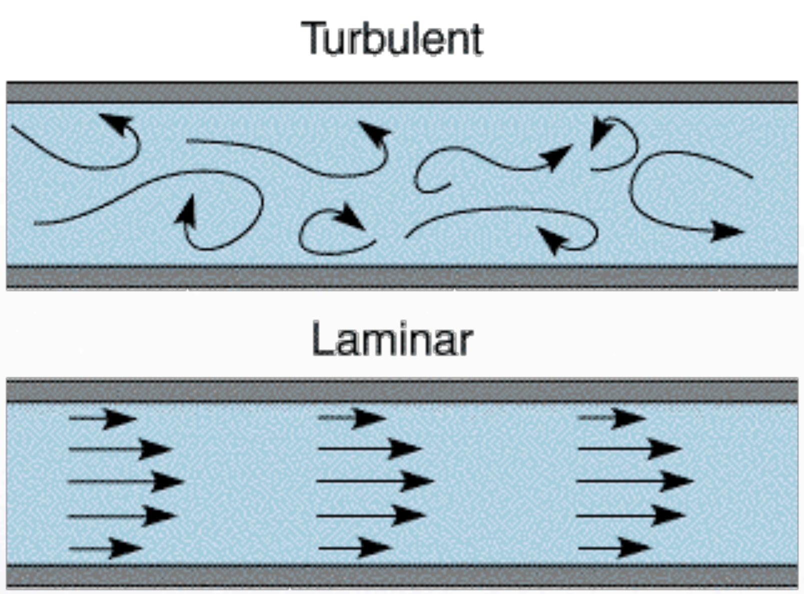
Lord Rayleigh

hydrodynamic instabilities provide a way for eddies to gain energy from mean flow

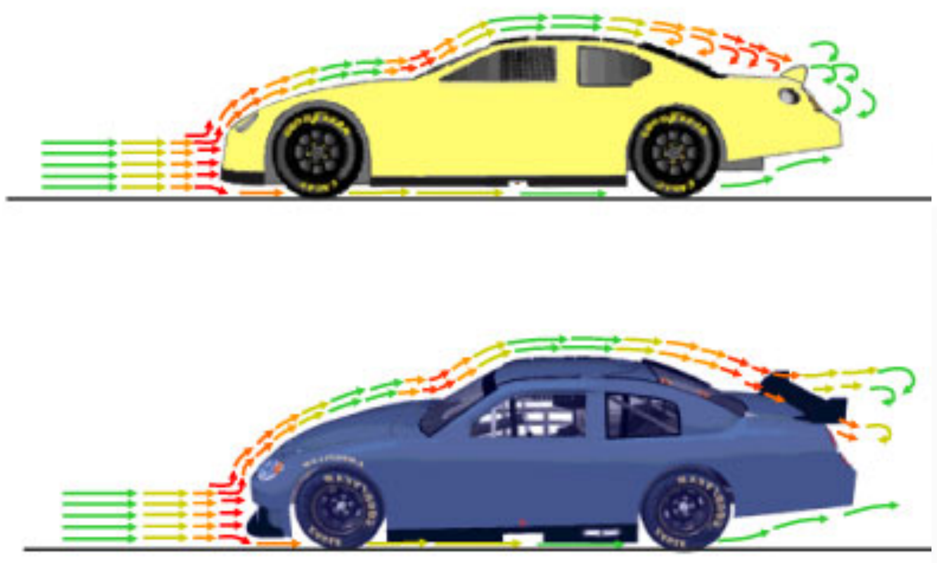
how about the opposite ?

can the mean flow gain energy from the eddies through an instability ?

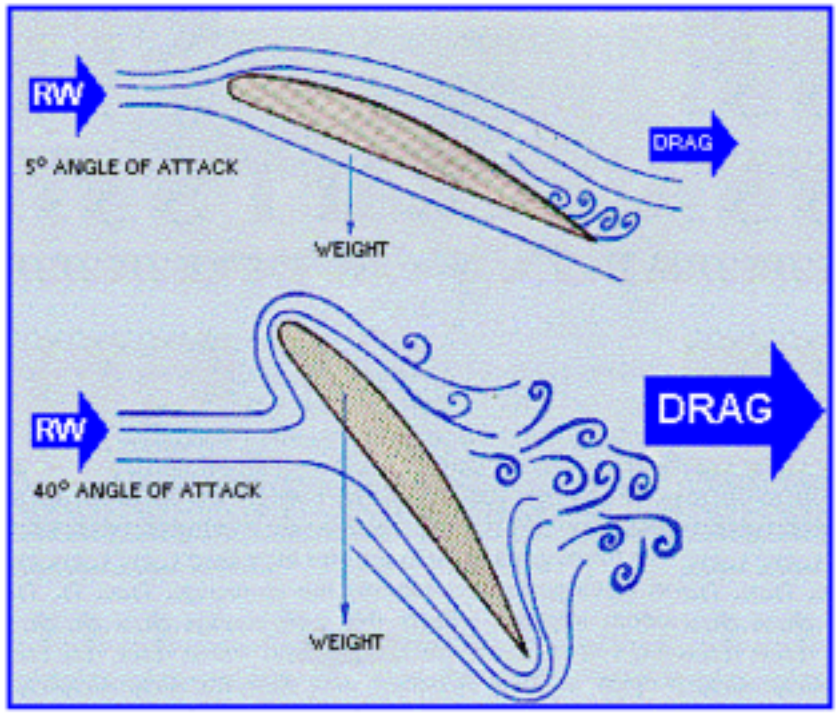
Turbulence (usually) acts as a drag



wall-bounded flow



airflow over vehicle



airflow over airfoil

can turbulence act to reinforce large scale flows?

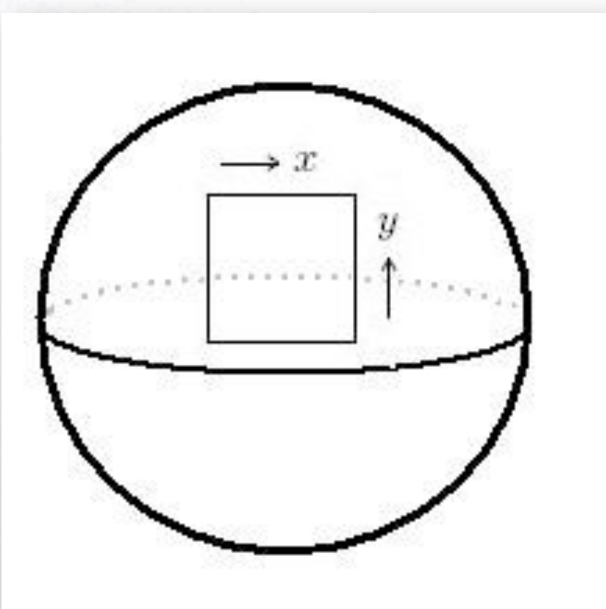
Our model:

