A barotropic process-model for eddy saturation

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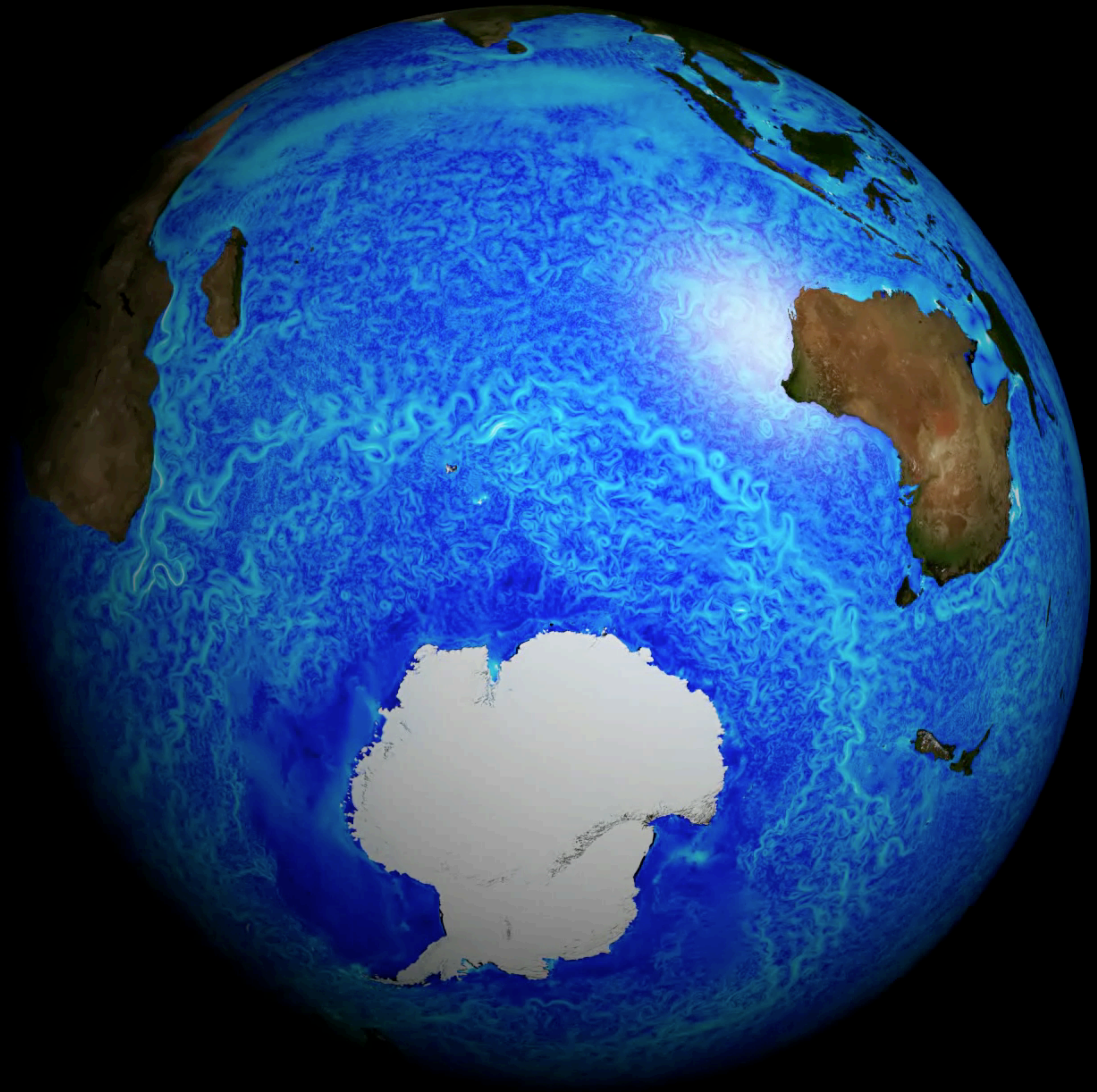


the Antarctic Circumpolar Current (**ACC**)

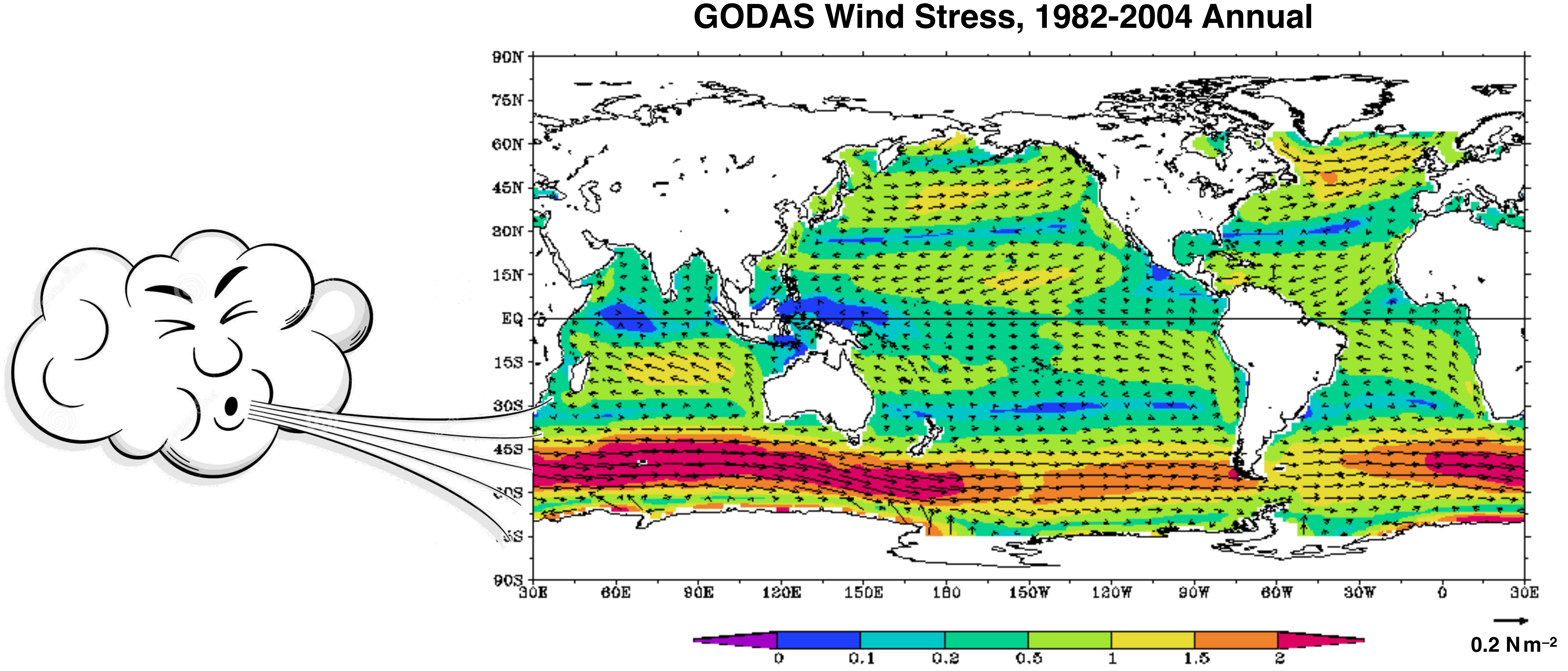
LLC4320 sea surface speed animation
by C. Henze and D. Menemenlis (NASA/JPL)
1/48th degree, 90 vertical levels
MITgcm spun up from ECCO v4 state estimate

What sets the strength of the current?

What are the interactions among
ACC – mesoscale eddies – bathymetric features?

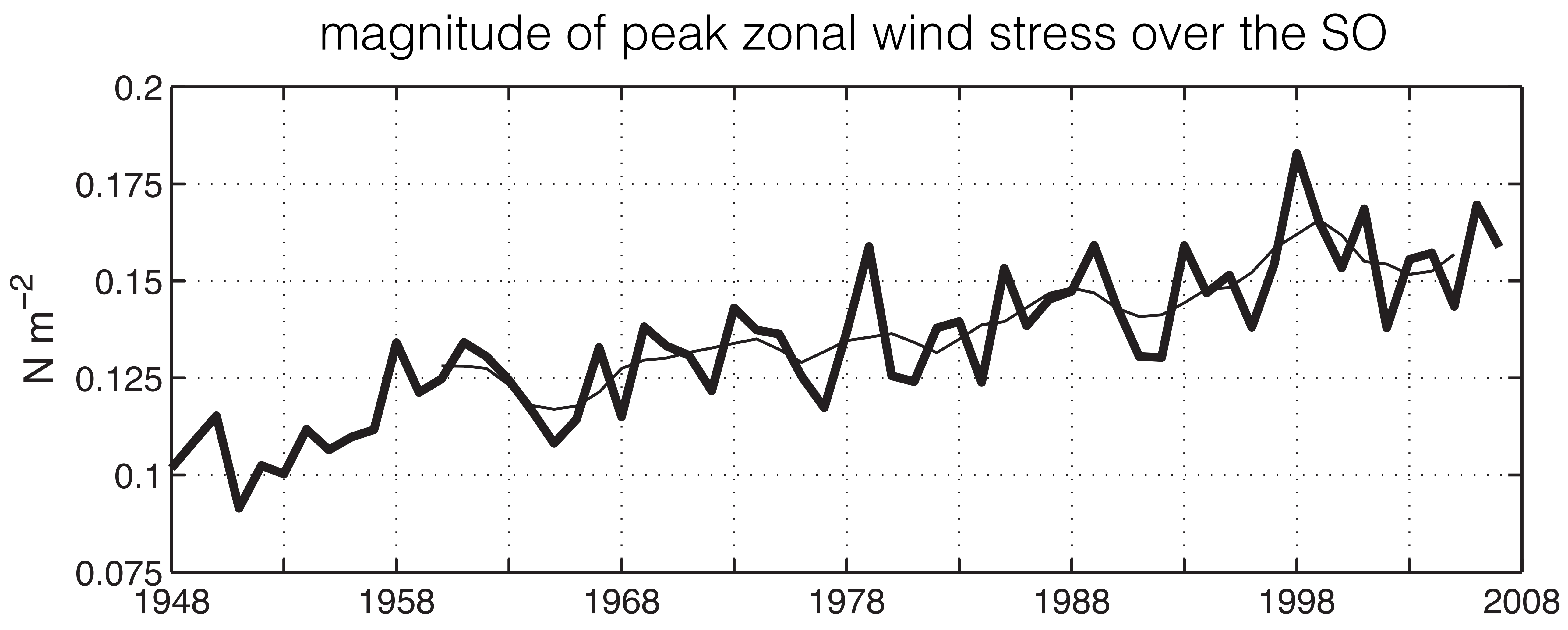
winds drive the Antarctic Circumpolar Current

