HELLENIC REPUBLIC
National and Kapodistrian
University of Athens

Formation of large-scale structure by turbulence in planetary atmospheres

Navid Constantinou
Physics Department

5 May 2015

structure of talk

- introduction to the physical problem
- formulation of the theory (S3T)
- study of the stability of homogeneous turbulent state
- comparison of S3T predictions with direct numerical simulations and verification of the theory
- stability of inhomogeneous turbulent states & relation with jet mergers
- relation of modulational instability of Rossby waves with S3T instability of homogeneous state (if time allows)
- summary

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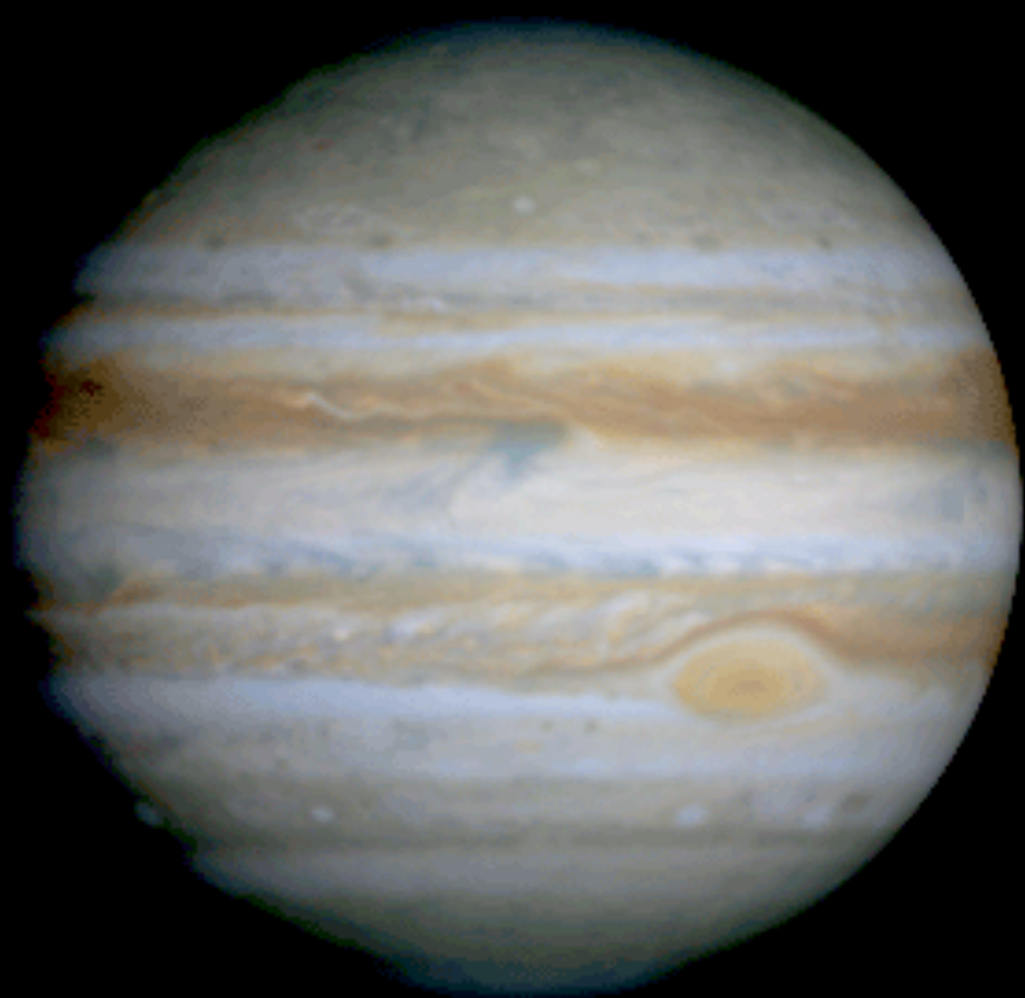
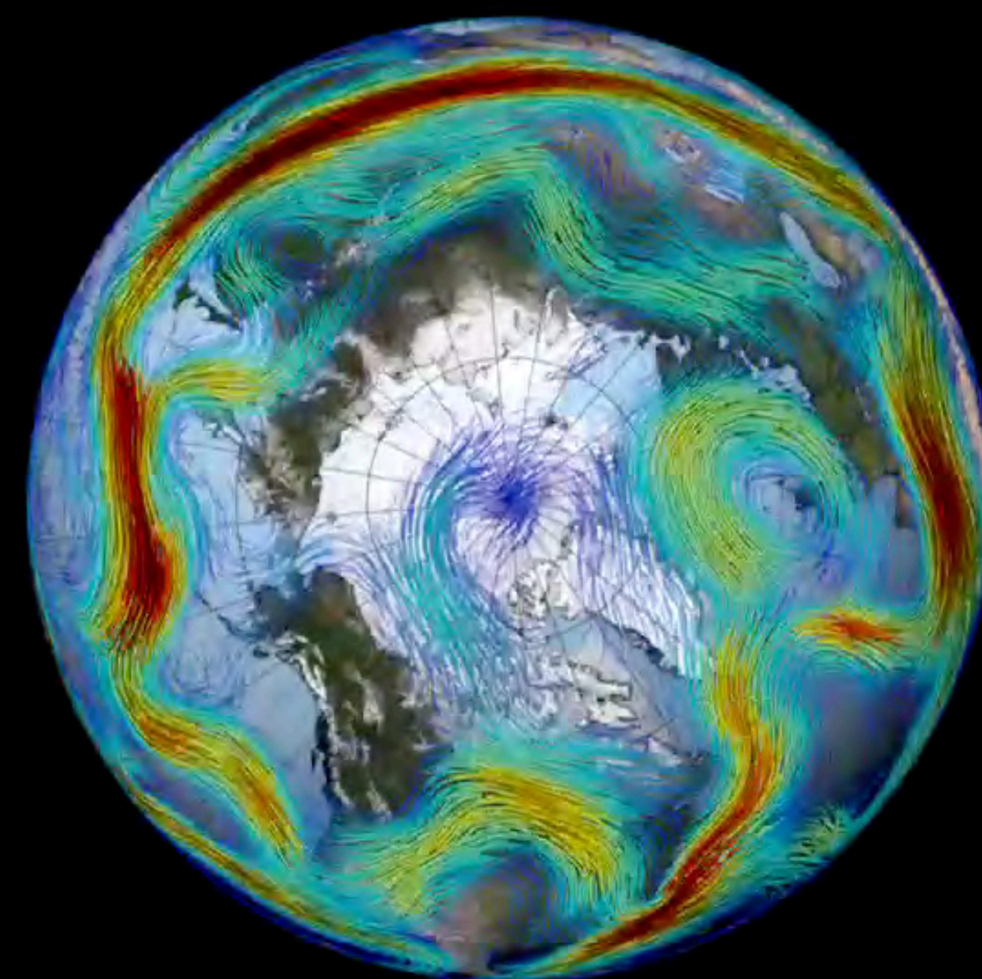
Planetary turbulence is anisotropic and inhomogeneous I



banded Jovian jets

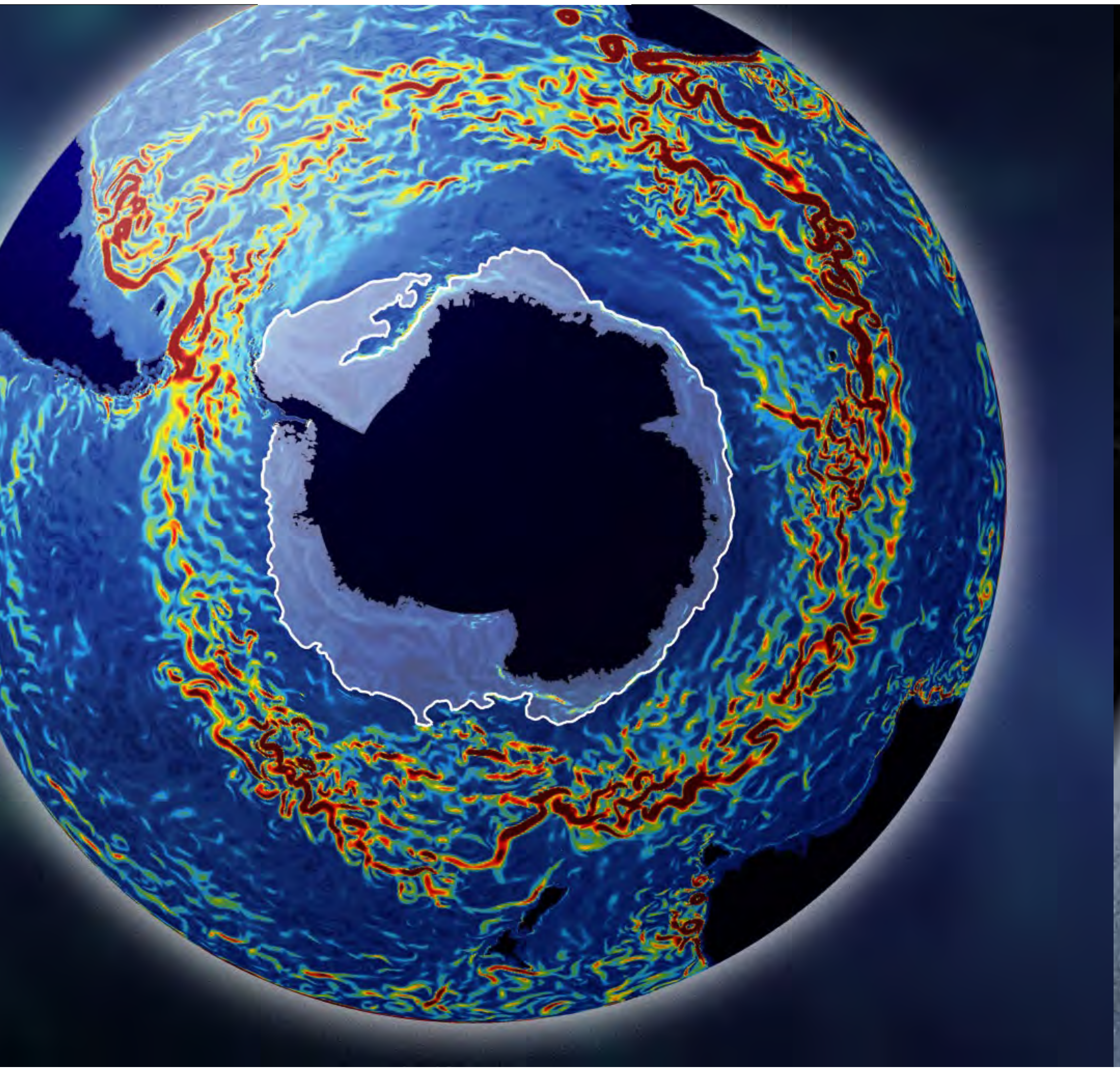
NASA/Cassini Jupiter Images



polar front jet

NASA/Goddard Space Flight Center

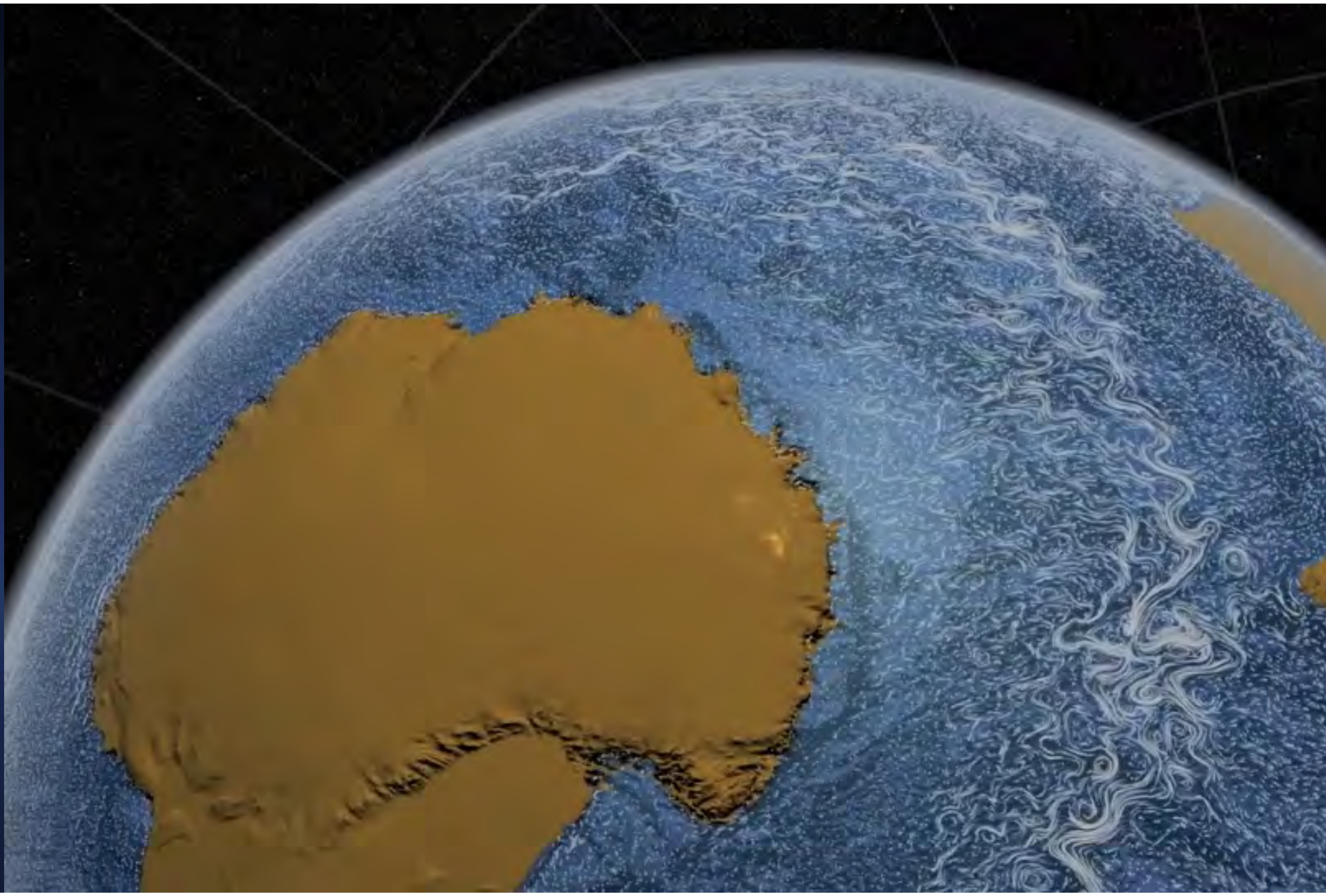
Planetary turbulence
is anisotropic and inhomogeneous II