Barotropic vorticity equation on a beta-plane

$$\partial_t\zeta + u\partial_x\zeta + v\partial_y\zeta + \beta v = -r\zeta + \sqrt{\varepsilon}\, f$$

dissipation

stochastic forcing



$$\nabla\cdot\vec{u} = 0 \;\Rightarrow\; \begin{aligned} u &= -\partial_y\psi \\ v &= \partial_x\psi \end{aligned}$$

$$\zeta \equiv \partial_x v - \partial_y u = \Delta\psi$$

Zonal - Eddy field decomposition

$$\varphi(x,y,t) = \Phi(y,t) + \varphi'(x,y,t)$$

zonal mean

eddy

where $\Phi(y,t) = \overline{\varphi}(y,t) = \dfrac{1}{L_x}\displaystyle\int_0^{L_x} \varphi(x',y,t)\,\mathrm{d}x'$

NL (nonlinear) System

$$\partial_t U = \overline{v'\zeta'} - rU$$

$$\partial_t \zeta' = -U\partial_x \zeta' - (\beta - U_{yy})v' - r\zeta' + F_e + \sqrt{\varepsilon}\, f$$

$$F_e = \left[\partial_y(\overline{v'\zeta'}) - \partial_y(v'\zeta')\right] - \partial_x(u'\zeta')$$

eddy-eddy
interaction term

QL (quasi-linear) System

$$\partial_t U = \overline{v'\zeta'} - rU$$

$$\partial_t \zeta' = -U\partial_x \zeta' - (\beta - U_{yy})v' - r\zeta' + \cancel{F_e} + \sqrt{\varepsilon}\, f$$

$$F_e = \cancel{\left[\partial_y(\overline{v'\zeta'}) - \partial_y(v'\zeta')\right] - \partial_x(u'\zeta')}$$

eddy-eddy
interaction term

QL (quasi-linear) System

$$\partial_t U = \overline{v'\zeta'} - rU$$

$$\partial_t \zeta' = \mathcal{A}(U)\,\zeta' + \sqrt{\varepsilon}\,f$$

where

$$\mathcal{A}(U) = -U\partial_x - (\beta - U_{yy})\partial_x\Delta^{-1} - r$$

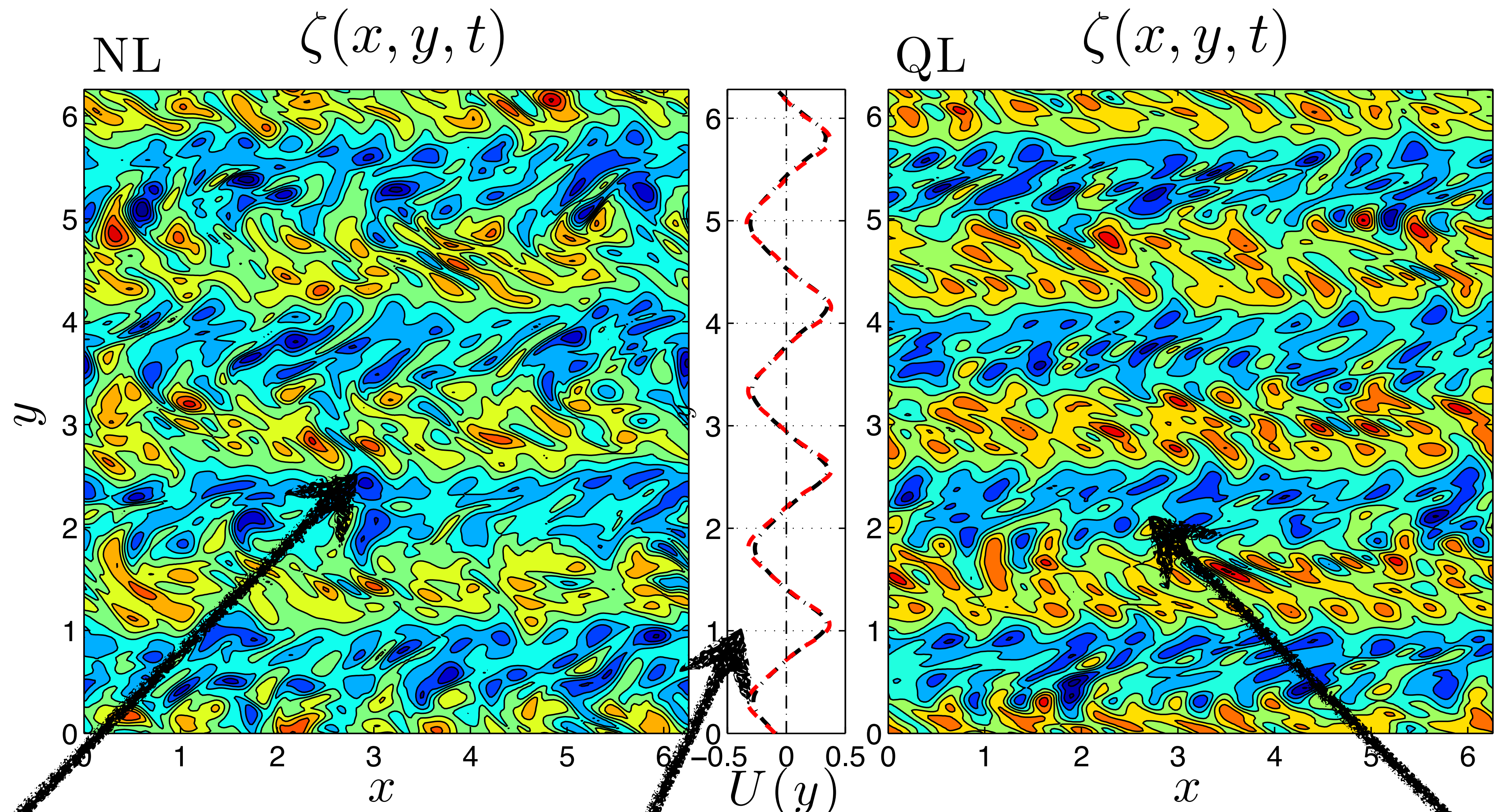
QL does NOT include

- turbulent cascades
- wave breaking
- nonlinear vorticity mixing

The great wave off Kanagawa

by Hokusai

QL captures the NL dynamics



NL vorticity snapshot

mean flow comparison

QL vorticity snapshot

Our goal

While QL captures and elucidates the jet-eddy dynamics it does not provide a predictive theory.

Can we construct a theory that:

- Predicts when organized flows will emerge / describes jet formation as a bifurcation.
- Predicts the structure and the stability of the emergent zonal flows.
- Describes the jet merger dynamics

? ? ?

Our goal

While QL captures and elucidates the jet-eddy dynamics it does not provide a predictive theory.

Can we construct a theory that:

- Predicts when organized flows will emerge / describes jet formation as a bifurcation.
- Predicts the structure and the stability of the emergent zonal flows.
- Describes the jet merger dynamics

Such a theory can be constructed. It is based on the statistical dynamics associated with the QL equations.