Climate Prediction Center



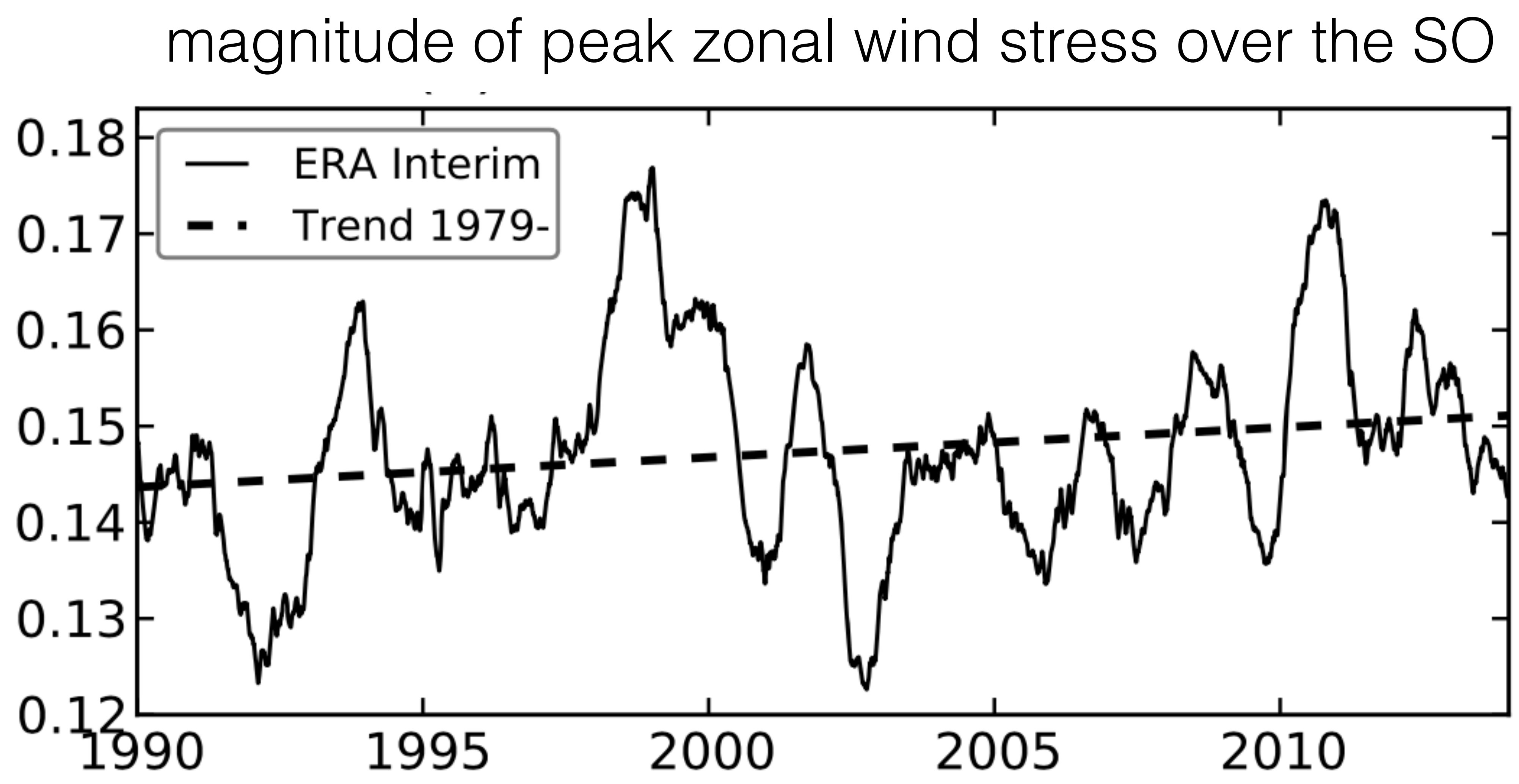
strong westerly winds blow over the Southern Ocean
transferring momentum through wind stress at the surface

winds over Southern Ocean are getting stronger



Farneti et al. 2015

results from
inter-annual
CORE-II simulations



Hogg et al. 2015

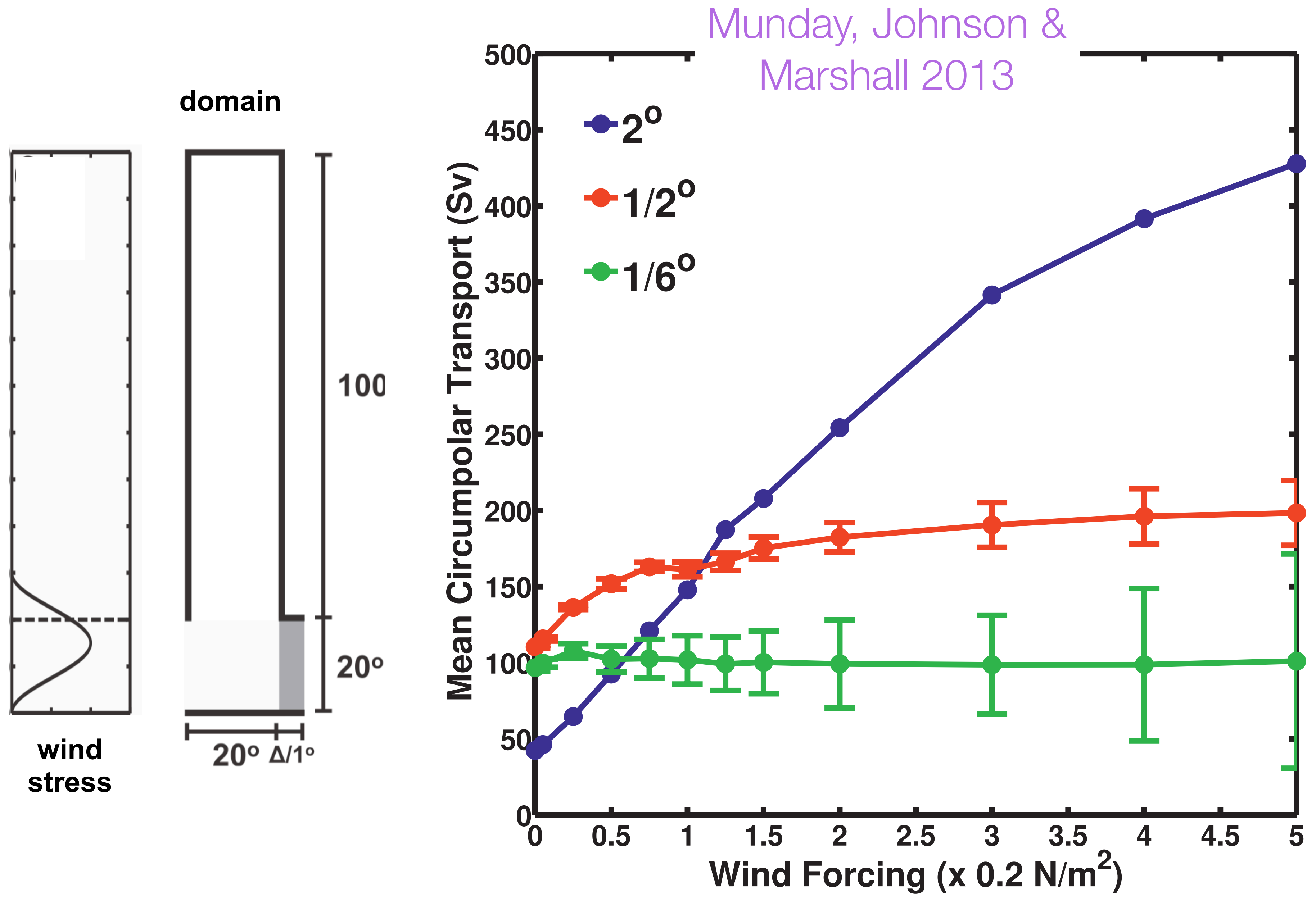
how will the ACC respond?

does doubling the winds implies double the ACC transport?

not always — “eddy saturation”

what is “eddy saturation”?