computer simulation

San Diego Supercomputer Center, UCSD

ACC

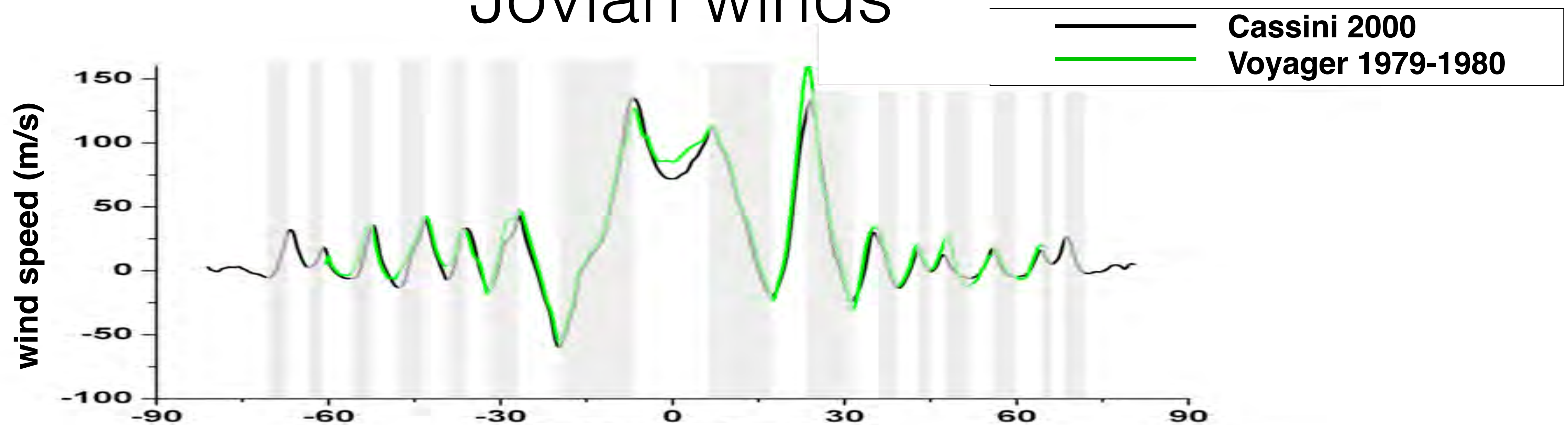


satellite observations

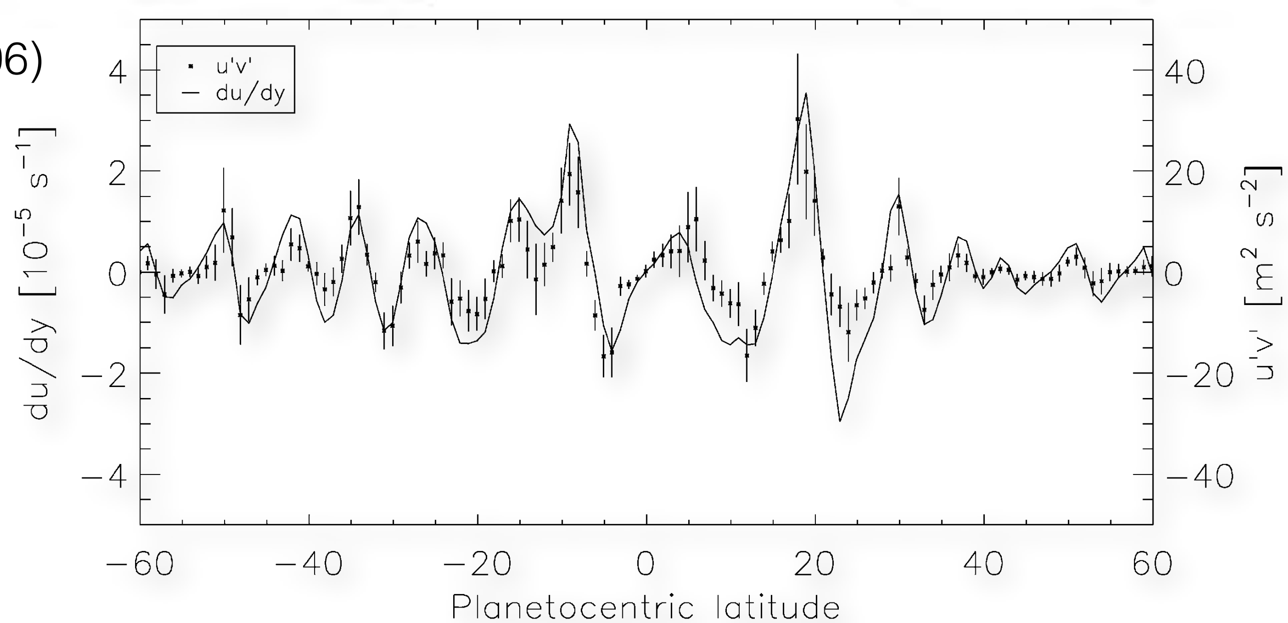
NASA/Goddard Space Flight Center

Jets appear "steady" and are eddy-driven

Jovian winds



(Salyk et. al. 2006)



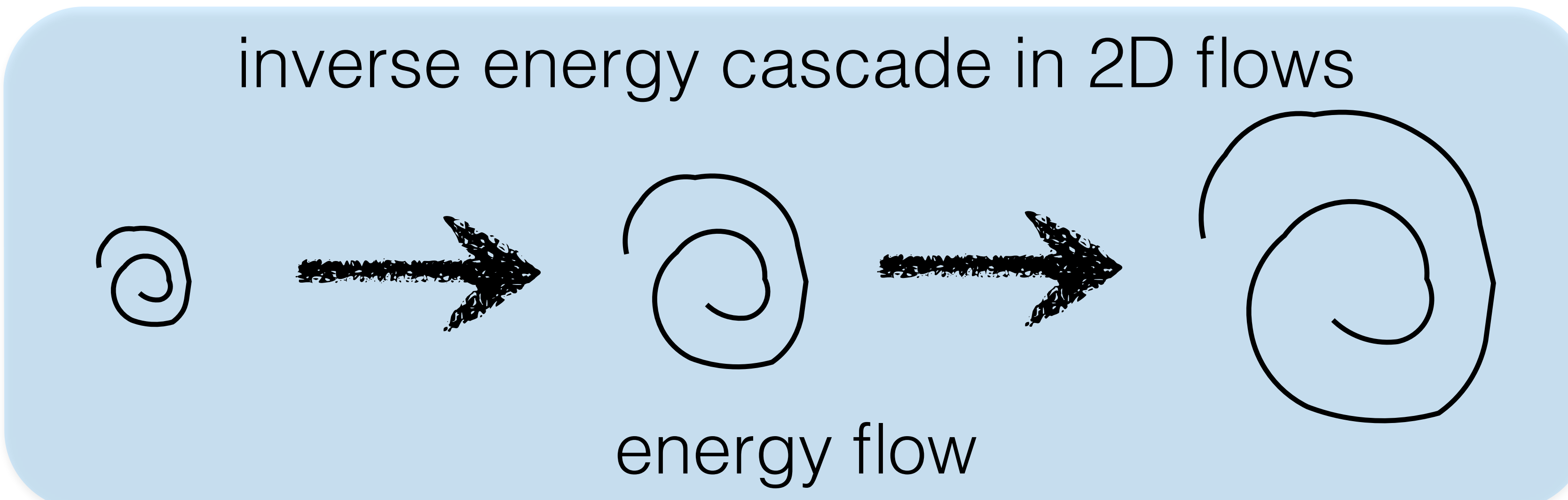
$$\overline{u'v'} = \kappa \frac{\partial U}{\partial y}$$

$$\kappa \approx 10^6 \ \mathrm{m^2\,s^{-1}}$$

$$\partial_t U = -\partial_y \overline{u'v'} = -\kappa \frac{\partial^2 U}{\partial y^2}$$

anti-diffusion!!

Classical phenomenology attributes
large-scale structure formation to turbulent cascades
(inverse energy transfer from smaller to large scales)



We will deploy a statistical theory
for the description of turbulent flows

Emergence of large-scale structure
out of homogeneous turbulence will be understood
as a statistical instability of the turbulent flow

Classical hydrodynamic stability

Lord Rayleigh

Lord Rayleigh taught us how to study the stability of a laminar flow to infinitesimal eddies

Can we study the stability of turbulent flows?

Classical hydrodynamic stability

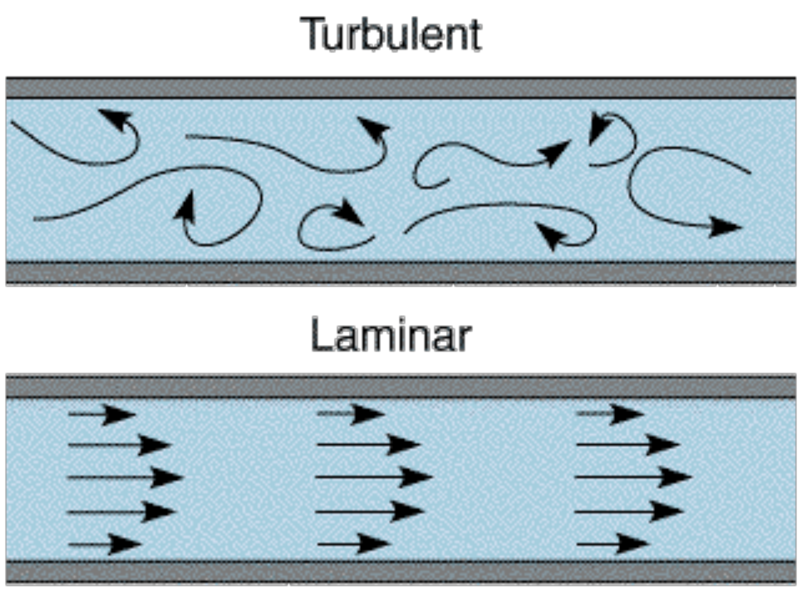
Lord Rayleigh

Hydrodynamic instabilities provide a way for eddies to gain energy from mean flow

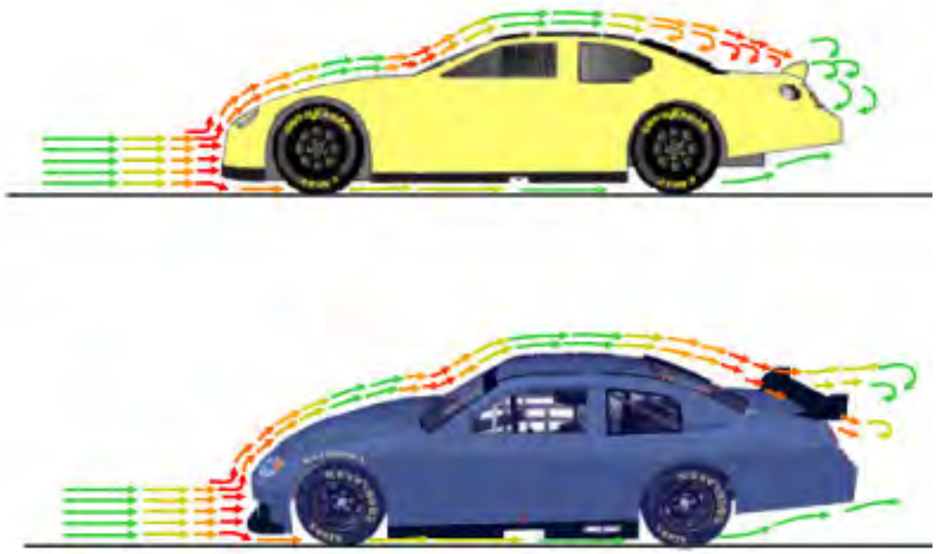
How about the opposite ?

Can the mean flow gain energy from the eddies through an instability ?

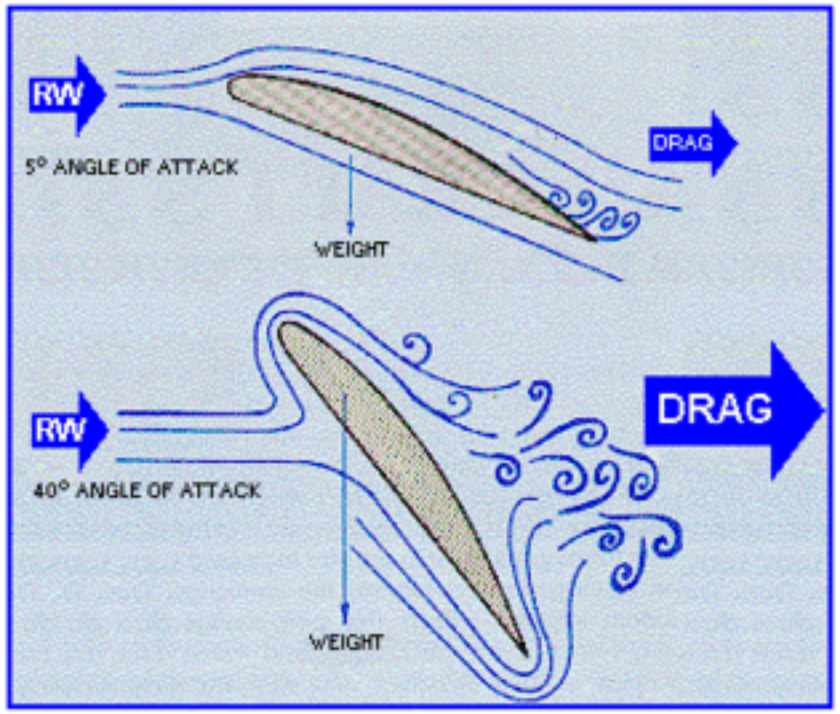
Turbulence (usually) acts as a drag



wall-bounded
flow



airflow over
vehicle



airflow over airfoil

can turbulence act to reinforce large scale flows?