The theory:
Stochastic Structural Stability Theory (S3T)

Consider the first two equal-time cumulants:

$$U(x,y,t) = \Big\langle u(x,y,t) \Big\rangle$$

$$Z(x,y,t) = \Big\langle \zeta(x,y,t) \Big\rangle$$

$$\zeta'(x,y,t) = \zeta(x,y,t) - Z(x,y,t)$$

$$C(x_1,x_2,y_1,y_2,t) = \Big\langle \zeta'(x_1,y_1,t)\zeta'(x_2,y_2,t) \Big\rangle$$

$\langle \bullet \rangle =$ ensemble average over realizations of the excitation

Ergodic assumption

$\langle \bullet \rangle =$ zonal average of a zonally unbounded single realization of the stochastic excitation

Then we have:

$$U(y,t) = \overline{u(x,y,t)}$$

$$Z(y,t) = \overline{\zeta(x,y,t)}$$

$$\zeta'(x,y,t) = \zeta(x,y,t) - Z(y,t)$$

$$C(x_1 - x_2, y_1, y_2, t) = \Big\langle \zeta'(x_1,y_1,t)\zeta'(x_2,y_2,t) \Big\rangle$$

The theory:

Stochastic Structural Stability Theory (S3T)

$$\partial_t U = \langle v'\zeta' \rangle - rU$$

$$\partial_t C = (\mathcal{A}_1 + \mathcal{A}_2)C + \varepsilon Q$$

$$Q(x_1 - x_2, y_1 - y_2) = \\ = \Big\langle f(x_1, y_1, t) f(x_2, y_2, t) \Big\rangle$$

$$\mathcal{A}_j(U) = -U(y_j)\partial_{x_j} + (\beta - U''(y_j))\,\partial_{x_j}\Delta_j^{-1} - r$$

$$\overline{v'\zeta'} = \langle v'\zeta' \rangle = \mathcal{R}(C) \qquad (j = 1, 2)$$

QL system $U\ ,\ \zeta'$ → S3T system $U\ ,\ C$ = ensemble average dynamics of the QL system

We have three dynamical systems

NL	→	QL	→	S3T
simulation		simplified simulation		theory

S3T equilibria

$$\partial_t U = \mathcal{R}(C) - rU$$

$$\partial_t C = (\mathcal{A}_1 + \mathcal{A}_2)C + \varepsilon Q$$

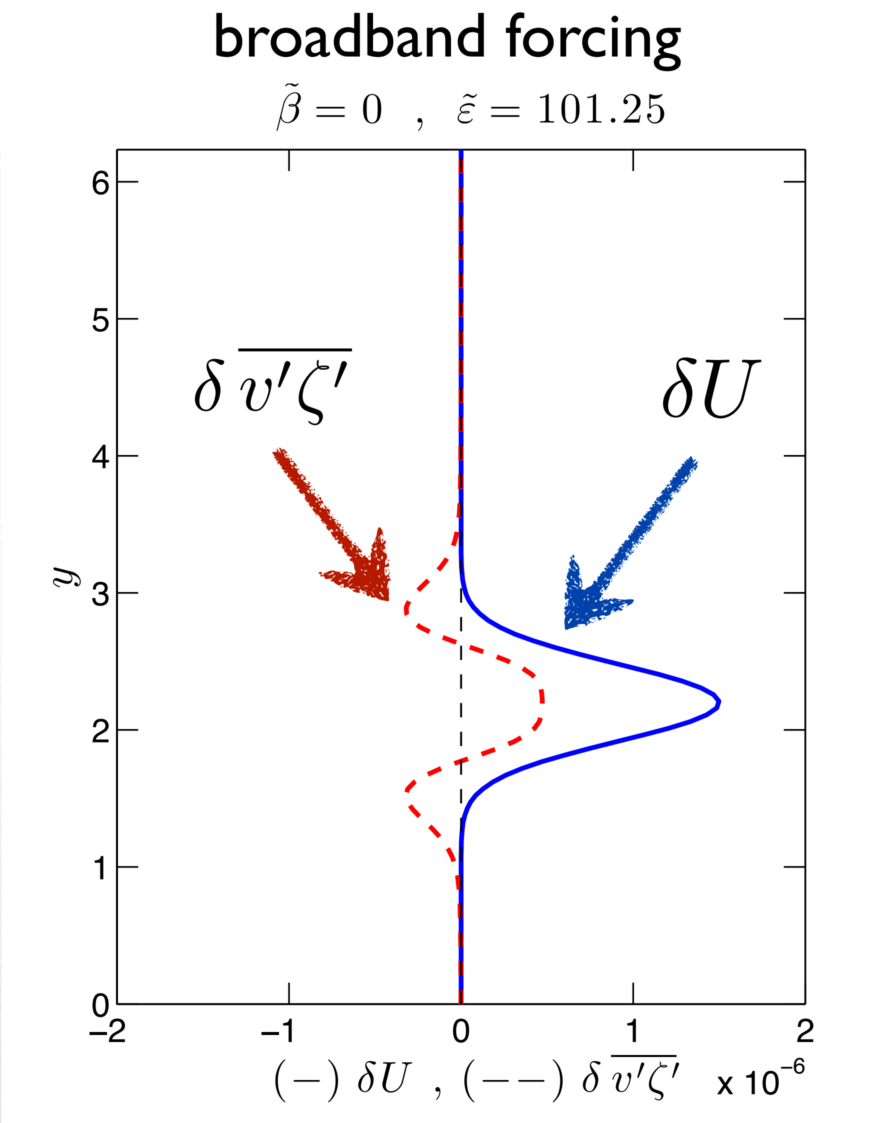
S3T system admits equilibria (U^E, C^E)

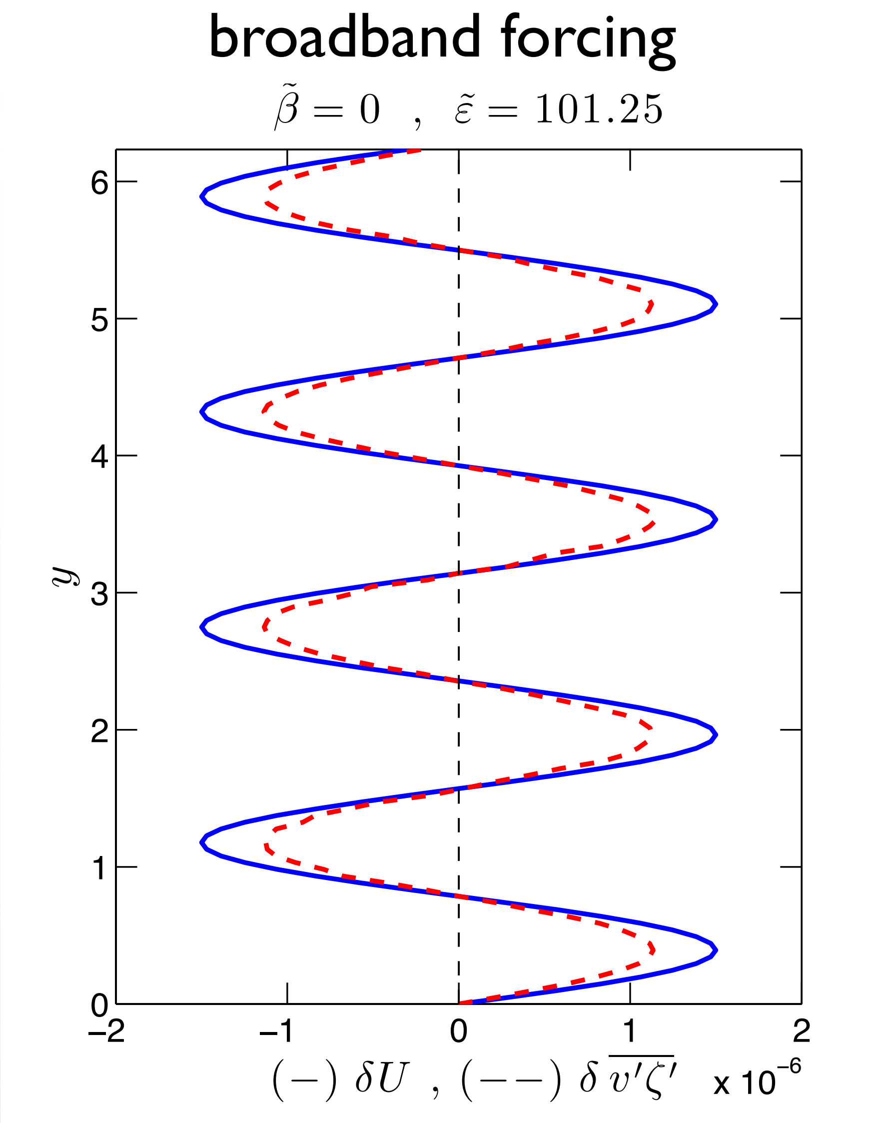
For example, when we have homogeneity then