The *insensitivity* of the total ACC volume transport
to wind stress increase.



Eddy saturation is seen in eddy-resolving ocean models.

Higher resolution ⟶ eddy saturation “occurs”

Eddy saturation was theoretically predicted by Straub (1993);
the explanation was based *entirely* on **baroclinicity**.
(based on vertical momentum transfer interfacial eddy form stress)

[There are many other examples: Hallberg & Gnanadesikan 2001, Tansley & Marshall 2001, Hallberg & Gnanadesikan 2006, Hogg et al. 2008, Nadeau & Straub 2009, Farneti et al. 2010, Nadeau & Straub 2012, Meredith et al. 2012, Morisson & Hogg 2013, Abernathey & Cessi 2014, Farneti et al. 2015, Nadeau & Ferrari 2015, Marshall et al. 2017.]

momentum comes in at the surface through wind stress

how is this momentum balanced?


W.H. Munk
1917 - last week

Note on the Dynamics of the Antarctic Circumpolar Current

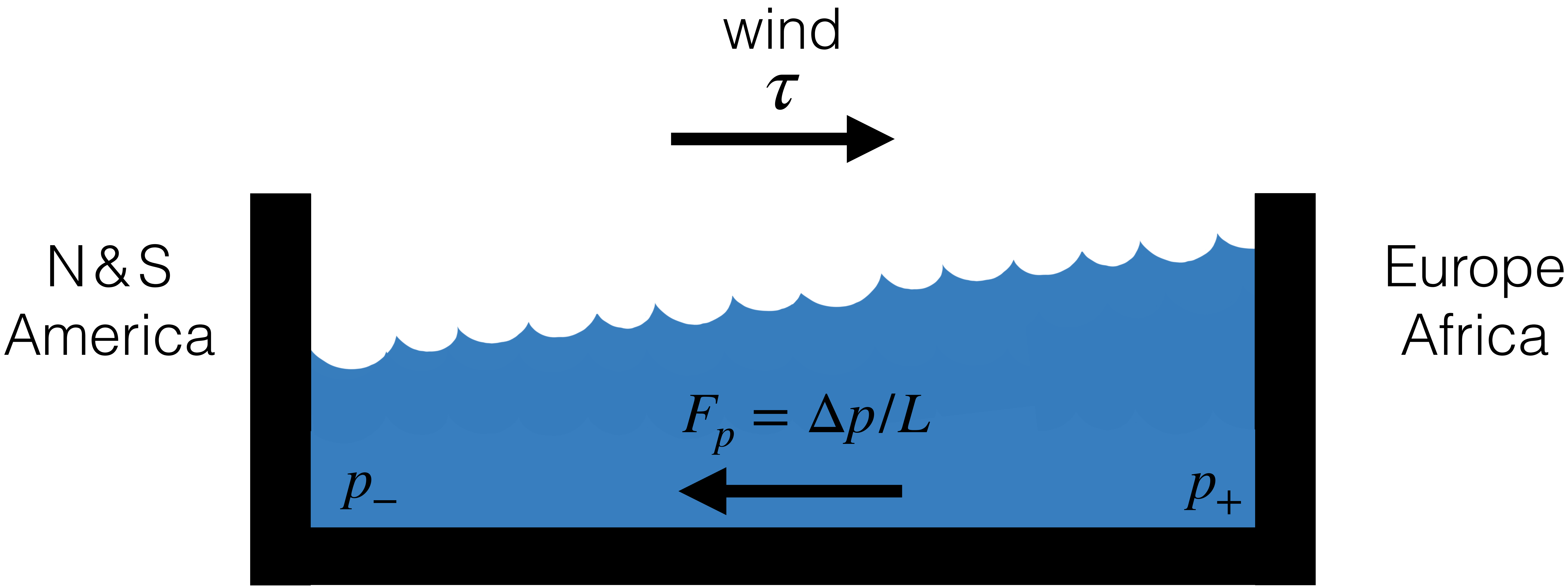
By W. H. MUNK and E. PALMÉN

1951

Abstract

Unlike all other major ocean currents, the Antarctic Circumpolar Current probably does not have sufficient frictional stress applied at its lateral boundaries to balance the wind stress. The balancing stress is probably applied at the bottom, largely where the major submarine ridges lie in the path of the current. The meridional circulation provides a mechanism for extending the current to a large enough depth to make this possible.

ocean with continental boundaries (e.g., Atlantic)



the surface of the ocean tilts and creates
an east-west pressure gradients that
mostly balances the momentum input

(the ocean 'leans' onto the eastern boundary)

ocean without continental boundaries
(e.g. Southern Ocean)

the flow over ocean ridges creates pressure differences
that counterbalance the momentum input

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start with the zonal
angular momentum
equation

$f(y)$ is the Coriolis parameter
$f = 2\Omega \sin\vartheta$

$$(\partial_t + u\partial_x + v\partial_y + w\partial_z)\underbrace{\left(u - \int^y f(y')\,\mathrm{d}y'\right)}_{\stackrel{\text{def}}{=} a} + p_x = \tau_z$$

angular momentum

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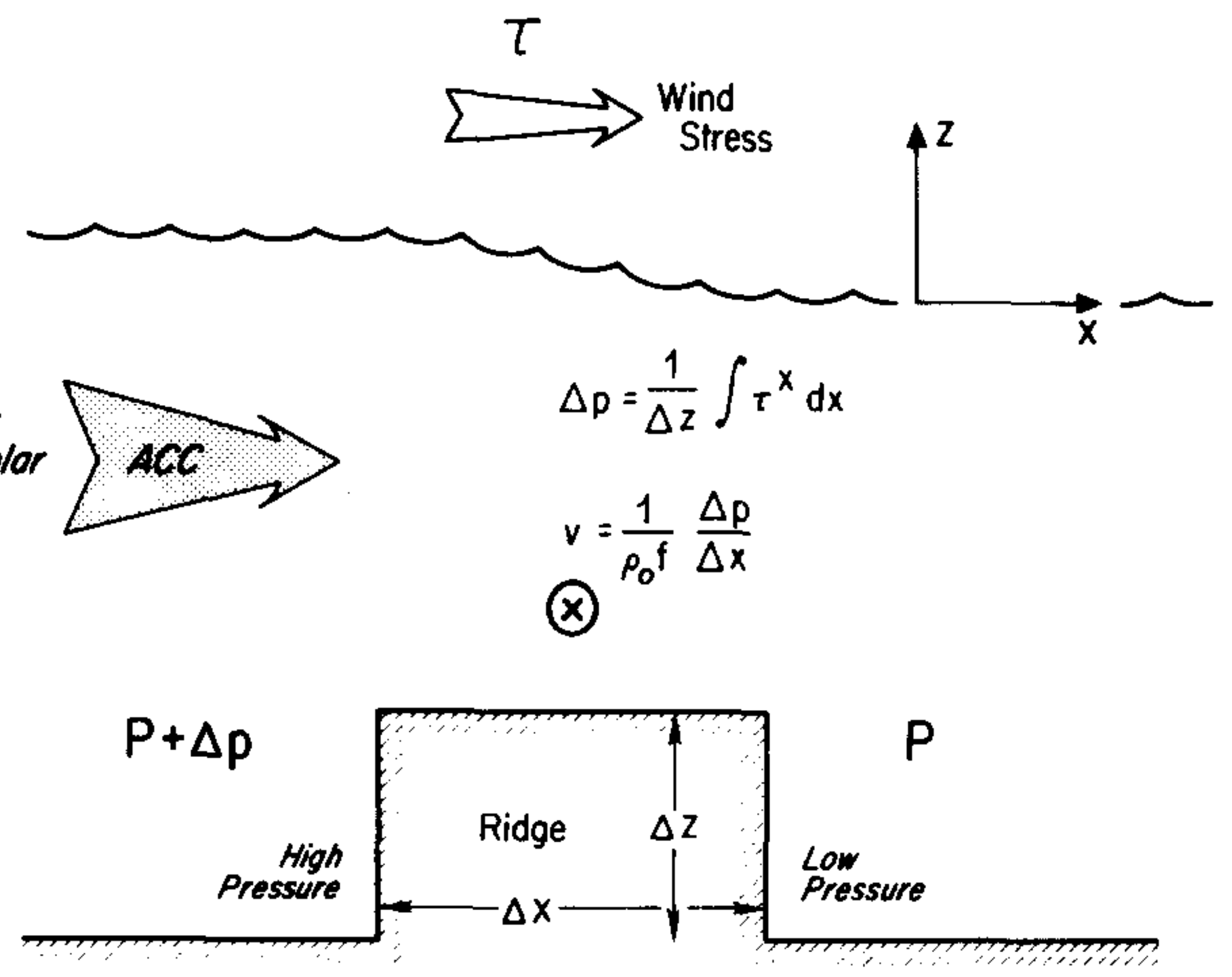
$$(\partial_t + u\partial_x + v\partial_y + w\partial_z)\underbrace{\left(u - \int^y f(y')\,\mathrm{d}y'\right)}_{\substack{\stackrel{\text{def}}{=} a \\ \text{angular momentum}}} + p_x = \tau_z$$

vertically integrate,
top $z = 0$ to bottom $z = -h(x,y)$

$$\partial_t \int_{-h}^{0} a\,\mathrm{d}z + \partial_x\left[\int_{-h}^{0} ua + p\,\mathrm{d}z\right] + \partial_y \int_{-h}^{0} va\,\mathrm{d}z = $$
$$= \underbrace{\tau(0)}_{\text{wind stress}} - \underbrace{\tau(-h)}_{\text{bottom drag}} + \underbrace{h_x p(-h)}_{\text{form stress}}$$

we've used integration by parts: $\int_{-h}^{0} p_x\,\mathrm{d}z = \partial_x \int_{-h}^{0} p\,\mathrm{d}z - h_x p(-h)$

topographic form stress



Schematic presentation of bottom form drag or mountain drag. Wind stress imparted eastward momentum in the water column is removed by the pressure difference across the ridge.