Barotropic vorticity equation on a beta-plane

$$\partial_t \zeta + \mathbf{u} \cdot \nabla \zeta + \mathbf{u} \cdot \boldsymbol{\beta} = -r\zeta + \sqrt{\varepsilon}\,\xi$$

anisotropicity due to rotation

linear dissipation at rate *r*

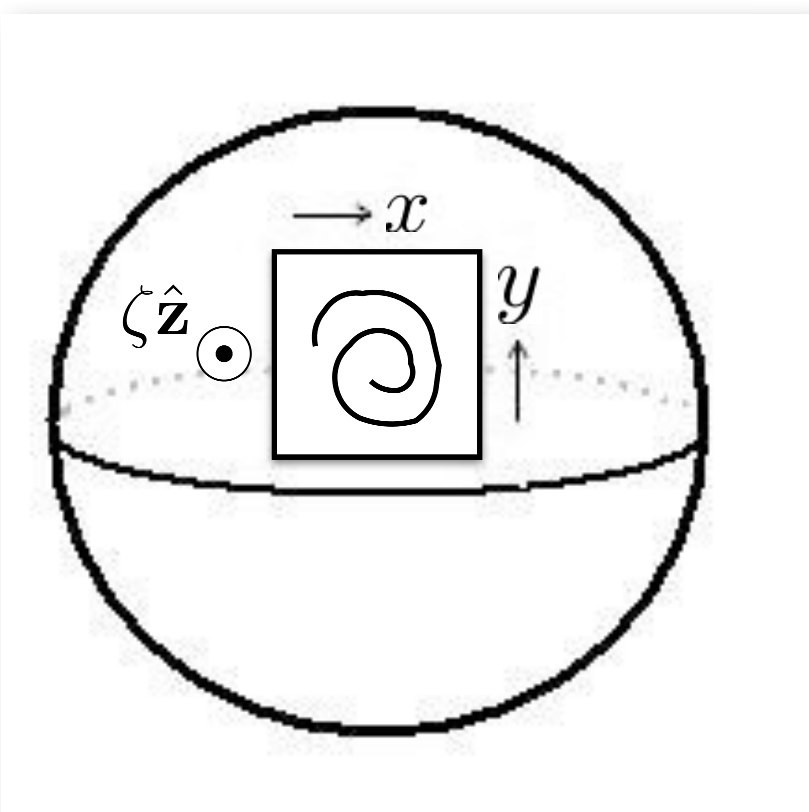
stochastic forcing

$$\nabla \cdot \mathbf{u} = 0$$

$$\nabla \times \mathbf{u} = \zeta\,\hat{\mathbf{z}}$$

$$\mathbf{u} = \hat{\mathbf{z}} \times \Delta^{-1}\zeta$$

$$(\Delta \equiv \nabla \cdot \nabla)$$



$$\boldsymbol{\beta} = (0, \beta)$$

$\boldsymbol{\beta}$ is the gradient of the planetary vorticity

$$\langle \xi(\mathbf{x}_a, t)\xi(\mathbf{x}_b, t')\rangle = Q(\mathbf{x}_a - \mathbf{x}_b)\,\delta(t - t')$$

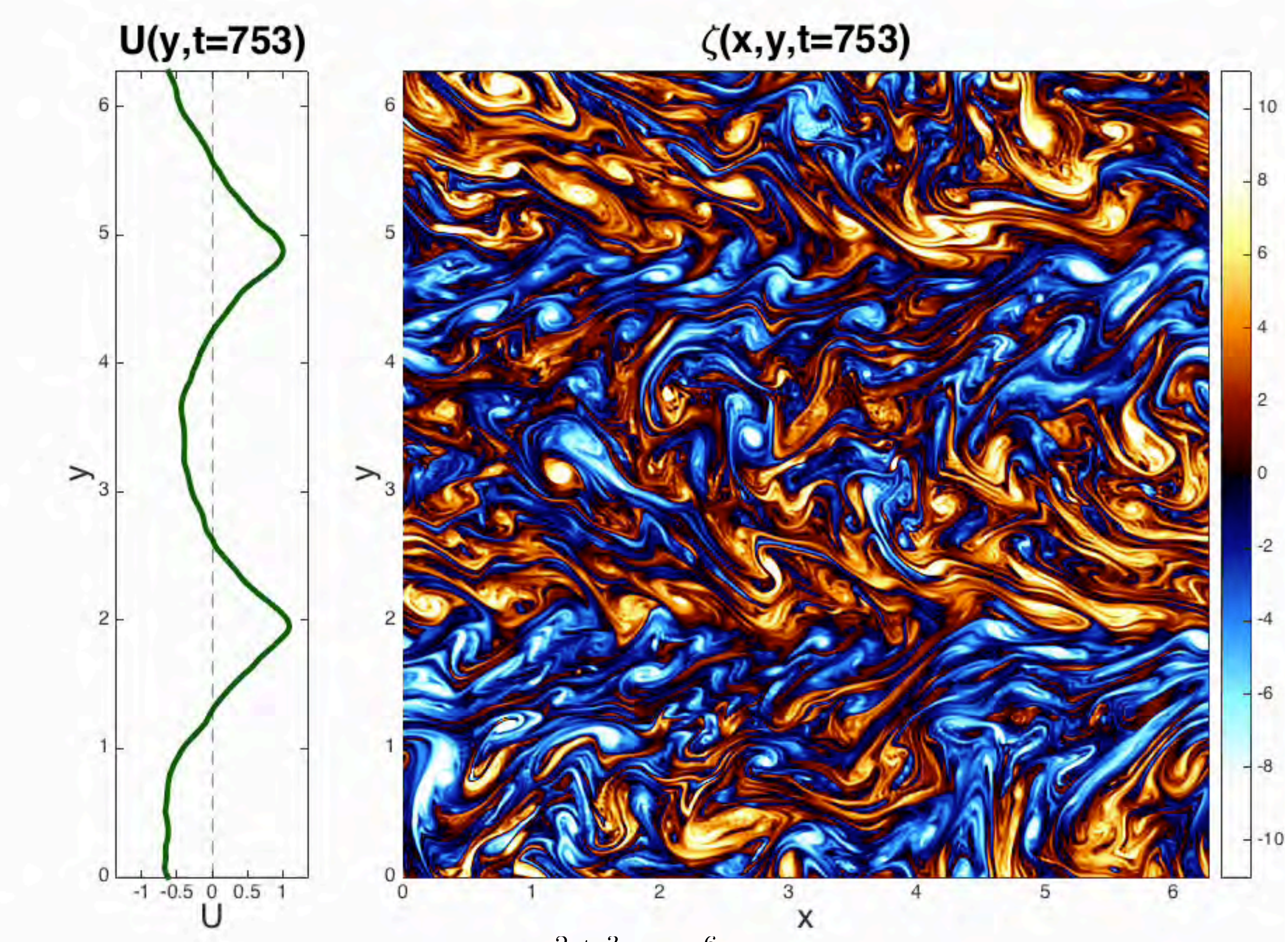
$\boldsymbol{\xi}$ is statistically homogeneous

we have two non-dimensional parameters

$\varepsilon k_f^2 / r^3$

$\beta/(k_f r)$

Barotropic β-plane turbulence exhibits large-scale structure formation



statistically homogeneous forcing

(no inhomogeneity is imposed by the forcing)

initial random flow inhomogeneities organize the turbulence in a manner so that they are reinforced

$$\varepsilon k_f^2/r^3 = 10^6$$

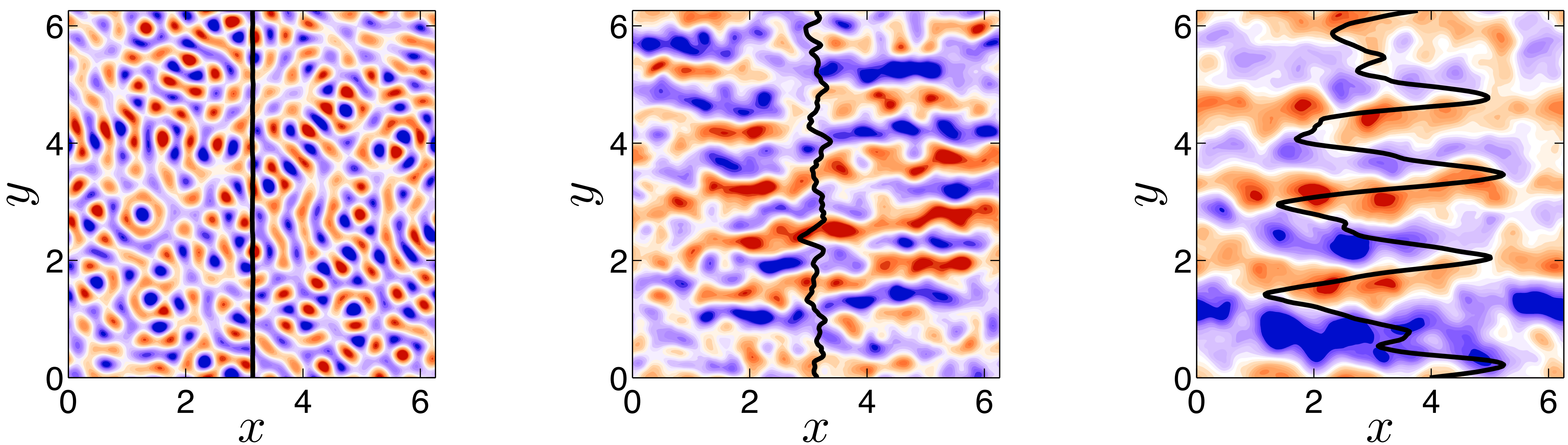
$$\beta/(k_f r) = 67$$

http://www.youtube.com/watch?v=ZtzW25NooDk

β-plane turbulence shows flows at statistically steady state: homogeneous — traveling waves — zonal jets

$$\beta/(k_f r) = 67$$

$\varepsilon k_f^2/r^3 \; = 10^2$ $\qquad$ 5×10^3 $\qquad$ 5×10^4



this suggests that there is some kind of transition as ε is increased

[shown are snapshots of the streamfunction field $\psi(\mathbf{x},t)$
with instantaneous zonal mean flow $U(y,t)$]

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Barotropic vorticity equation on a beta-plane

Using decomposition: $$\zeta(\mathbf{x},t) = \underbrace{\mathcal{T}\left[\zeta(\mathbf{x},t)\right]}_{Z(\mathbf{x},t)} \ + \ \zeta'(\mathbf{x},t)$$

mean flow — eddies

$$\partial_t Z + \mathbf{U}\cdot\nabla Z + \mathbf{U}\cdot\boldsymbol{\beta} = -\mathcal{T}\left[\mathbf{u}'\cdot\nabla\zeta'\right] - rZ$$

$$\partial_t \zeta' = \mathcal{A}(\mathbf{U})\,\zeta' + \mathcal{T}\left[\mathbf{u}'\cdot\nabla\zeta'\right] - \mathbf{u}'\cdot\nabla\zeta' + \sqrt{\varepsilon}\,\xi$$

with

$$\mathcal{A}(\mathbf{U}) \equiv -\mathbf{U}\cdot\nabla + \Big[(\Delta\mathbf{U}) - \boldsymbol{\beta}\cdot(\hat{\mathbf{z}}\times\nabla)\Big]\,\Delta^{-1} - r$$

$\mathcal{T}$ →

- average over the zonal direction *x*
- Reynolds over an intermediate time scale or length scale
  (larger than the time scale or length scale of the turbulent motions
  and smaller than the time scale or length scale of mean field)

NL system

$$\partial_t Z + \mathbf{U}\cdot\nabla Z + \mathbf{U}\cdot\boldsymbol{\beta} = -\mathcal{T}\left[\mathbf{u}'\cdot\nabla\zeta'\right] - rZ$$

$$\partial_t \zeta' = \mathcal{A}(\mathbf{U})\,\zeta' + \mathcal{T}\left[\mathbf{u}'\cdot\nabla\zeta'\right] - \mathbf{u}'\cdot\nabla\zeta' + \sqrt{\varepsilon}\,\xi$$

NL system

restrict nonlinearity by *not* allowing
eddy-eddy → eddy interactions (QL)

$$\partial_t Z + \mathbf{U}\cdot\nabla Z + \mathbf{U}\cdot\boldsymbol{\beta} = -\mathcal{T}\left[\mathbf{u}'\cdot\nabla\zeta'\right] - rZ$$

$$\partial_t \zeta' = \mathcal{A}(\mathbf{U})\,\zeta' + \mathcal{T}\left[\mathbf{u}'\cdot\nabla\zeta'\right] - \mathbf{u}'\cdot\nabla\zeta' + \sqrt{\varepsilon}\,\xi$$

QL system

restrict nonlinearity by *not* allowing eddy-eddy → eddy interactions (QL)

$$\partial_t Z + \mathbf{U} \cdot \nabla Z + \mathbf{U} \cdot \boldsymbol{\beta} = -\mathcal{T}\left[\mathbf{u}' \cdot \nabla \zeta'\right] - rZ$$

$$\partial_t \zeta' = \mathcal{A}(\mathbf{U})\, \zeta' + \sqrt{\varepsilon}\, \xi$$

QL allows ***only*** the direct, two-way interaction of the eddies and the mean flow

QL does ***not*** include turbulent cascades

S3T system

Under the ergodic assumption that the average *T* is equal to ensemble average over forcing realizations:

$$\mathcal{T}(\,\bullet\,) \;=\; \langle\,\bullet\,\rangle$$

we derive from QL a *closed* system for the evolution of the first two statistical moments of the flow

$$Z(\mathbf{x},t) = \langle \zeta(\mathbf{x},t) \rangle \quad , \quad C(\mathbf{x}_a,\mathbf{x}_b,t) = \langle \zeta'(\mathbf{x}_a,t)\zeta'(\mathbf{x}_b,t) \rangle$$

S3T system

$$\partial_t Z + \mathbf{U}\cdot\nabla Z + \mathbf{U}\cdot\boldsymbol{\beta} = \mathcal{R}(C) - rZ$$

$$\partial_t C_{ab} = \left[\mathcal{A}_a(\mathbf{U}) + \mathcal{A}_b(\mathbf{U})\right] C_{ab} + \varepsilon\, Q_{ab}$$

with

$$C_{ab} \equiv C(\mathbf{x}_a, \mathbf{x}_b, t) = \langle \zeta'(\mathbf{x}_a, t)\zeta'(\mathbf{x}_b, t)\rangle$$

$Q_{ab} \equiv Q(\mathbf{x}_a - \mathbf{x}_b) \longrightarrow$ the spatial covariance of the statistically homogeneous stochastic forcing

$$\mathcal{R}(C) \equiv -\langle \mathbf{u}'\cdot\nabla\zeta'\rangle = -\nabla\cdot\left[\frac{\hat{\mathbf{z}}}{2}\times(\nabla_a\Delta_a^{-1} + \nabla_b\Delta_b^{-1})C_{ab}\right]_{a=b}$$

(the Reynolds stresses are given as a linear function of *C*)

S3T system

$$\partial_t Z + \mathbf{U} \cdot \nabla Z + \mathbf{U} \cdot \boldsymbol{\beta} = \mathcal{R}(C) - rZ$$

$$\partial_t C_{ab} = [\mathcal{A}_a(\mathbf{U}) + \mathcal{A}_b(\mathbf{U})]\, C_{ab} + \varepsilon\, Q_{ab}$$

Neglect of the eddy-eddy term in NL is equivalent with neglect of third and higher-order statistical moments.