$$U^E = 0 \quad \text{and} \quad C^E = \frac{\varepsilon}{2r} Q$$

is an equilibrium for all β, dissipation values $r > 0$ and energy input rates $\varepsilon > 0$

Eddies tend to reinforce zonal flow inhomogenuities





Stability of S3T equilibria

perturbing the S3T equilibrium $(U^E + \delta U, C^E + \delta C)$

$$\partial_t \delta U = \mathcal{R}(\delta C) - r\,\delta U$$

$$\partial_t \delta C = (\mathcal{A}_1 + \mathcal{A}_2)\delta C + (\delta\mathcal{A}_1 + \delta\mathcal{A}_2) C^E$$

with $\delta\mathcal{A}_j = \mathcal{A}_j(U^E + \delta U) - \mathcal{A}_j(U^E)\ , \quad j = 1, 2$

Stability of S3T equilibria

perturbing the S3T equilibrium $(U^E + \delta U, C^E + \delta C)$

$$\partial_t \delta U = \mathcal{R}(\delta C) - r\,\delta U$$
$$\partial_t \delta C = (\mathcal{A}_1 + \mathcal{A}_2)\delta C + (\delta\mathcal{A}_1 + \delta\mathcal{A}_2)C^E$$

with $\delta\mathcal{A}_j = \mathcal{A}_j(U^E + \delta U) - \mathcal{A}_j(U^E)\ ,\quad j = 1, 2$

searching for eigensolutions $(\delta U, \delta C) = (\delta\hat{U}, \delta\hat{C})e^{\sigma t}$

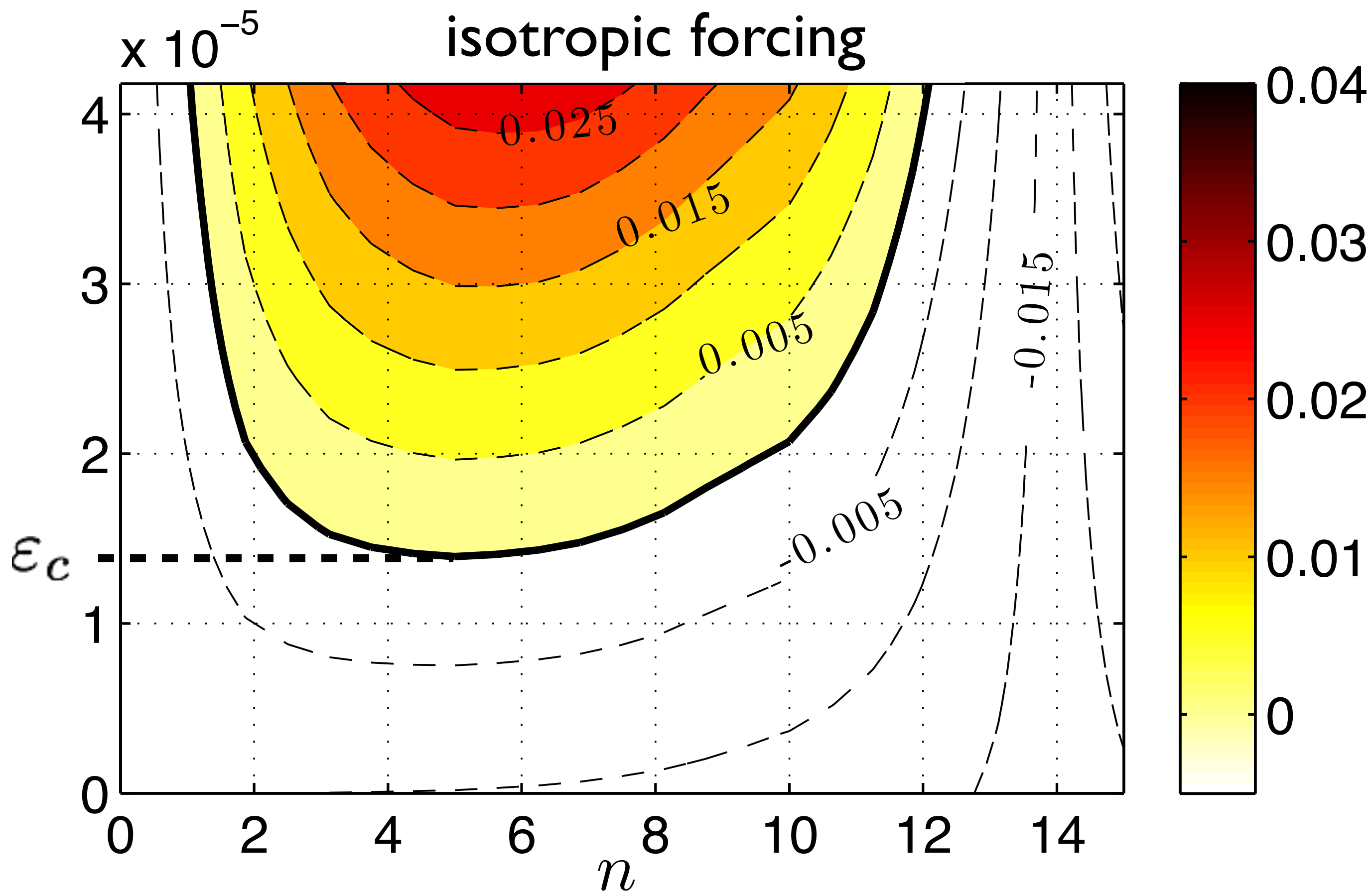
$$\sigma \begin{pmatrix} \delta\hat{U} \\ \delta\hat{C} \end{pmatrix} = \mathbb{L} \begin{pmatrix} \hat{U} \\ \delta\hat{C} \end{pmatrix}$$

$\mathbb{L} = \mathbb{L}(U^E, C^E)$ (for $N_y = 128 \Rightarrow \dim(\mathbb{L}) \approx 5 \cdot 10^5$)