Johnson & Bryden 1989

$$\partial_t \int_{-h}^{0} a\,\mathrm{d}z + \partial_x \left[\int_{-h}^{0} ua + p\,\mathrm{d}z\right] + \partial_y \int_{-h}^{0} va\,\mathrm{d}z = $$
$$= \underbrace{\tau(0)}_{\text{wind stress}} - \underbrace{\tau(-h)}_{\text{bottom drag}} + \underbrace{h_x p(-h)}_{\text{form stress}}$$

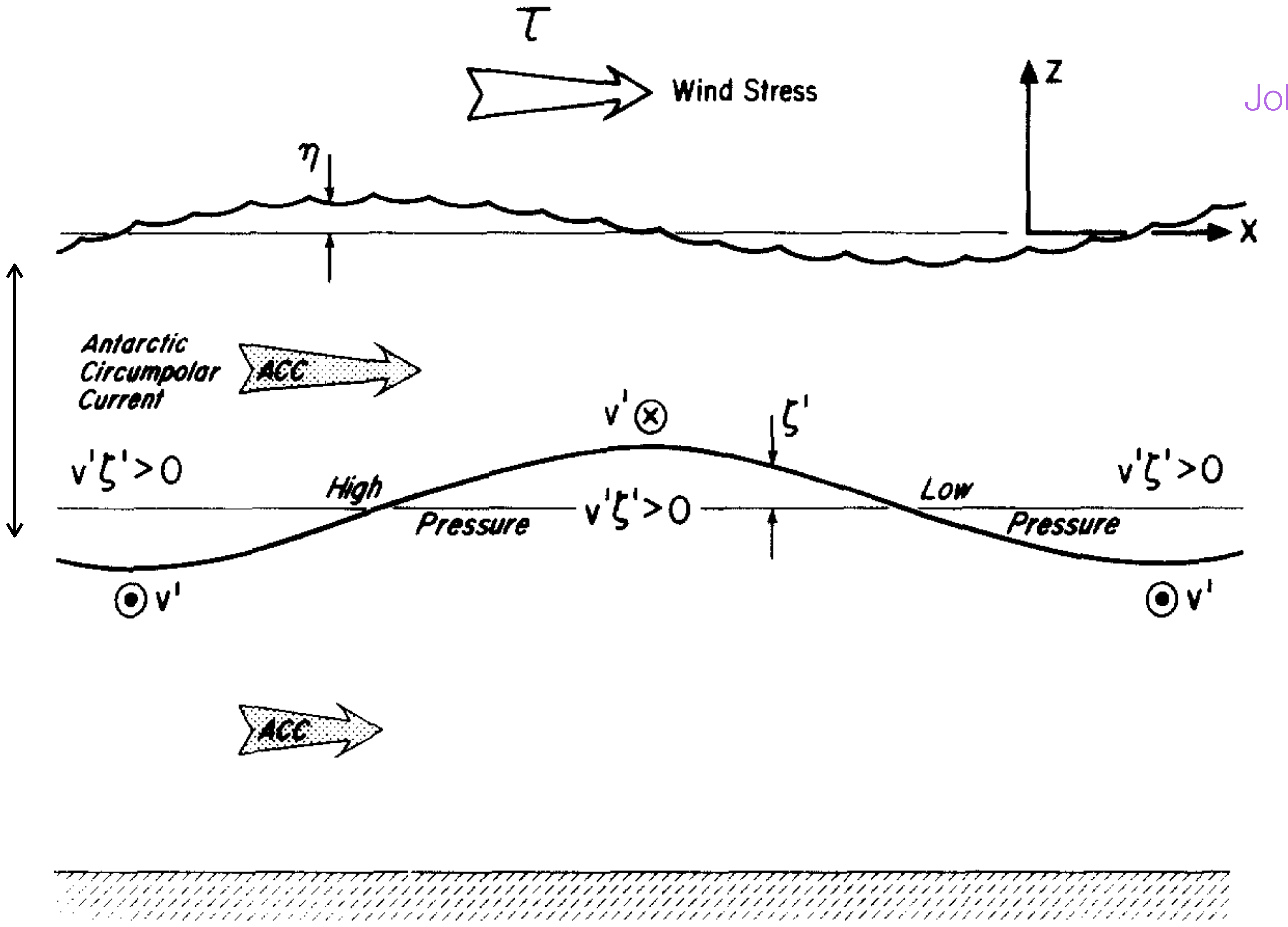
Topographic form stress is a purely **barotropic** process.

interfacial form stress

Johnson & Bryden 1989

vertically integrate from the sea-surface down to a moving buoyancy surface

(i.e., integrate within a layer of constant density)



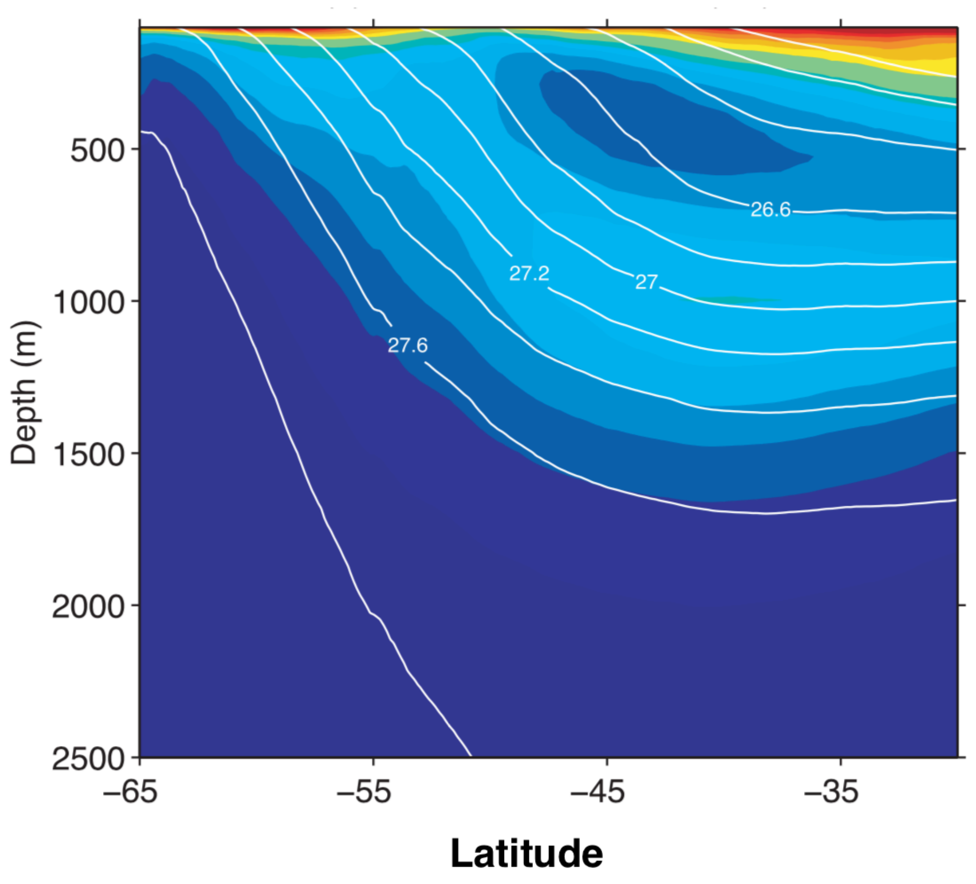
Schematic presentation of interfacial form drag. Correlations of perturbations in the interface height, ζ′, and the meridional velocity, V′ (⊙ indicating poleward flow and ⊗ indicating equatorward flow), which are related to pressure perturbations by geostrophy, allow the upper layer to exert an eastward force on the lower layer and the lower layer to exert a westward force on the upper layer; thus effecting a downward flux of zonal momentum.

Interfacial form stress requires **baroclinicity**.

the most popular scenario for the momentum balance

- momentum in imparted at the surface by wind,
- isopycnals slope $\longrightarrow$ baroclinic instability,
- momentum is transferred downwards by interfacial eddy form stress
- momentum reaches the bottom where is transferred to the solid Earth by topographic form stress.