S3T system

(the theory)

$$\partial_t Z + \mathbf{U}\cdot\nabla Z + \mathbf{U}\cdot\boldsymbol{\beta} = \mathcal{R}(C) - rZ$$

$$\partial_t C_{ab} = \left[\mathcal{A}_a(\mathbf{U}) + \mathcal{A}_b(\mathbf{U})\right] C_{ab} + \varepsilon\, Q_{ab}$$

The S3T system

- autonomous
- deterministic
- admits fixed point solutions consisting of a mean flow and second-order eddy statistics $\left(\mathbf{U}^e(\mathbf{x}), C^e(\mathbf{x}_a, \mathbf{x}_b)\right)$
- allows the study of the stability of such equilibrium solutions

stability of S3T equilibria

$$\Big(\mathbf{U}^e(\mathbf{x}), C^e(\mathbf{x}_a, \mathbf{x}_b)\Big)$$

perturbations ($\delta Z,\ \delta C$) about an equilibrium satisfy the linearized S3T equations:

$$\partial_t \delta Z = \mathcal{A}^e\, \delta Z + \mathcal{R}(\delta C)$$

$$\partial_t \delta C_{ab} = (\mathcal{A}^e_a + \mathcal{A}^e_b)\delta C_{ab} + (\delta\mathcal{A}_a + \delta\mathcal{A}_b) C^e_{ab}$$

$$\mathcal{A}^e \equiv \mathcal{A}(\mathbf{U}^e)$$

eigenanalysis of this system determines the stability of $\Big(\mathbf{U}^e(\mathbf{x}), C^e(\mathbf{x}_a, \mathbf{x}_b)\Big)$

stability of S3T equilibria

$$\Big(\mathbf{U}^e(\mathbf{x}), C^e(\mathbf{x}_a, \mathbf{x}_b)\Big)$$

perturbations ($\delta Z,\ \delta C$) about an equilibrium satisfy the linearized S3T equations:

hydrodynamic stability

$$\partial_t \delta Z = \mathcal{A}^e\, \delta Z + \mathcal{R}(\delta C)$$

$$\partial_t \delta C_{ab} = (\mathcal{A}^e_a + \mathcal{A}^e_b)\delta C_{ab} + (\delta\mathcal{A}_a + \delta\mathcal{A}_b) C^e_{ab}$$

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for statistically homogeneous forcing there exists *always* a statistically homogeneous S3T equilibrium

$$\mathbf{U}^e = 0 \quad , \quad C^e(\mathbf{x}_a - \mathbf{x}_b) = \frac{\varepsilon\, Q}{2r}$$

(for any ε, β and homogeneous Q)

for statistically homogeneous forcing there exists *always* a statistically homogeneous S3T equilibrium

$$\mathbf{U}^e = 0 \quad , \quad C^e(\mathbf{x}_a - \mathbf{x}_b) = \frac{\varepsilon\, Q}{2r}$$

(for any ε, β and homogeneous Q)

eigenfunctions:

$$\delta Z = e^{\mathrm{i}\mathbf{n}\cdot\mathbf{x}}\, e^{(\sigma - \mathrm{i}\omega_{\mathbf{n}})t} \qquad \text{(plane wave)}$$

$$\delta C_{ab} = e^{\mathrm{i}\mathbf{n}\cdot(\mathbf{x}_a + \mathbf{x}_b)/2}\, \delta C^{(\mathrm{h})}(\mathbf{x}_a - \mathbf{x}_b)\, e^{(\sigma - \mathrm{i}\omega_{\mathbf{n}})t}$$

Instability occurs when Re(σ)>0 for at least one plane wave **n**

Eigenvalues σ satisfy:

$$\sigma + r = \varepsilon \int \frac{\mathrm{d}^2\mathbf{k}}{(2\pi)^2} \frac{|\mathbf{n}\times\mathbf{k}|^2 (k_p^2 - k^2)(k^2 - n^2)}{k_p^2 k^4 n^2 \left[\sigma + 2r + \mathrm{i}\left(\omega_{\mathbf{k}+\mathbf{n}} - \omega_{\mathbf{k}} - \omega_{\mathbf{n}}\right)\right]} \frac{\hat{Q}(\mathbf{k})}{2r}$$

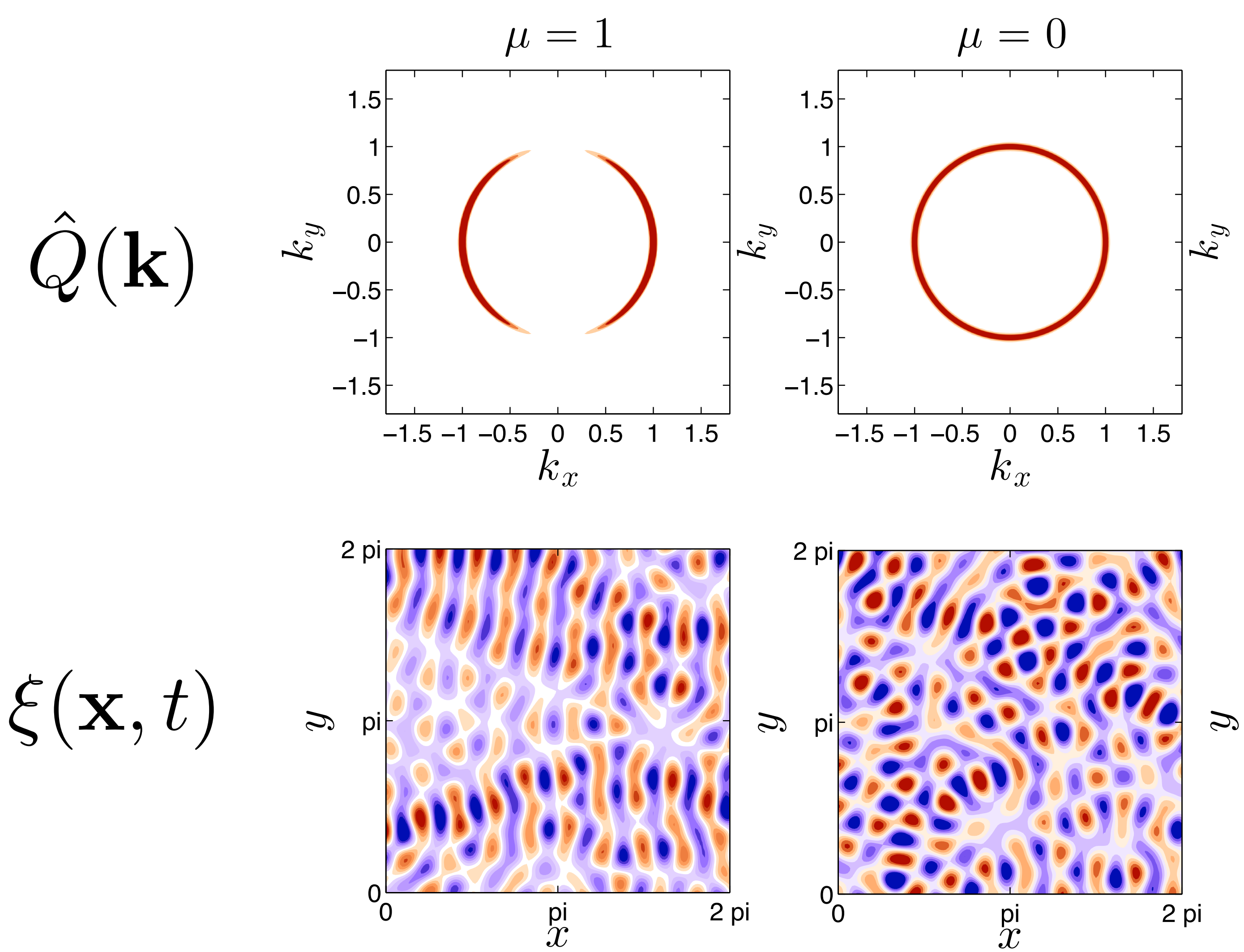
$\mathbf{k}_p = \mathbf{k} + \mathbf{n}$, $k_p = |\mathbf{k}_p|$, $k = |\mathbf{k}|$, $\omega_{\mathbf{n}} = \frac{-\beta n_x}{n^2}$ Rossby wave frequency , $\hat{Q}(\mathbf{k}) = \int \frac{\mathrm{d}^2\mathbf{k}}{(2\pi)^2} Q(\mathbf{x}_a - \mathbf{x}_b) e^{\mathrm{i}\mathbf{k}\cdot(\mathbf{x}_a - \mathbf{x}_b)}$

take forcing prescribed with spatial covariance with spectrum

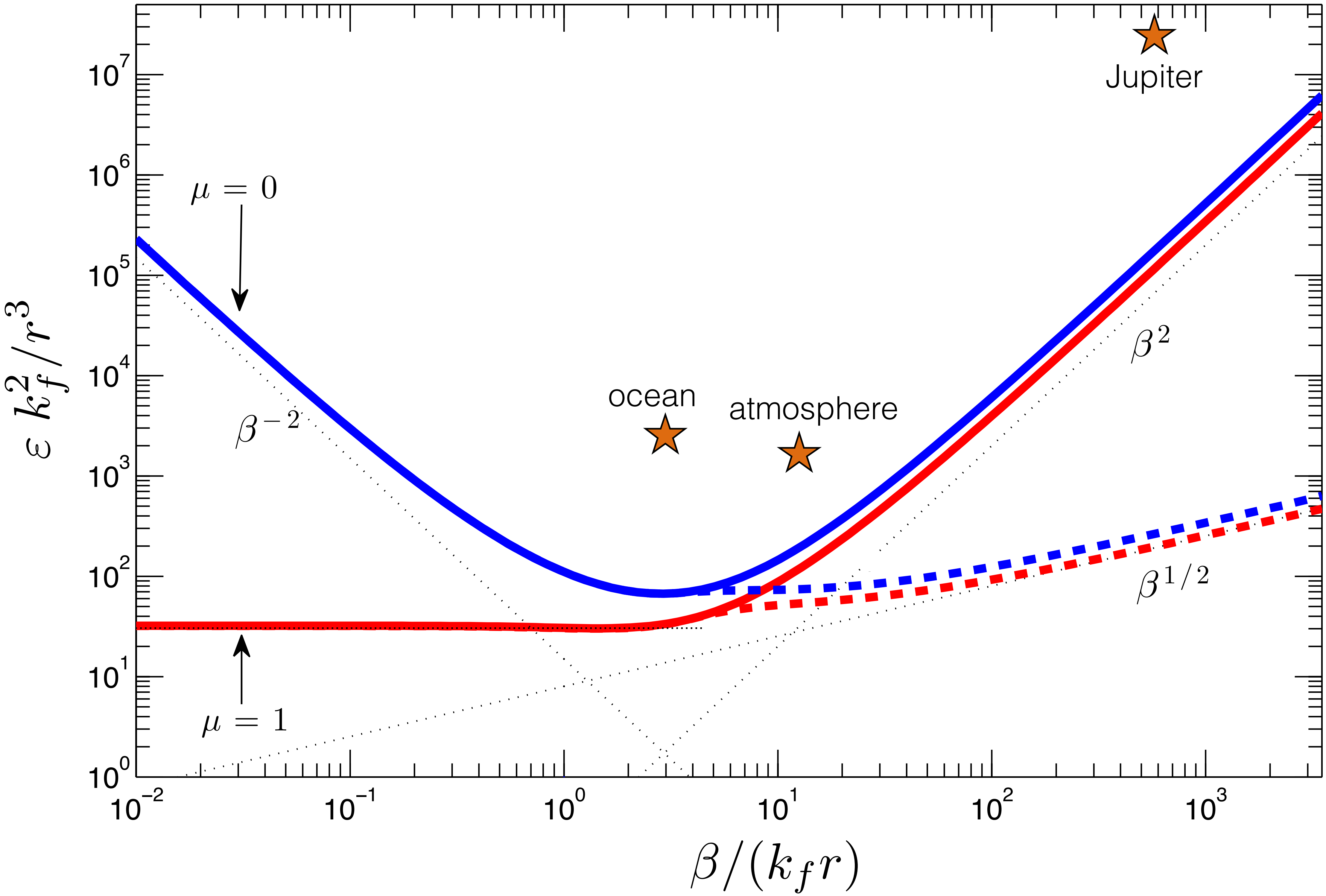
$$\hat{Q}(\mathbf{k}) \sim \delta(k - k_f)\ \left[1 + \mu \frac{k_x^2 - k_y^2}{k^2}\right] \qquad |\mu| \leq 1$$

anisotropic (baroclinic) [≈Earth]

isotropic (convection) [≈Jupiter]