Stability of S3T homogeneous equilibrium

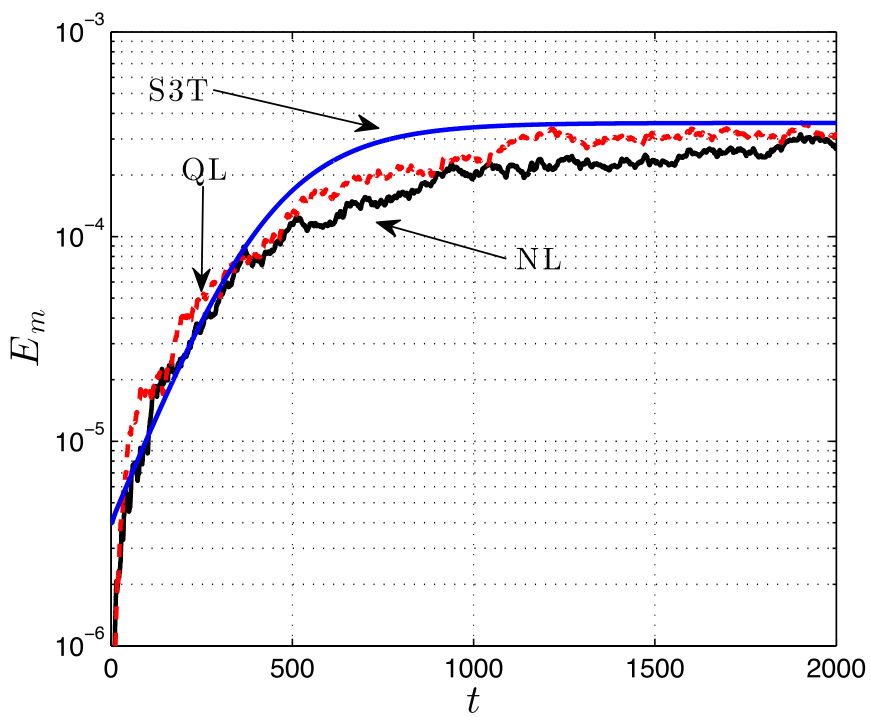
for the homogeneous equilibrium $U^E = 0$, $C^E = \dfrac{\varepsilon}{2r} Q$

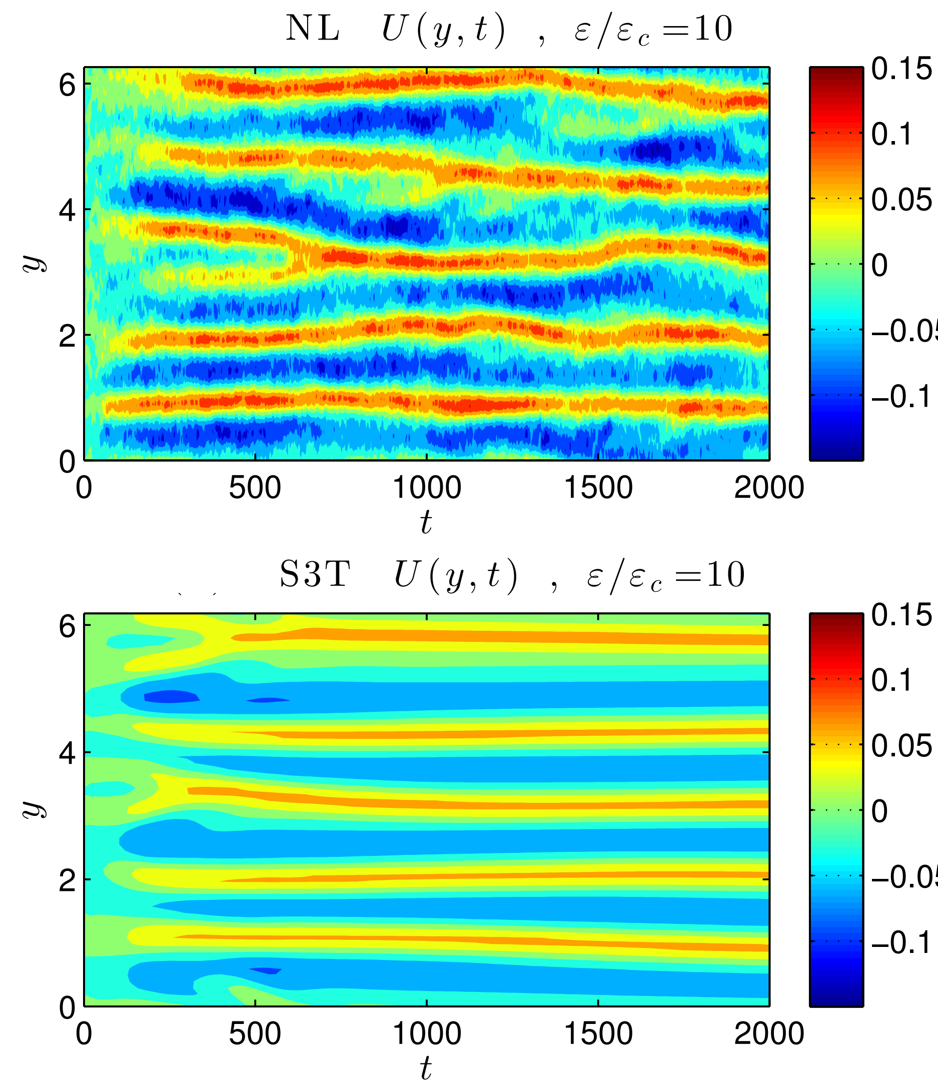
eigensolutions are harmonics: $\delta \hat{U} = e^{\mathrm{i}ny}$

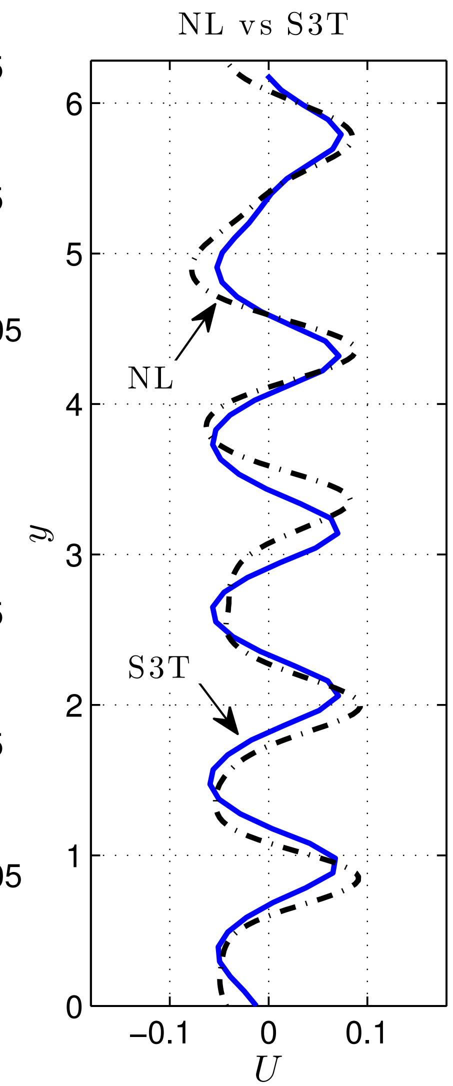


(contours of $\mathrm{Re}(\sigma)$)