Johnson & Bryden 1989



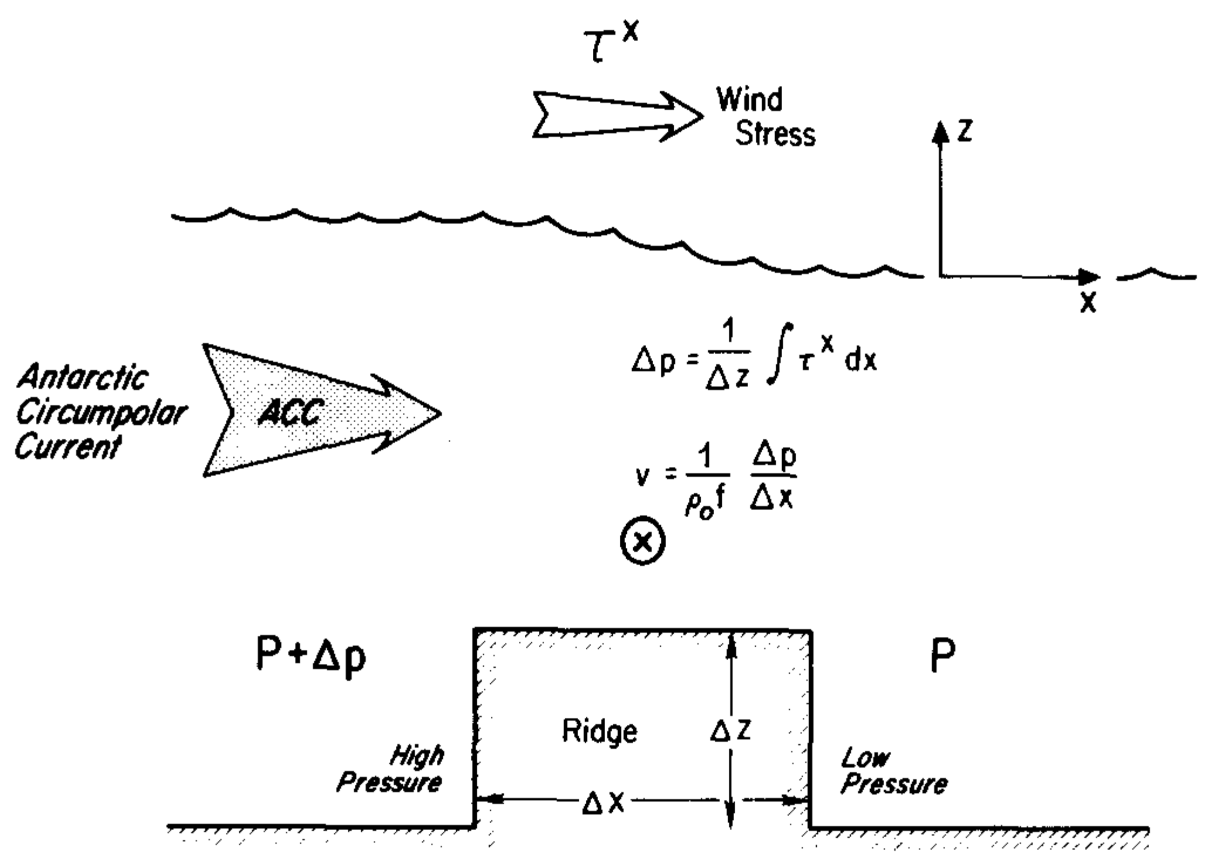
Meredith et al. 2012

$$\text{isopycnal slope} = \left[-\frac{\tau}{f\,\kappa} \right]^{1/2}$$

Marshall & Radko 2003

This baroclinic scenario sets up the ACC transport
(e.g., the transport through Drake Passage).

but what about barotropic dynamics?

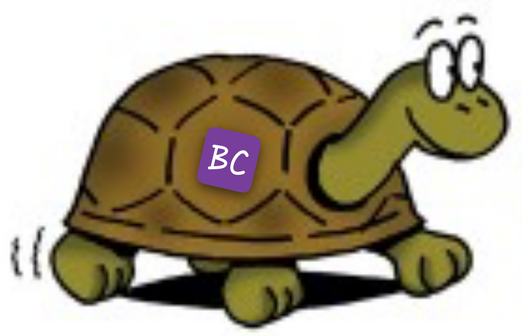


The sea surface pressure gradient can be *directly* communicated to the bottom.

And it will be, unless compensated by internal isopycnal gradients.

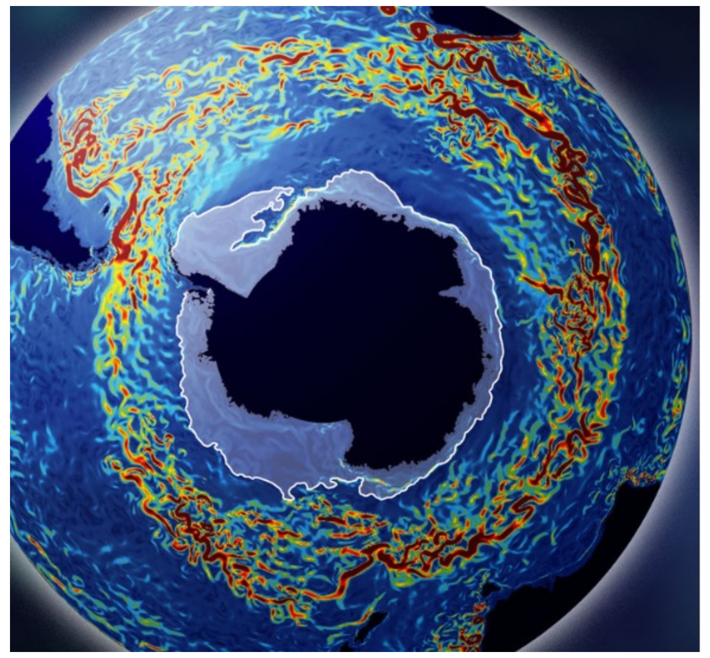
Isn't barotropic "communication" much "easier"?

wind stress is *rapidly* communicated to the bottom through barotropic processes

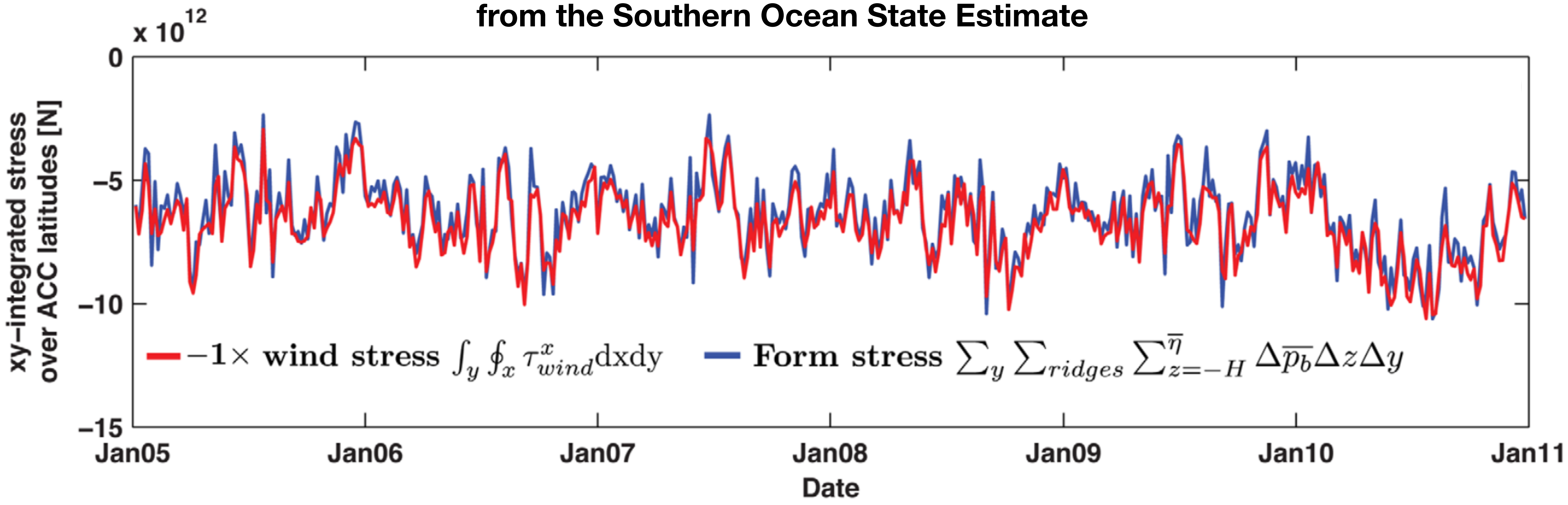


Barotropic processes are fast (~days).

Baroclinic processes are much slower (~years).



SOSE, UCSD



Masich, Chereskin, and Mazloff 2015

~90% of variance in the topographic form stress signal is explained by the **0-day** time lag.