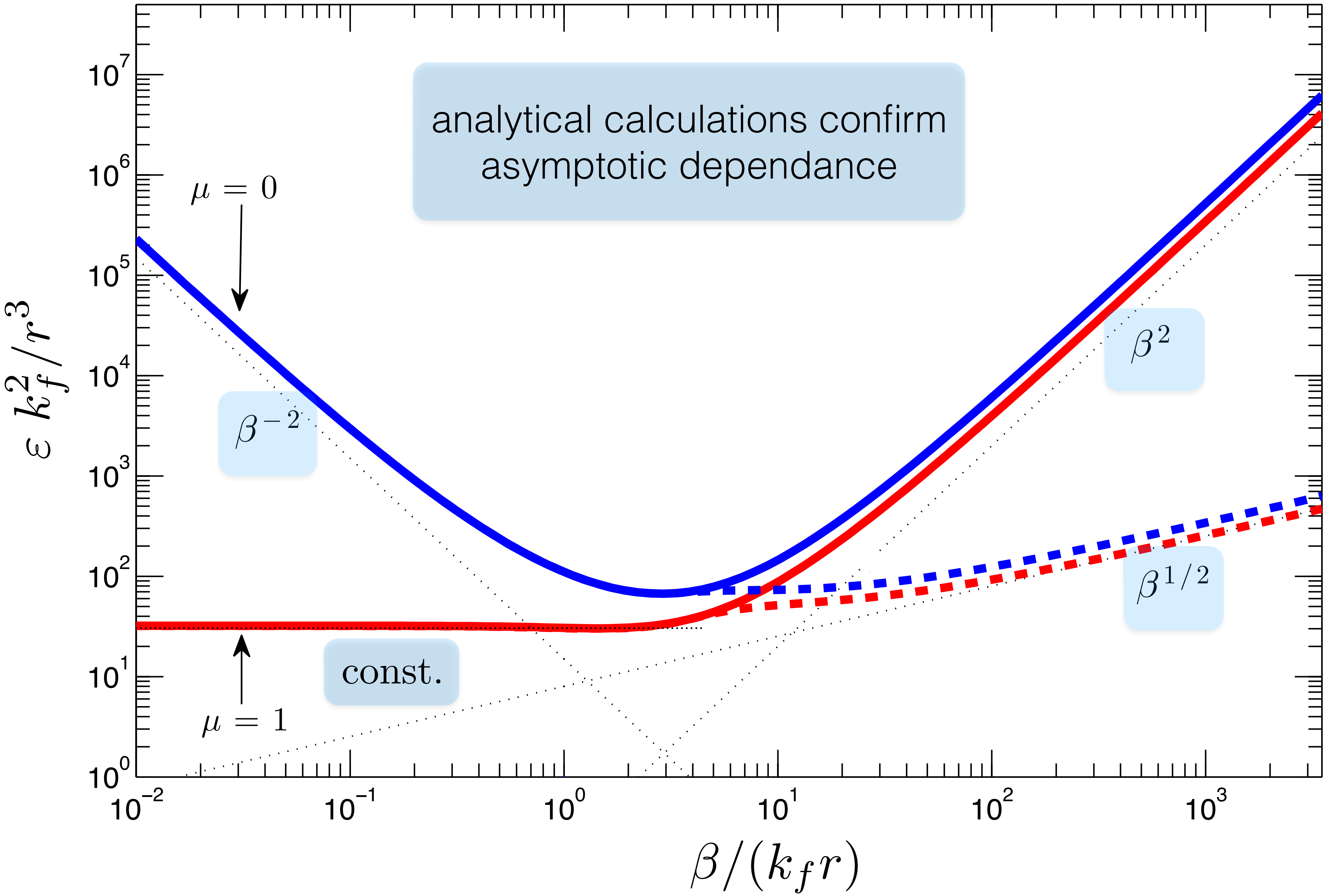


Critical ε for S3T instability of the homogeneous state



for large β the critical ε scaling is independent of the forcing structure

Critical ε for S3T instability of the homogeneous state



for large β the critical ε scaling is independent of the forcing structure

instability of the homogeneous turbulent state

$$\sigma + r = \underbrace{\varepsilon \int \frac{\mathrm{d}^2\mathbf{k}}{(2\pi)^2} \frac{|\mathbf{n}\times\mathbf{k}|^2 (k_p^2 - k^2)(k^2 - n^2)}{k_p^2 k^4 n^2 \left[\sigma + 2r + \mathrm{i}\left(\omega_{\mathbf{k}+\mathbf{n}} - \omega_{\mathbf{k}} - \omega_{\mathbf{n}}\right)\right]} \frac{\hat{Q}(\mathbf{k})}{2r}}_{f}$$

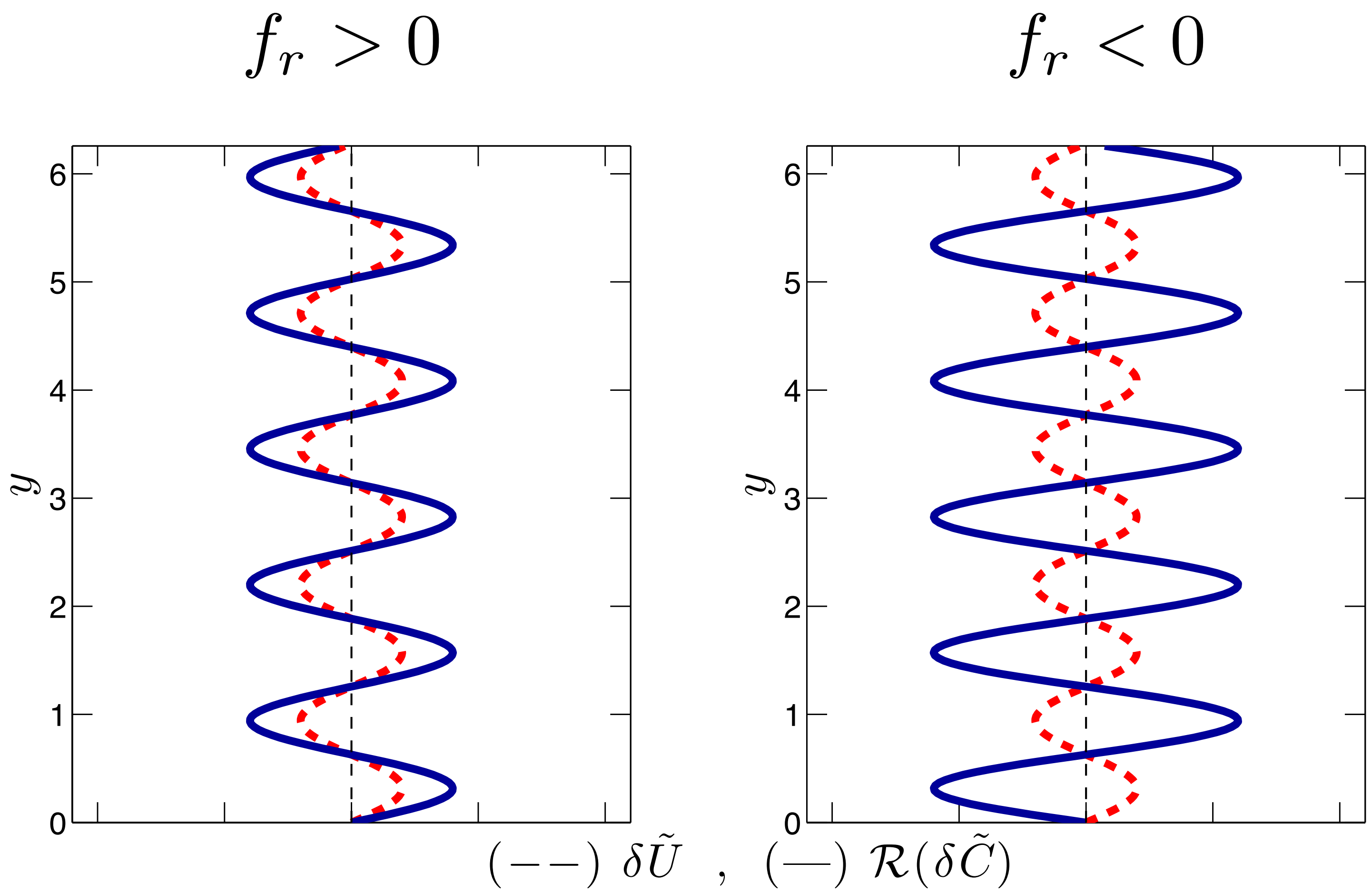
mean flow perturbation equation

$$(\partial_t - \mathcal{A}^e)\,\delta Z = \mathcal{R}(\delta C)$$

f is the sum of the contributions of the spectral components of Q (or C^e) to the perturbation Reynolds stress divergence

Instability requires $\quad f_r \equiv \mathrm{Re}(f)\Big|_{\varepsilon_c} > 0 \qquad\qquad \varepsilon_c = \dfrac{r}{f_r}$

f_r determines if turbulence will act so as to reinforce or diminish the infinitesimal mean flow



turbulence acts as: anti-diffusion diffusion

analytic calculations for f_r show that turbulence acts anti-diffusively even for infinitesimal mean flows!

small β

anisotropic forcing ($\mu \neq 0$) → $f_r \sim \mu$ → $\varepsilon_c \sim 1/\mu$

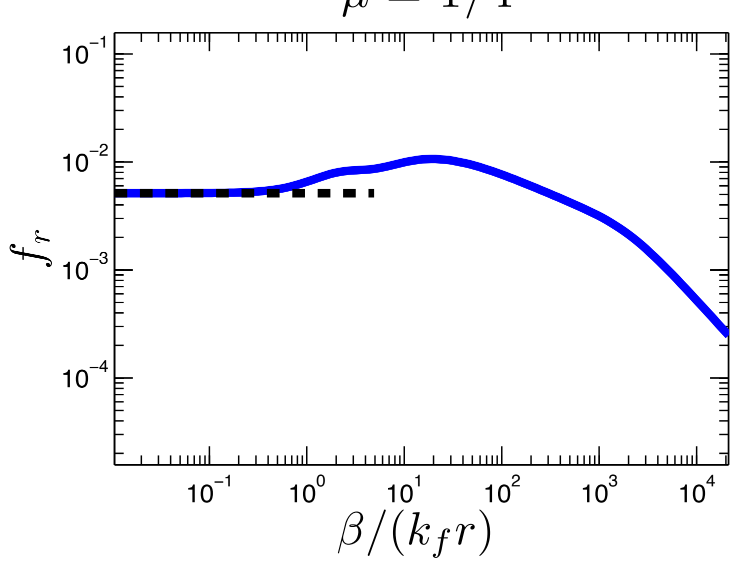
isotropic forcing ($\mu = 0$) → $f_r \sim \beta^2$ → $\varepsilon_c \sim \beta^{-2}$

$$(\partial_t + r)\delta U = -\partial_y \, \delta\langle u'v'\rangle = -C\,\mu\partial^2_{yy}\delta U + D\,\beta^2\partial^4_{yyyy}\delta U \qquad C, D > 0$$

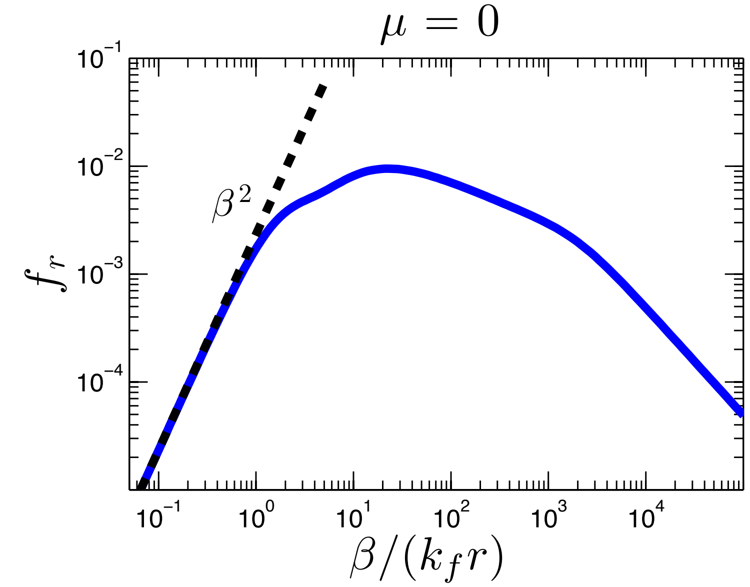
Jupiter's
finite amplitude jets
$\partial_y \, \overline{u'v'} = \kappa\,\partial^2_{yy} U$
$\kappa \approx 10^6 \ \mathrm{m^2\,s^{-1}}$