Comparison of S3T predictions with NL dynamics



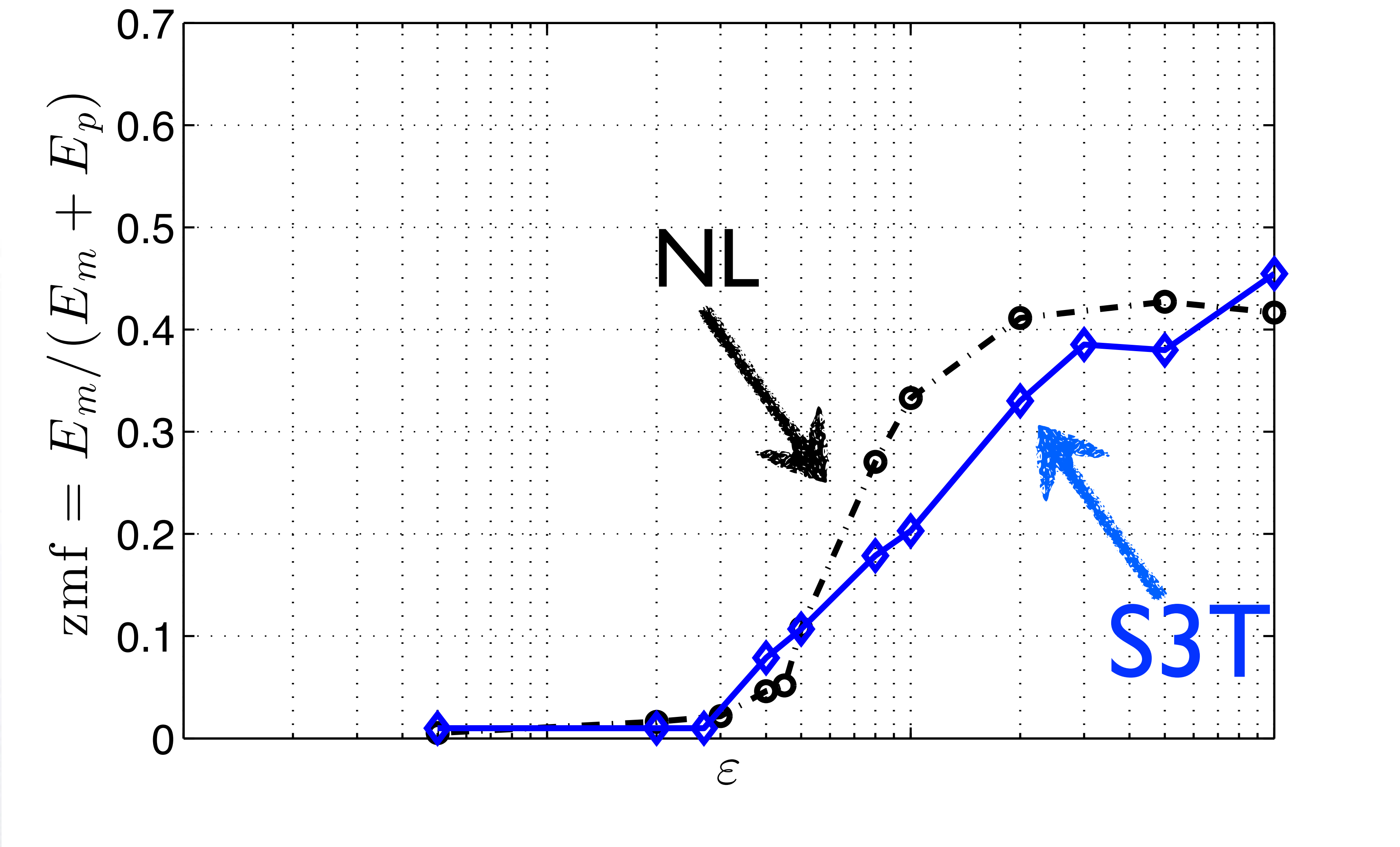




isotropic forcing

$$\tilde{\beta} = 71 \quad , \quad \tilde{\varepsilon} = 3.4 \times 10^5 = 10\tilde{\varepsilon}_c$$

S3T predicts jet formation bifurcation

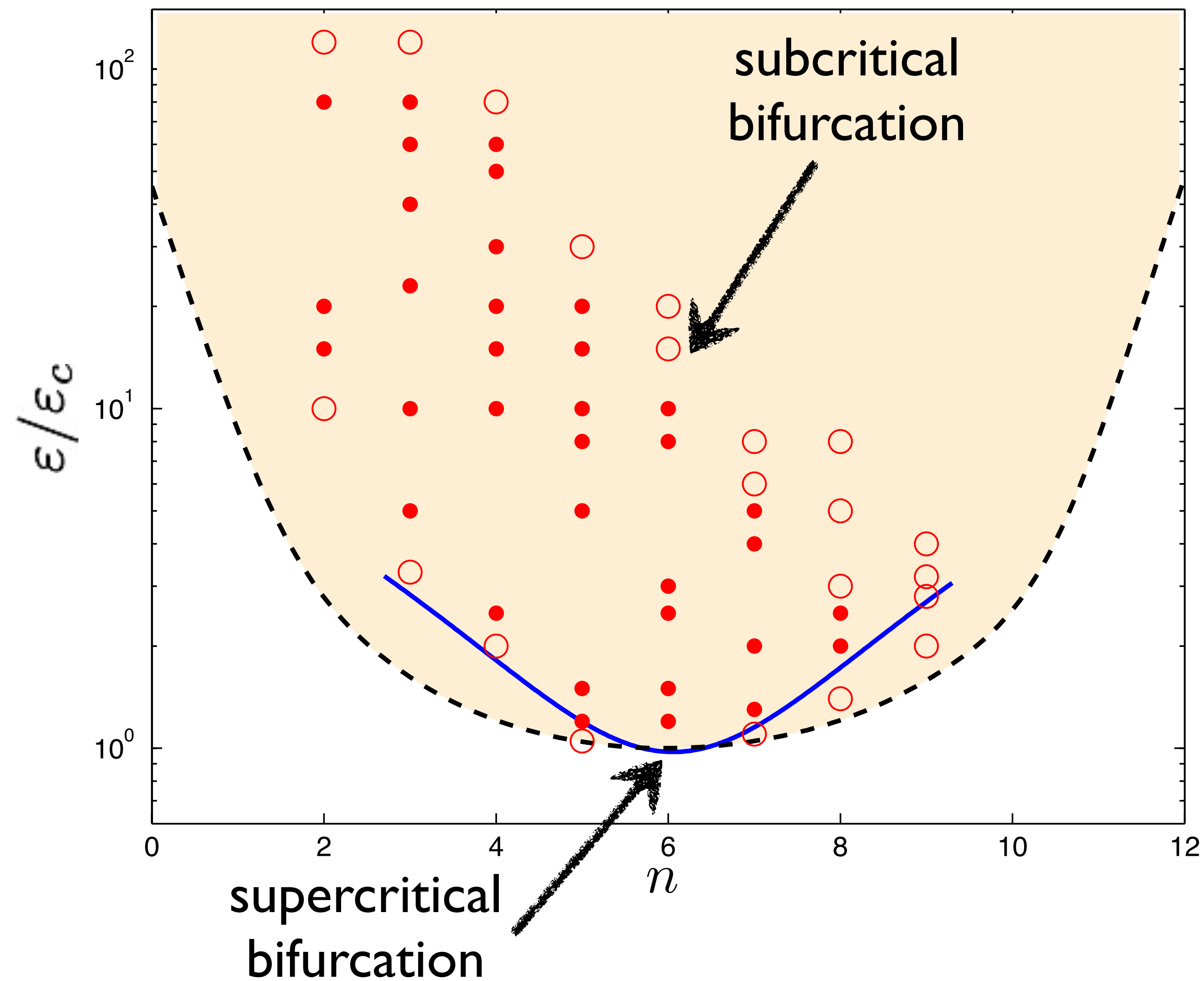


$$E_m : \ \text{zonal energy} \quad , \quad E_p : \ \text{eddy energy}$$

Stability of S3T equilibria

Stability analysis of the ideal states predicts:

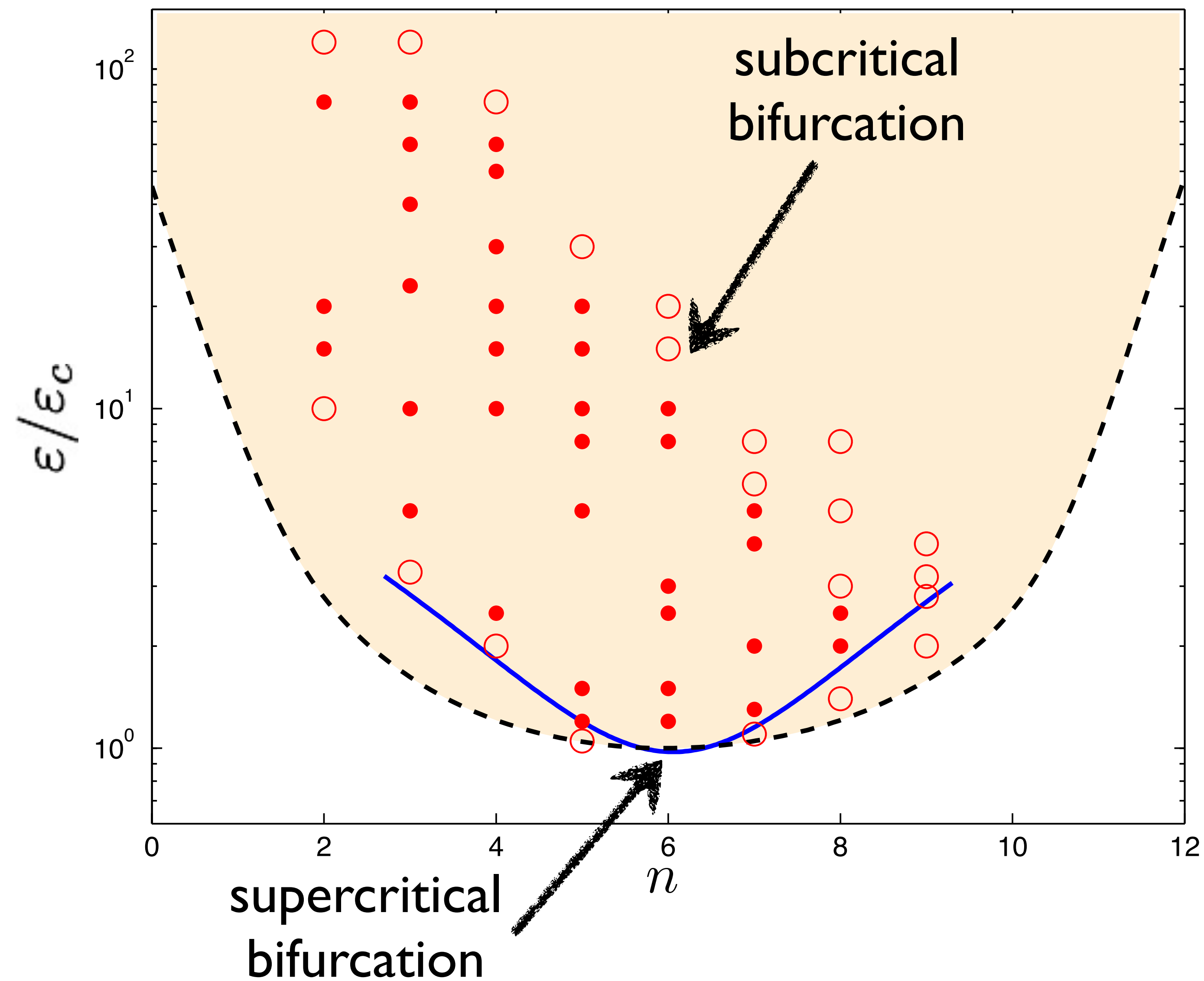
- formation of jets
- existence of multiple equilibria and their domain of attraction
- merging of jets



Stability of S3T equilibria

For higher energy input rates equilibria become S3T unstable and move towards the left of the diagram

S3T equilibria
(stable & unstable)
are hydrodynamically stable



Conclusions

- QL dynamics captures the jet formation process - The turbulent state is essentially determined by a wave/mean flow interaction
- S3T provides a closure of this turbulent system and a theory for the emergence, equilibration and the structural stability of the associated turbulent equilibria
- S3T introduces a new concept of instability arising from the interaction between turbulence with the large scale flow
- S3T predicts:
  - the formation of jets as an eddy/mean flow S3T instability
  - the existence of multiple equilibria as climate states and their stability
  - jet merger dynamics

Furthermore...