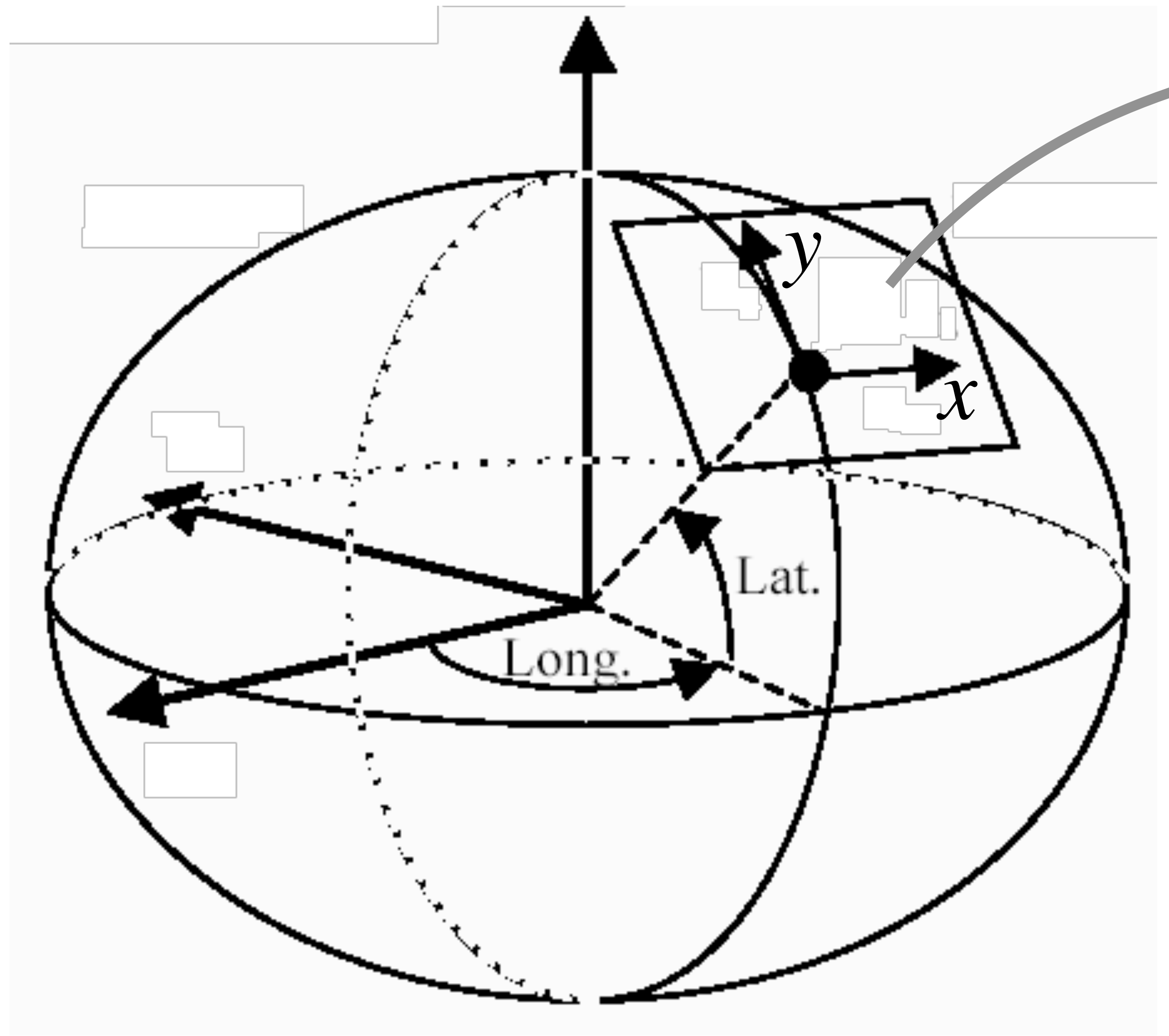
Similar statements also made by:

Straub 1993, Ward & Hogg 2011, Rintoul et al. 2014, Peña Molino et al. 2014, Donohue et al. 2016.

the plan

Revisit an old barotropic quasigeostrophic (QG) model on a beta-plane.

(Hart 1979, Davey 1980, Bretherton & Haidvogel 1976, Holloway 1987, Carnevale & Fredericksen 1987)



β plane

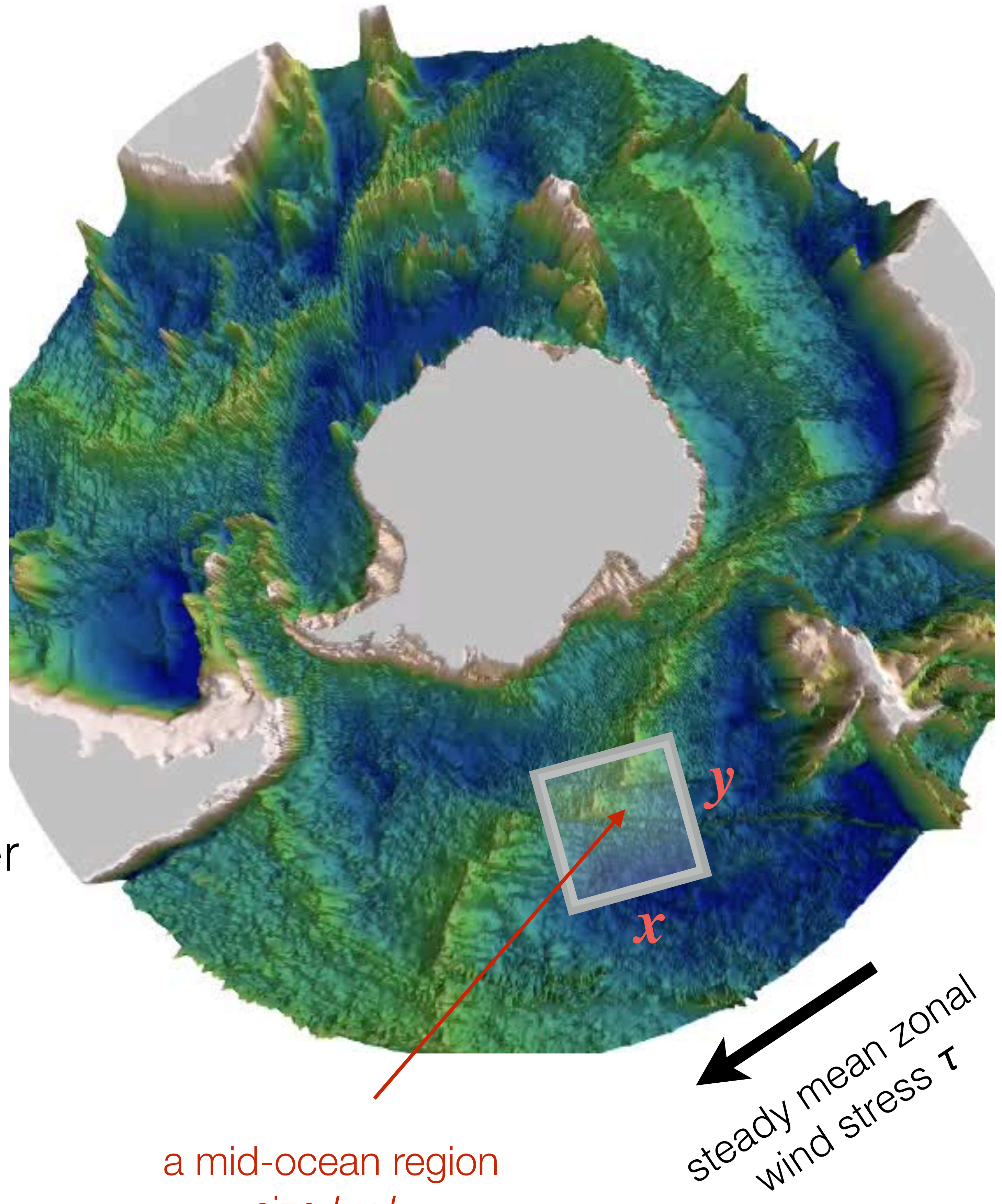
a local tangent plane on the surface of planet