$\mu > 0$	$\mu = 0$
2nd order *anti*-diffusion	4th order hyper-*anti*-diffusion

$\mu = 1/4$



$\mu = 0$



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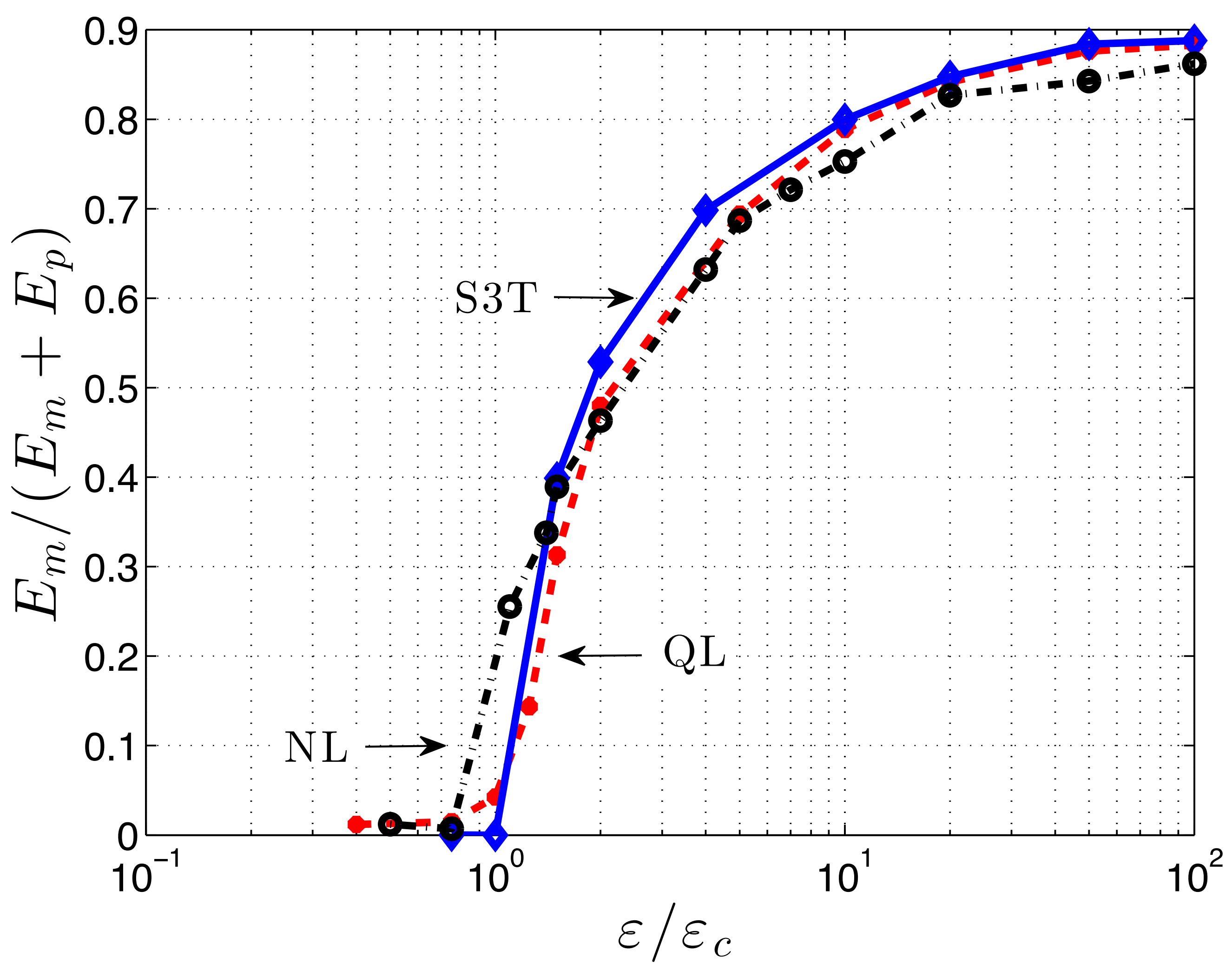
S3T predictions for jet formation and equilibration at finite amplitude

anisotropic forcing
[Earth-like]

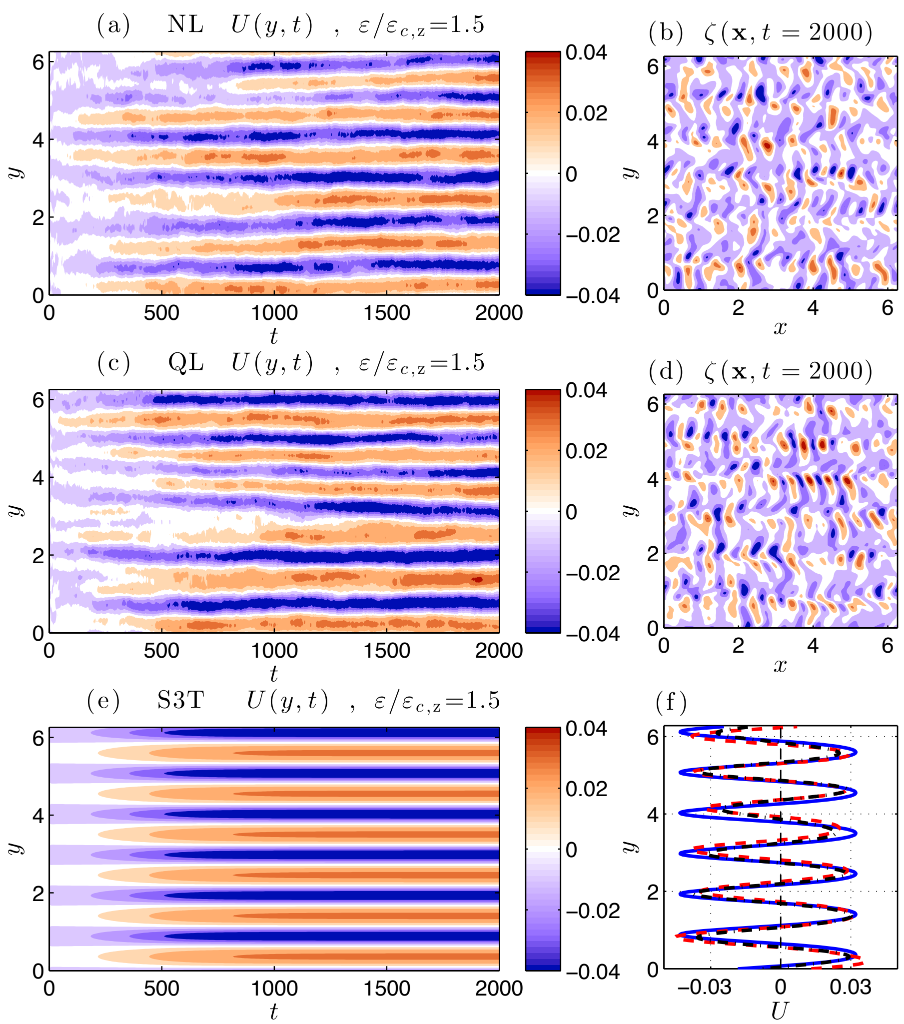
zonal mean flow
energy fraction



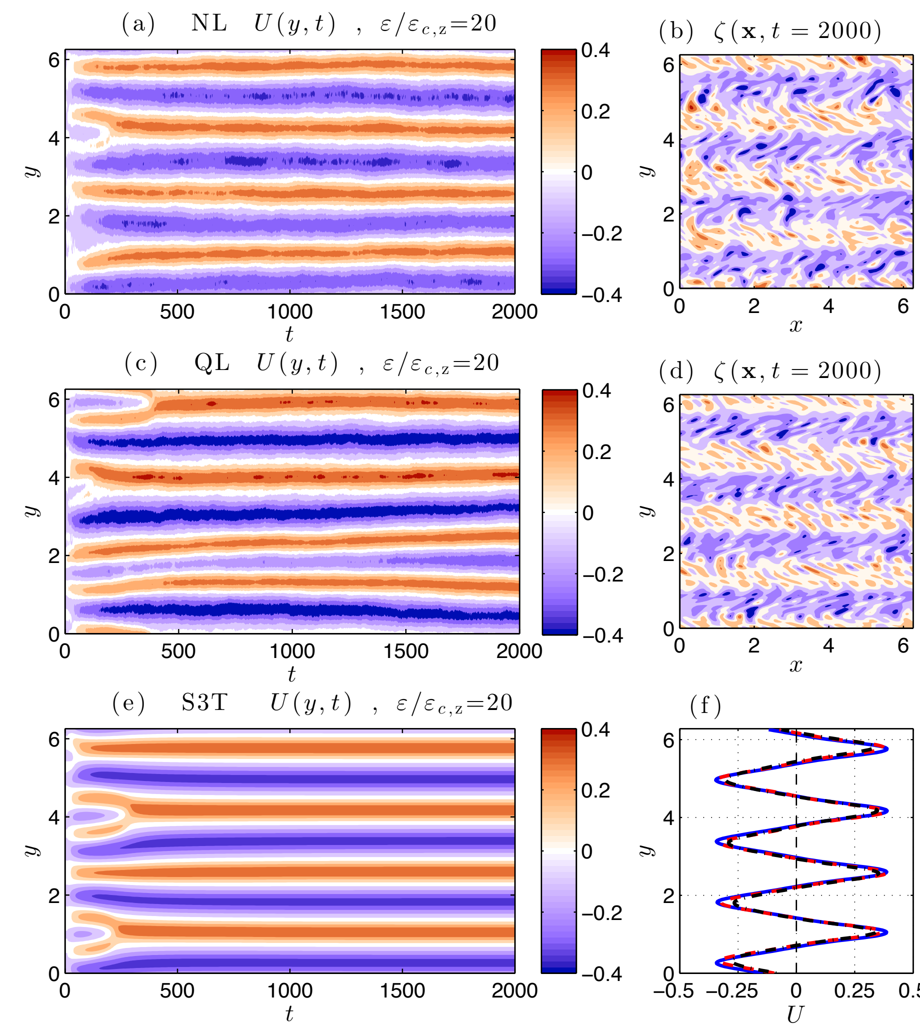
S3T predictions for jet formation and equilibration at finite amplitude

anisotropic forcing
[Earth-like]

$\varepsilon/\varepsilon_{c,z}$ = 1.5



$\varepsilon/\varepsilon_{c,z}$ = 20



statistical instabilities that are predicted by S3T show up in single NL/QL realizations of the flow

emergent instabilities grow and reach finite amplitude

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Zonal jet S3T equilibria

$$\mathbf{U}^e(\mathbf{x}) = \Big(U^e(y), 0\Big) \quad , \quad C^e(x_a - x_b, y_a, y_b)$$

We have developed numerical methods for

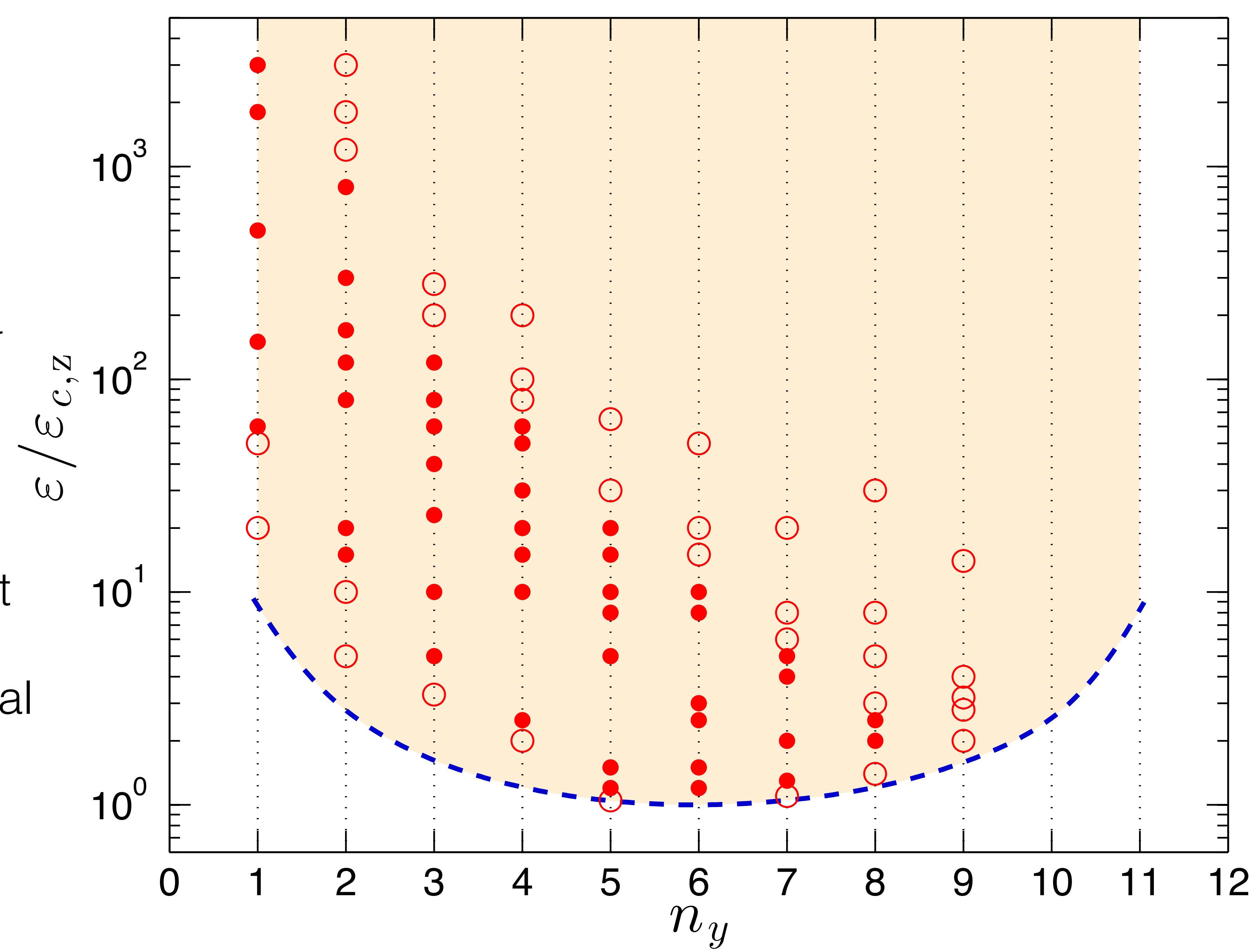
i) determining such equilibria with great accuracy and

ii) studying their S3T stability

Stability of zonal jet S3T equilibria to zonal jet perturbations

Stability analysis of inhomogeneous turbulent states with zonal jets predicts:

- existence of multiple equilibria and their domain of attraction
- merging of jets as ε increases
- finite amplitude equilibration at small supercriticality is described through the universal Eckhaus instability of the G-L amplitude equation



(= number of jets for a $2\pi \times 2\pi$ channel)