A

Ergodic assumption

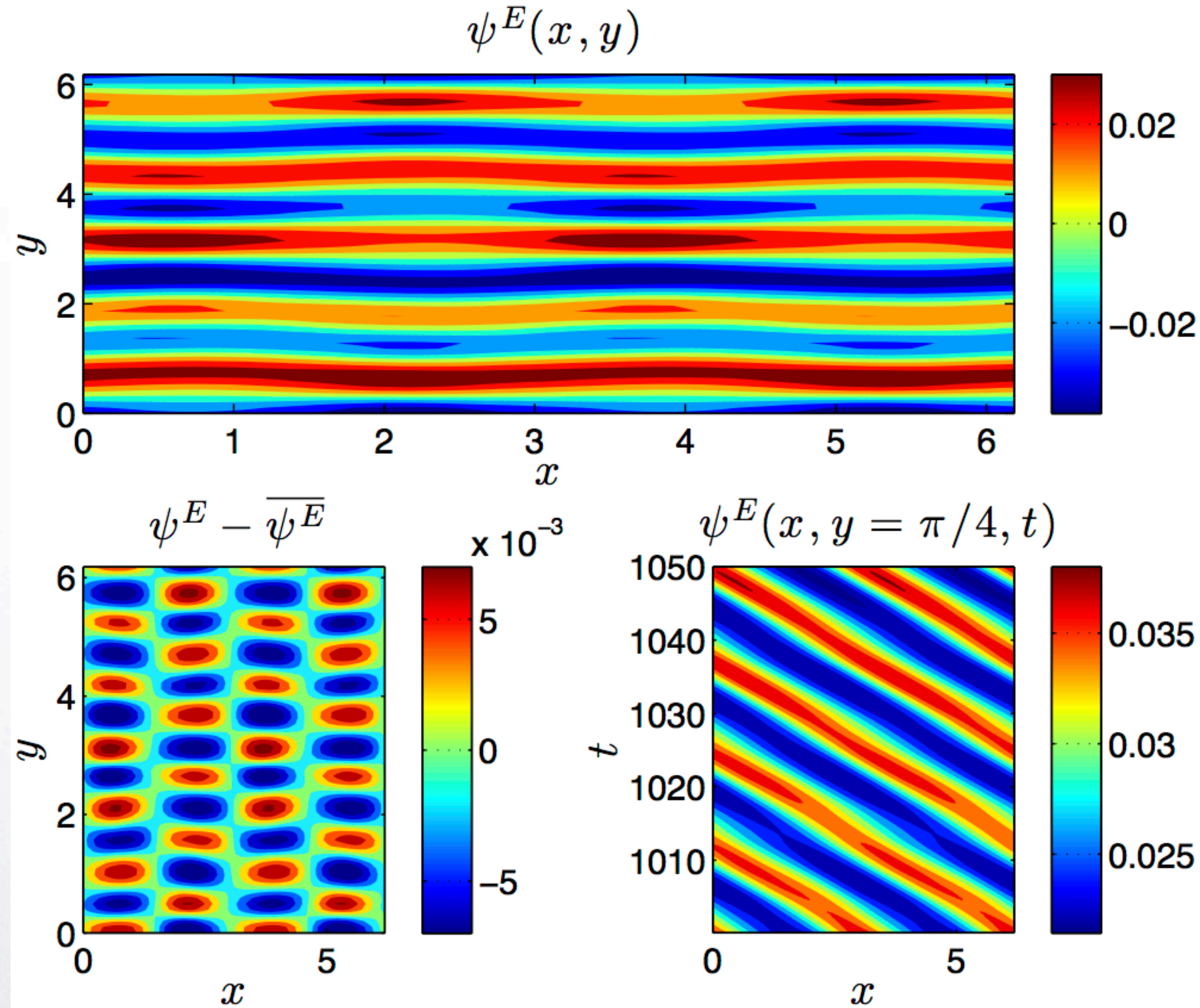
$$\langle \bullet \rangle =$$ Reynolds average over an intermediate time scale

It is possible to obtain non-zonal and even traveling wave finite amplitude S3T equilibria

Bakas and Ioannou, 2013: Emergence of large scale structure in barotropic beta-plane turbulence. *Phys. Rev. Lett.* **110**, 224501.

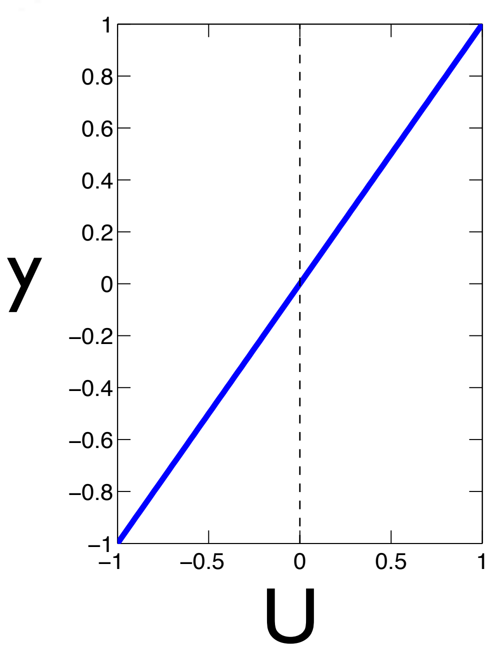
Generalized S3T equilibria

generalized S3T admits equilibria with zonal as well as non-zonal spectral components



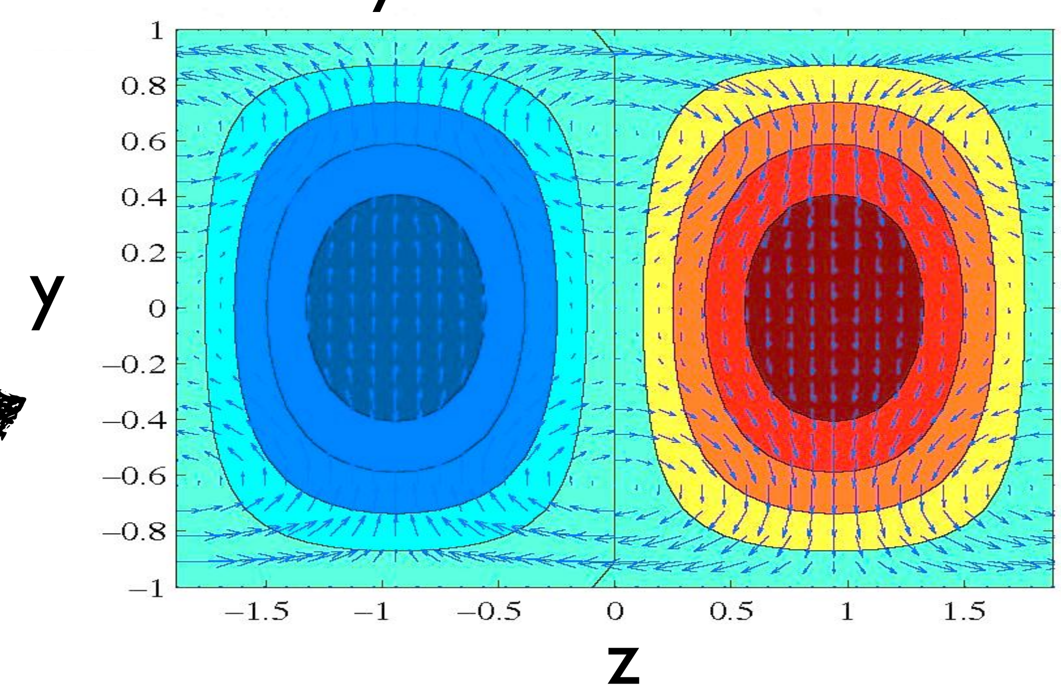
B S3T applied to wall-bounded shear flow

Formation of roll/streak structures in wall-bounded Couette/Poisseuille flow can be identified as ST3 instability



Couette flow
+
homogeneous turbulence

a test function perturbation streak induces a supporting roll



Farrell and Ioannou, 2012: Dynamics of streamwise rolls and streaks in turbulent wall-bounded shear flow. *J. Fluid Mech.* **708**, 149-196.

Constantinou et. al., 2013: Turbulence in the restricted dynamics of the S3T/RNL system: comparison with DNS. *J. Phys. Conf. Ser.* (to appear).

Ευχαριστώ

This work has been supported by Onassis Foundation

Constantinou, N.C., Farrell, B. F. and Ioannou, P.J., 2013: Emergence and equilibration of jets in beta-plane turbulence: applications of Stochastic Structural Stability Theory. *J. Atmos. Sci.*, doi:10.1175/JAS-D-13-076.1, in press.