variation of Coriolis parameter with latitude is included

$$f = f_0 + \beta y$$

x : longitude

y : latitude



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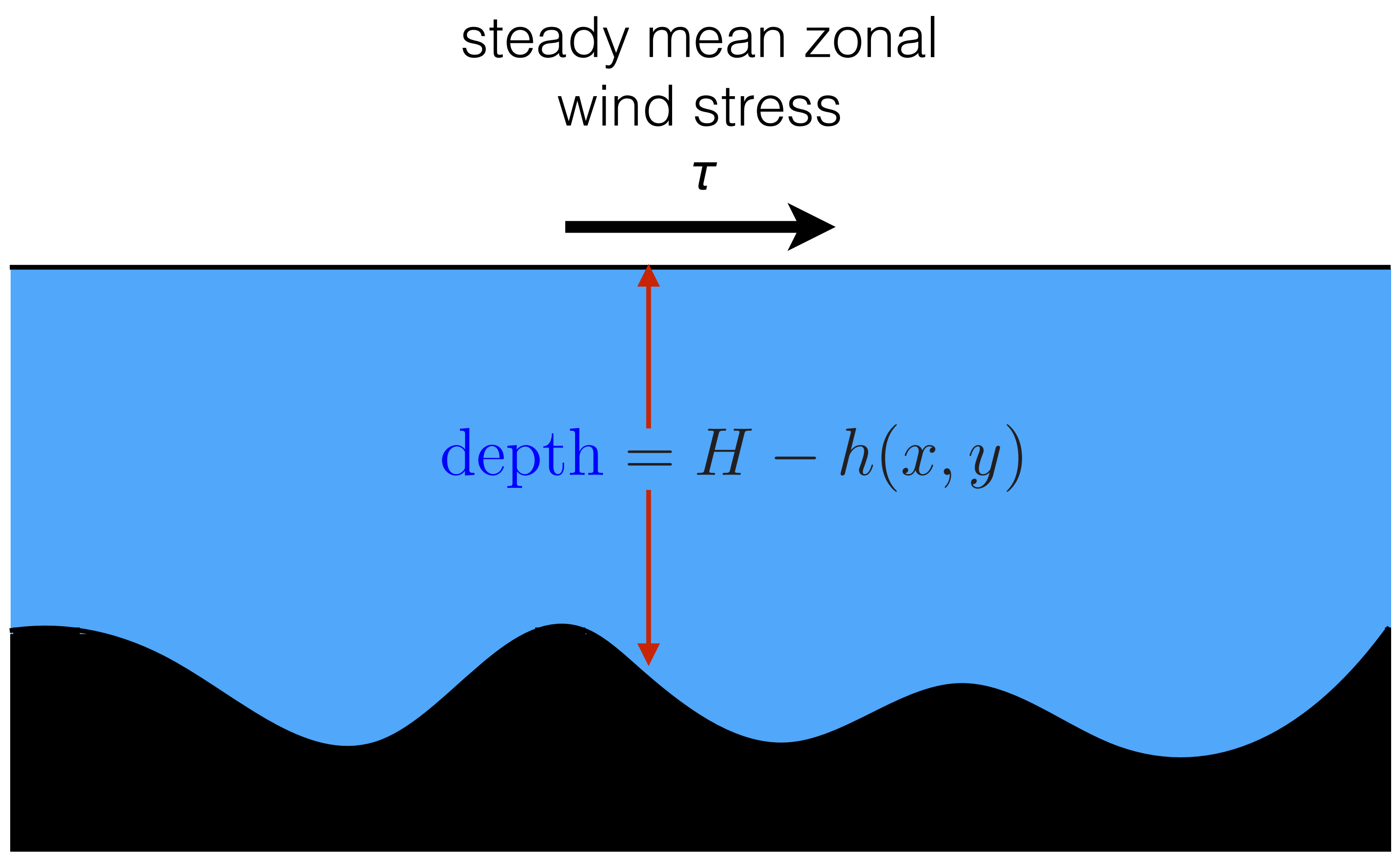
A distinctive feature of this model is a "large-scale barotropic flow" $U(t)$.

this is the ACC

Study how momentum is balanced by topographic form stress and investigate the requirements for eddy saturation.



topographic potential vorticity (PV) $\quad \eta = \dfrac{f_0 h}{H}$

QGPV $\quad \nabla^2 \psi + \eta + \beta y$

total streamfunction $\quad -U(t) y + \psi(x, y, t)$

total flow $$\Big(\underbrace{U(t) - \partial_y \psi(x, y, t)}_{\text{zonal}} \, , \, \underbrace{\partial_x \psi(x, y, t)}_{\text{meridional}} \Big)$$

a barotropic QG model for a mid-ocean region

total streamfunction $\quad -U(t)y + \psi(x, y, t)$

QGPV $\quad \nabla^2\psi + \eta + \beta y$

Material conservation of QGPV

$$\nabla^2\psi_t + U(\nabla^2\psi + \eta)_x + \mathsf{J}(\psi, \nabla^2\psi + \eta) + \beta\psi_x = -\mu\,\nabla^2\psi + \text{hyper visc.}$$

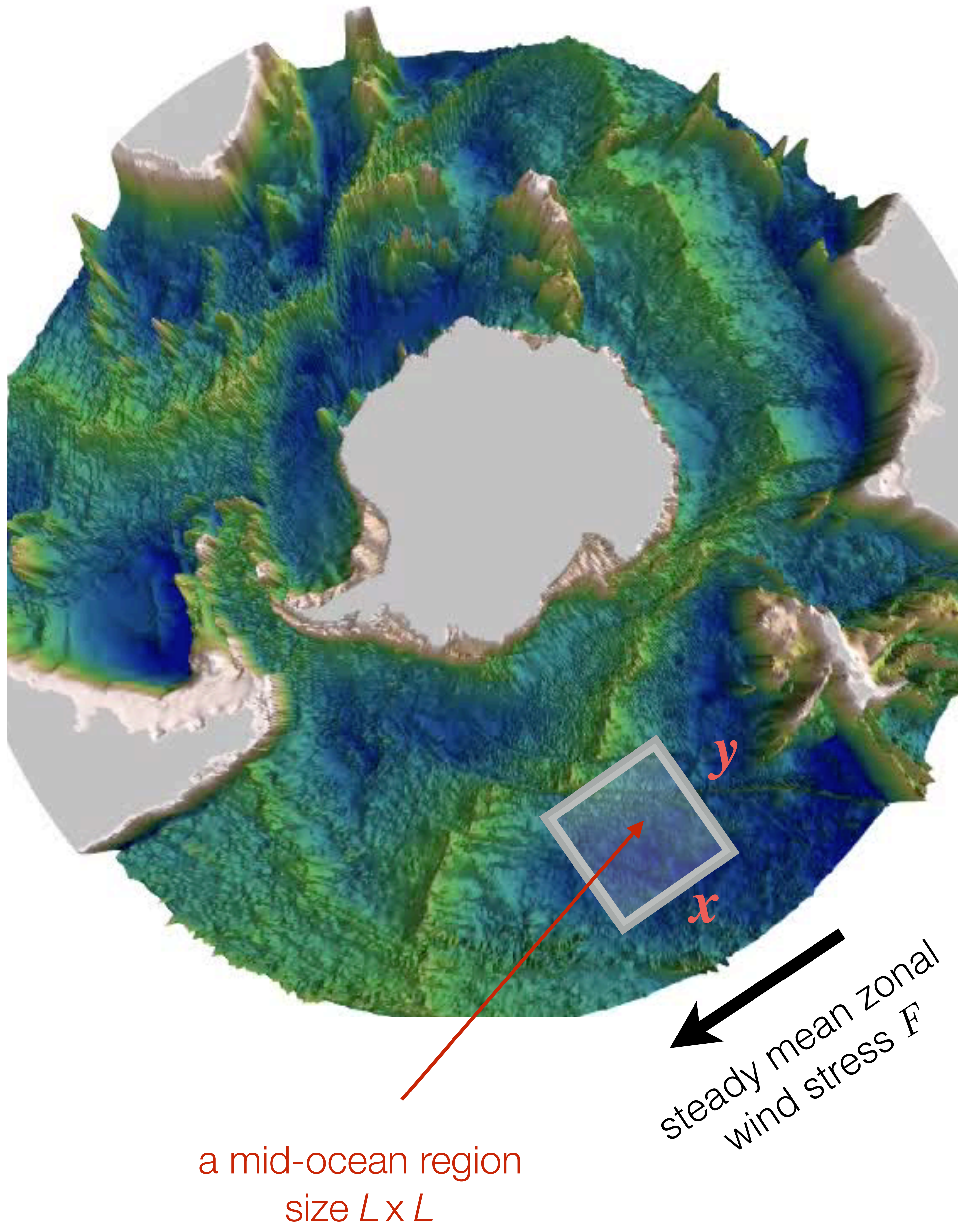
Large-scale zonal momentum

$$U_t = F - \mu U - \langle\psi\eta_x\rangle$$

topographic form stress

$\langle\ \rangle$ is domain average ; $\quad F = \dfrac{\tau}{\rho_0 H}$ wind stress forcing

periodic boundary conditions



(Hart 1979, Davey 1980, Bretherton & Haidvogel 1976, Holloway 1987, Carnevale & Fredericksen 1987, Constantinou & Young 2017, Constantinou 2018)

the large-scale flow equation: $U_t = F - \mu U - \langle \psi \eta_x \rangle$

zonal angular momentum density: $a(x,y,z,t) = u(x,y,z,t) - \int^y f(y')\,\mathrm{d}y'$

vertically integrated zonal angular momentum equation

$$\partial_t \int_{-h}^{0} a\,\mathrm{d}z + \cancel{\partial_x \left[\int_{-h}^{0} ua + p\,\mathrm{d}z\right] + \partial_y \int_{-h}^{0} va\,\mathrm{d}z} =$$

$$= \underbrace{\tau(0)}_{\text{wind stress}} - \underbrace{\tau(-h)}_{\text{bottom drag}} + \underbrace{h_x p(-h)}_{\text{form stress}}$$