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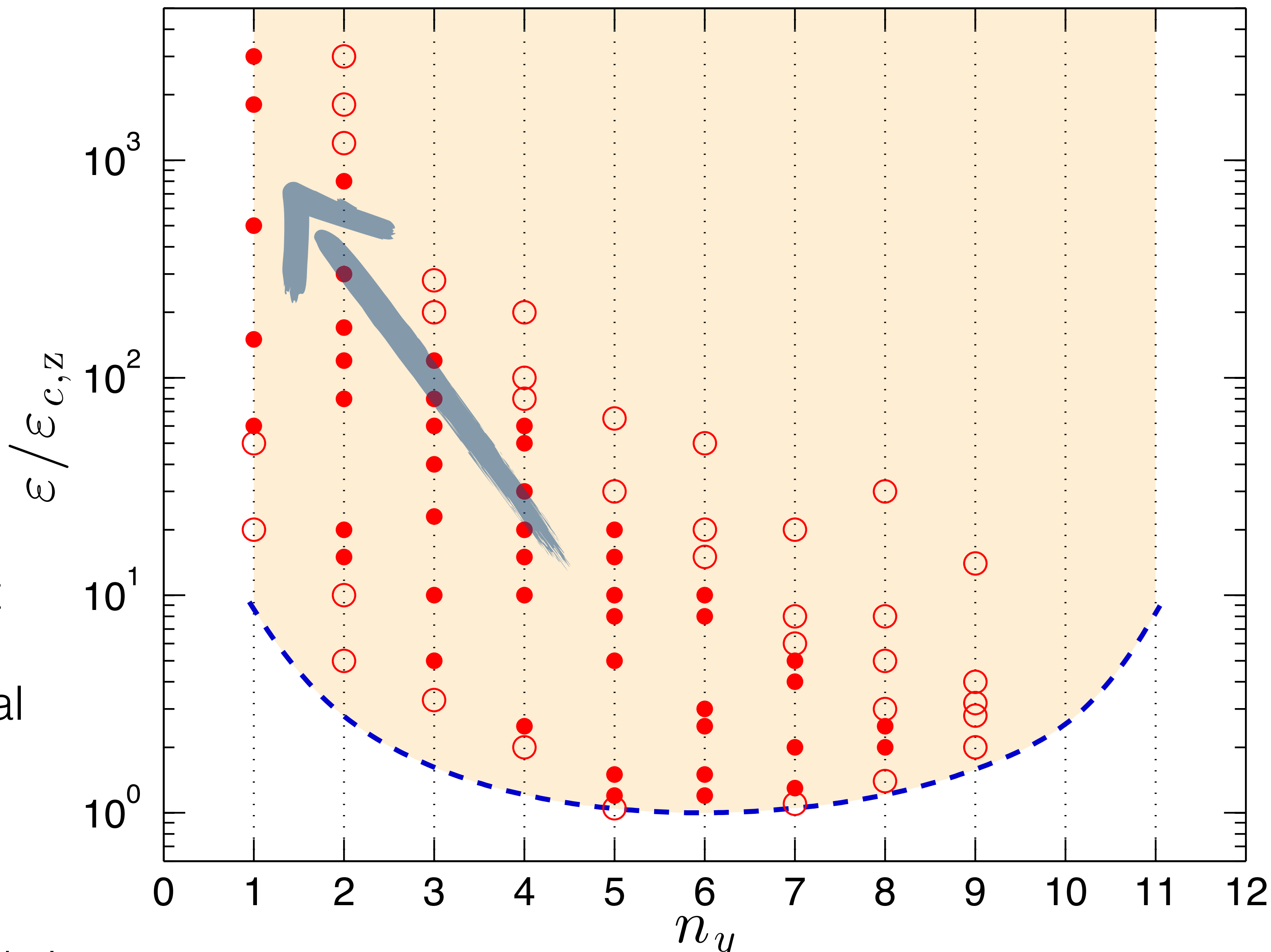
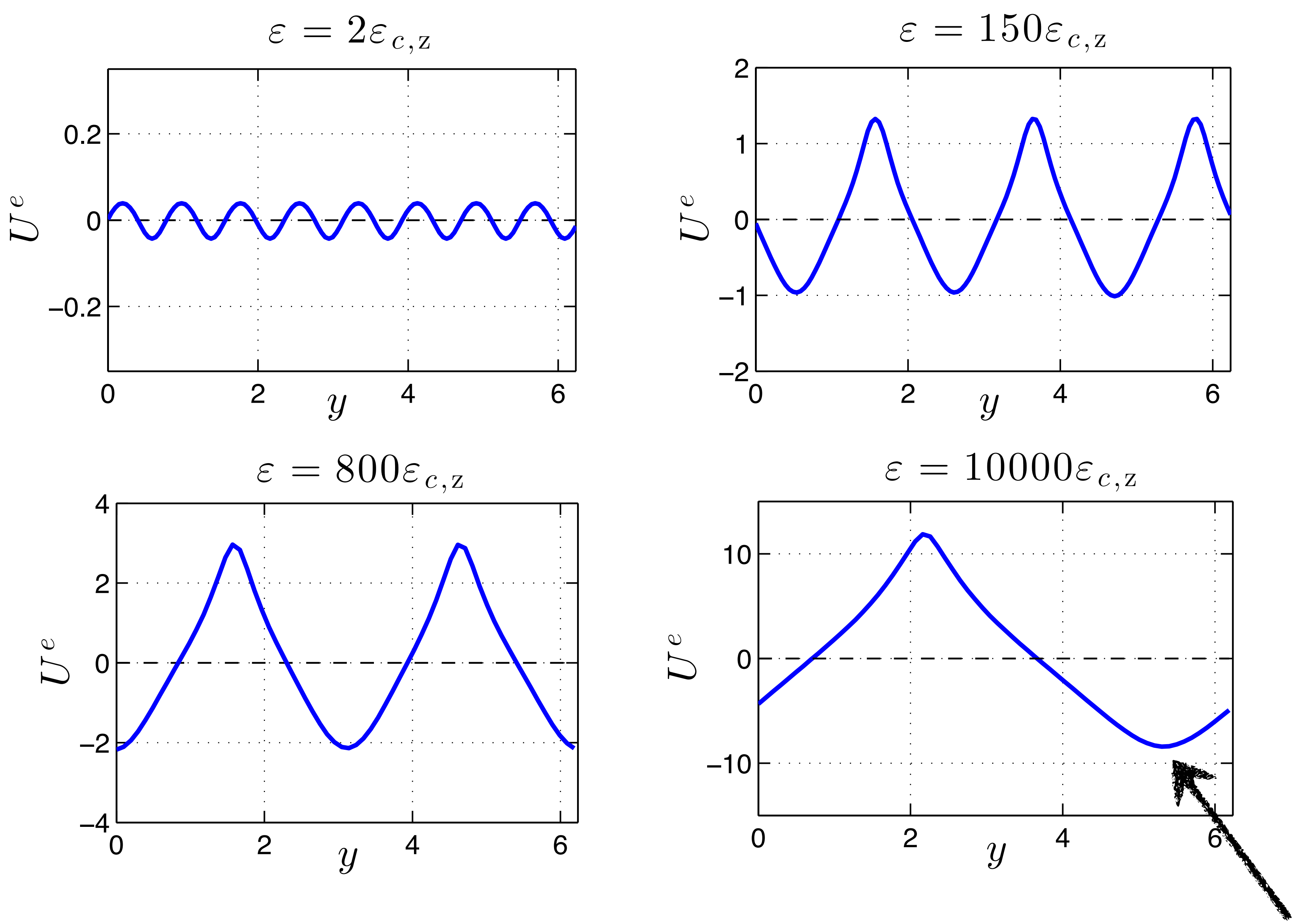


(= number of jets for a $2\pi \times 2\pi$ channel)

For higher energy input rates equilibria become S3T unstable and move towards the left of the diagram

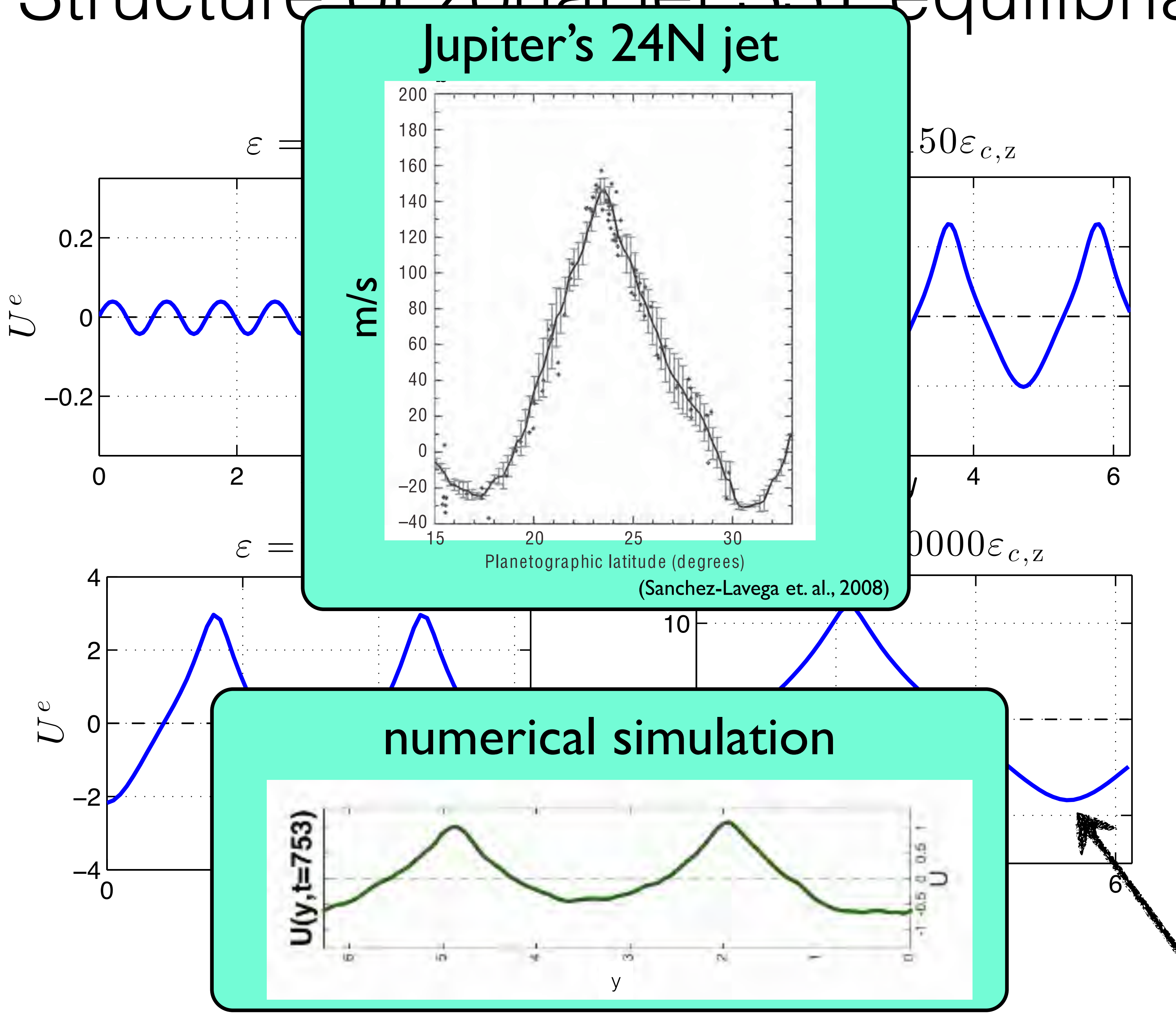
Structure of zonal jet S3T equilibria



$$\beta - \mathrm{d}^2U/\mathrm{d}y^2 \approx 0$$

Rayleigh-Kuo hydrodynamic stability criterion

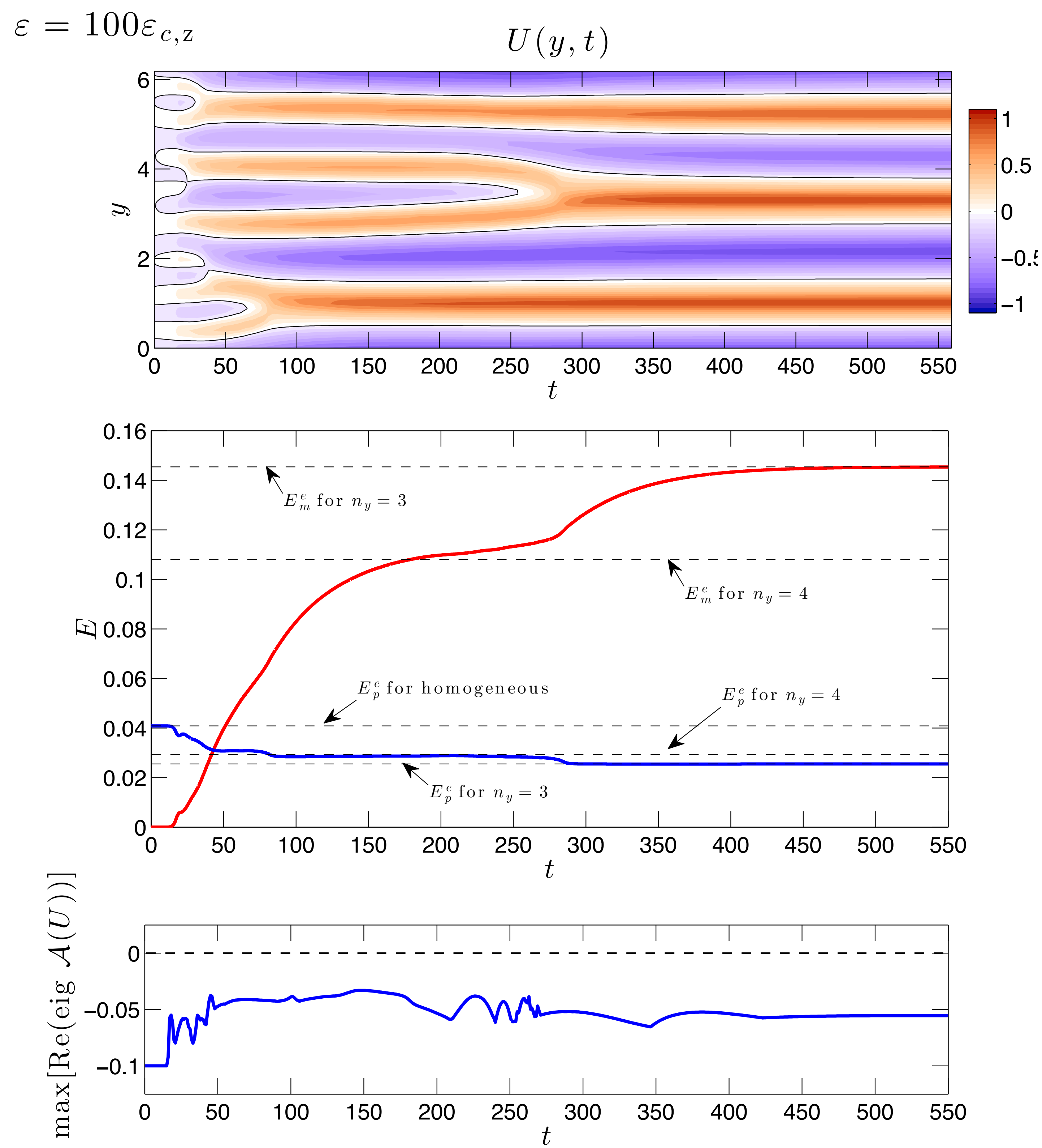
Structure of zonal jet S3T equilibria



$\beta - \mathrm{d}^2U/\mathrm{d}y^2 \approx 0$

Rayleigh-Kuo hydrodynamic stability criterion

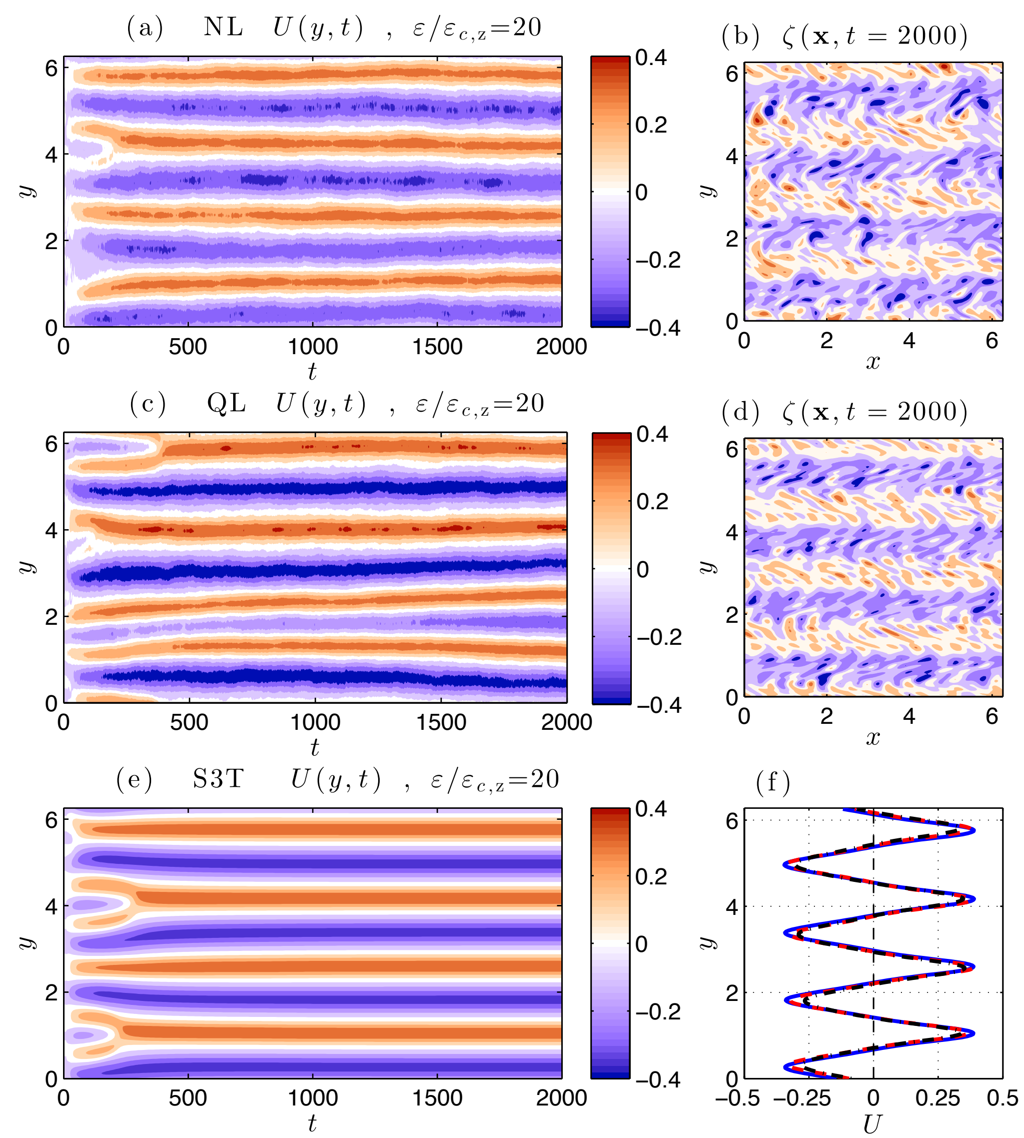
Jet mergers do *not* occur due to hydrodynamic instabilities



at this supercriticality the n_y=4 jet equilibrium is S3T unstable

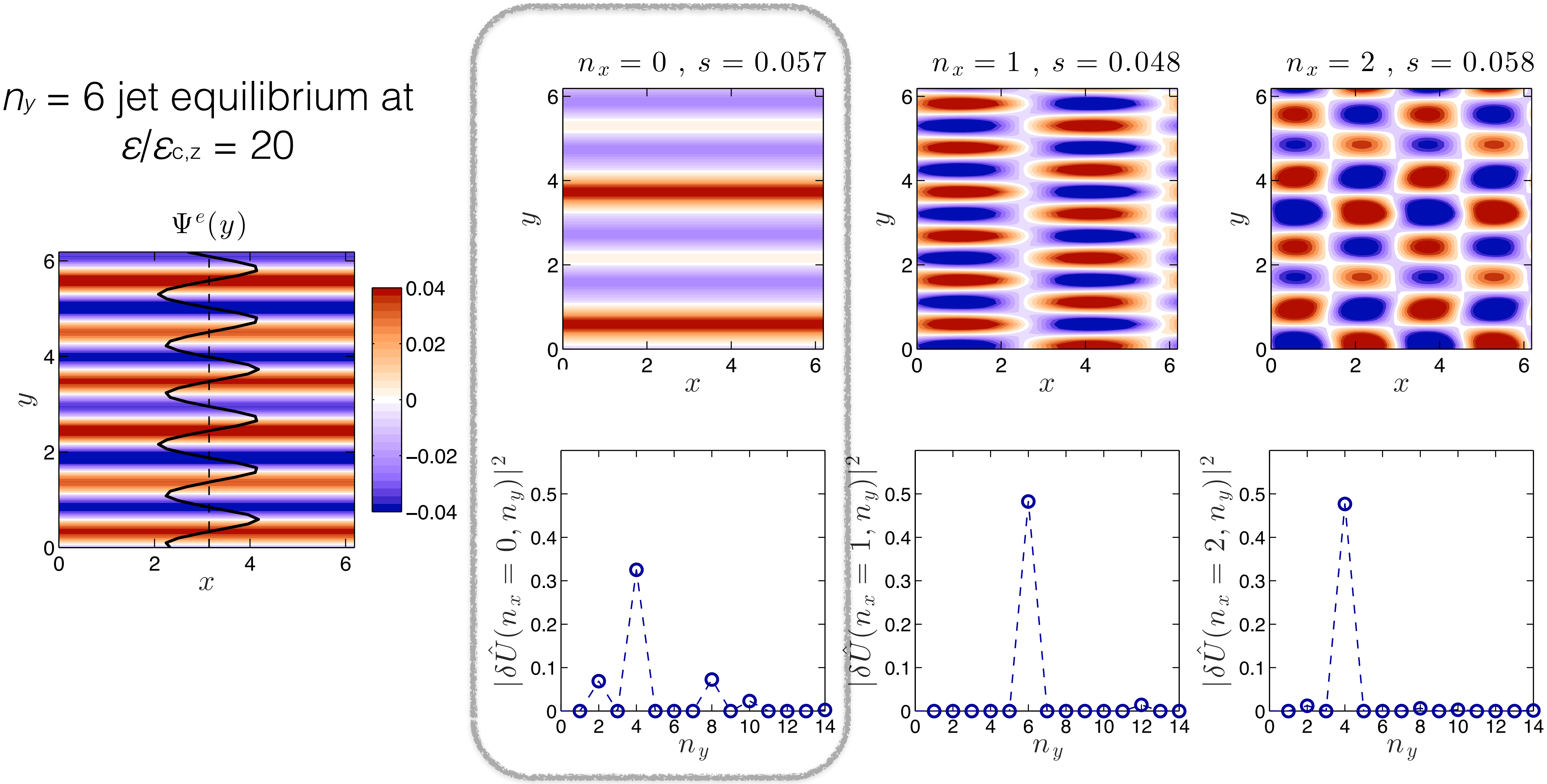
S3T stability analysis of the jet equilibria predicts the jet merger as the most unstable eigenfunction

Remember the case with $\varepsilon/\varepsilon_{c,z}$ = 20



S3T stability analysis of the jet equilibria predicts the jet merger as the most unstable eigenfunction

n_y = 6 jet equilibrium at $\varepsilon/\varepsilon_{c,z}$ = 20



structure of talk

- introduction to the physical problem
- formulation of the theory (S3T)
- study of the stability of homogeneous turbulent state
- comparison of S3T predictions with direct numerical simulations and verification of the theory
- stability of inhomogeneous turbulent states & relation with jet mergers
- relation of modulational instability of Rossby waves with S3T instability of homogeneous state (if time allows)
- summary

S3T generalizes the modulational instability of Rossby waves

MI is the hydrodynamic stability of finite amplitude Rossby waves (Lorenz 1972, Gill 1974, Connaughton et al. 2010)

$$ {}_{\mathbf{p}} = A\cos\left(\mathbf{p}\cdot\mathbf{x} - \omega_{\mathbf{p}} t\right) $$

We demonstrated that the problem of the hydrodynamic stability of ***any*** coherent nonlinear solution is mathematically equivalent to the S3T stability of the homogeneous equilibrium with the same eddy covariance spectrum.

$$ \hat{C}^e(\mathbf{k}) = (2\pi)^2 p^4 |A|^2 [\delta(\mathbf{k}-\mathbf{p}) + \delta(\mathbf{k}+\mathbf{p})] $$

However, the two problems are very different:

MI studies the stability of infinitely coherent solutions

S3T studies the statistical stability of forced—dissipative flows with a given turbulence spectrum

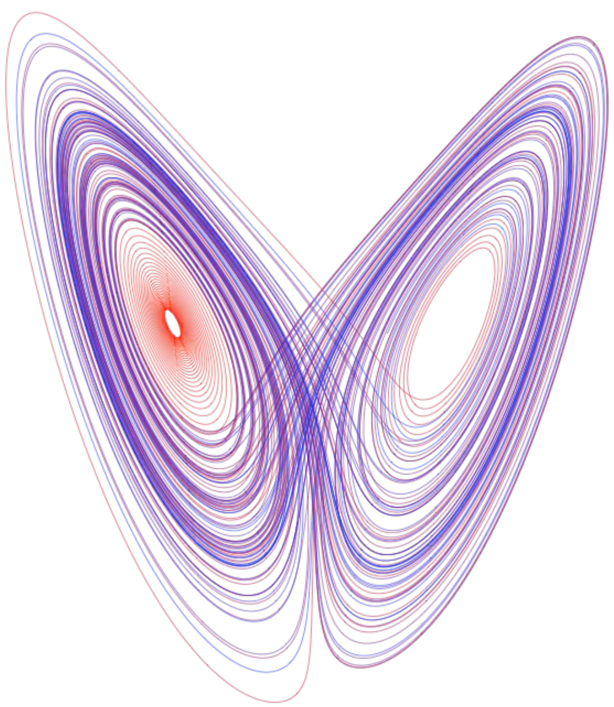
Conclusions

- S3T makes exact analytical predictions for the emergence of large-scale structure in planetary turbulence
- S3T predicts that the transition from a homogeneous to an inhomogeneous turbulent state occurs through a bifurcation of the statistical state dynamics
- S3T predicts the finite amplitude of the emergent large-scale structure
- The stability of inhomogeneous statistical turbulent equilibria (i.e. as in the Earth or Jupiter) can be studied within S3T framework and thus the sensitivity of the climate state of the planet can be determined

To understand turbulent flows one should adopt the perspective of the statistical state dynamics

Lorenz's vision

Ed Lorenz



"More than any other theoretical procedure, numerical integration is also subject to the criticism that it yields little insight into the problem. The computed numbers are not only processed like data but they look like data, and a study of them may be no more enlightening than a study of real meteorological observations. An alternative procedure which does not suffer this disadvantage consists of deriving *a new system of equations whose unknowns are the statistics themselves*."

The Nature and Theory of the General Circulation of the Atmosphere,
by E. N. Lorenz, **1967**

S3T is a first step towards this *new system of equations*

References

- Constantinou, Farrell & Ioannou (2014) Emergence and equilibration of jets in beta-plane turbulence: applications of Stochastic Structural Stability Theory. *J. Atmos. Sci.*, **71 (5)**, 1818-1842.

- Bakas, Constantinou and Ioannou (2015) S3T stability of the homogeneous state of barotropic beta-plane turbulence, *J. Atmos. Sci.*, **71 (5)**, 1689-1712.

- Constantinou (2015) Formation of large-scale structures by turbulence in rotating planets, Ph.D. thesis, University of Athens


ευχαριστώ
thanks