horizontally integrate, drop the boundary fluxes, and divide by the volume

$$U_t = F - \mu U - \langle \psi \eta_x \rangle$$

$$U(t) \stackrel{\text{def}}{=} V^{-1} \iiint u(x,y,z,t)\,\mathrm{d}V$$

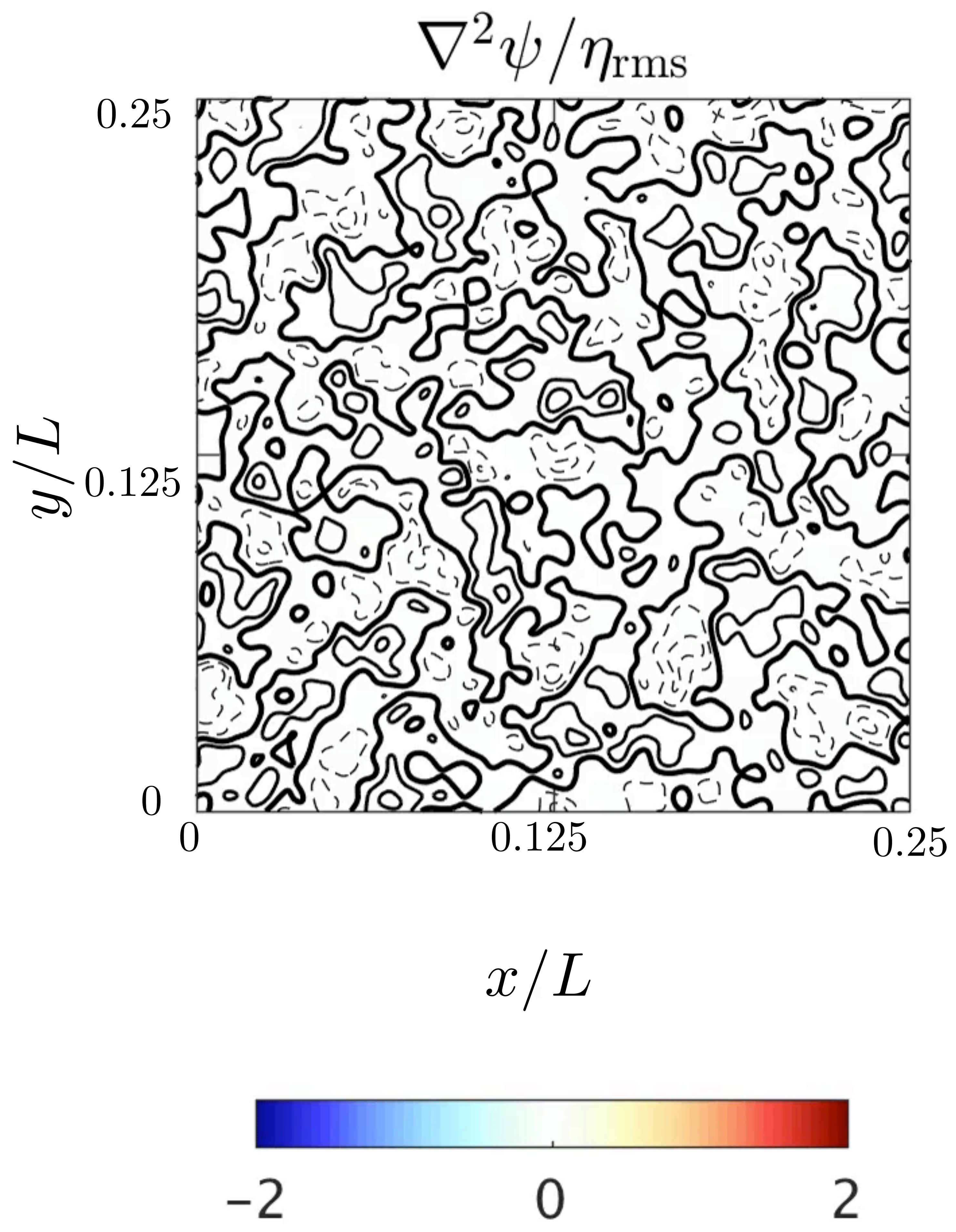
vertical & horizontal integral over a mid-ocean region (**not** a zonal average)

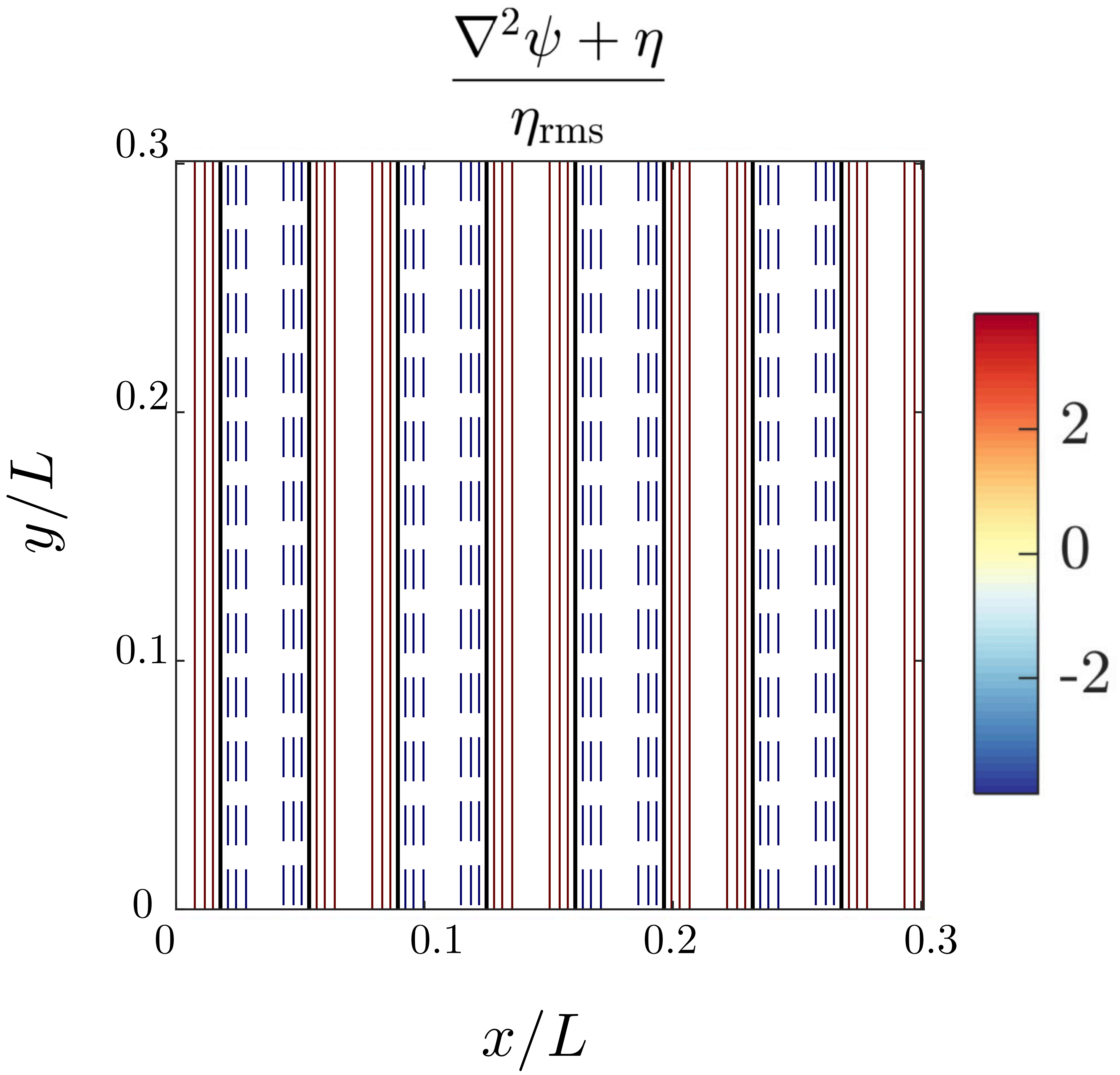
this barotropic QG model exhibits turbulence and eddies

random topography with k^{-2} spectrum

$\mu t = 0.00$

topography $\propto \cos(mx)$





let's put some "quasi-realistic" numbers

$$L = 4000\,\mathrm{km} \quad H = 4\,\mathrm{km} \quad \rho_0 = 1035 \text{ kg m}^{-3}$$

$$\text{lat} = 60\,\mathrm{S} \Rightarrow f_0 = -1.26 \times 10^{-4} \text{ s}^{-1} \text{ , } \beta = 1.14 \times 10^{-11} \text{ m}^{-1}\text{s}^{-1}$$

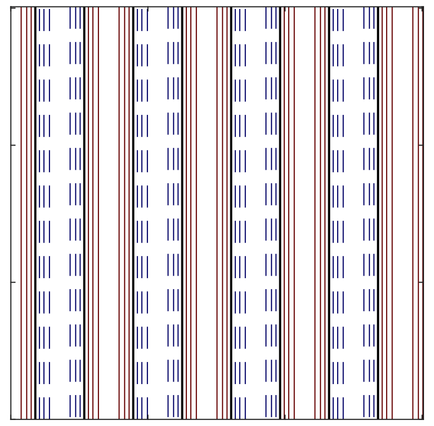
$$h_{\mathrm{rms}} = 200 \text{ m} \Rightarrow \eta_{\mathrm{rms}} = 6.3 \times 10^{-6} \text{ s}^{-1}$$

$$\mu = 6.3 \times 10^{-8} \text{ s}^{-1} \approx (180 \text{ days})^{-1}$$

a topographic length-scale: $\ell_\eta = \sqrt{\dfrac{\eta_{\mathrm{rms}}}{|\boldsymbol{\nabla}\eta|_{\mathrm{rms}}}} = 0.01L$

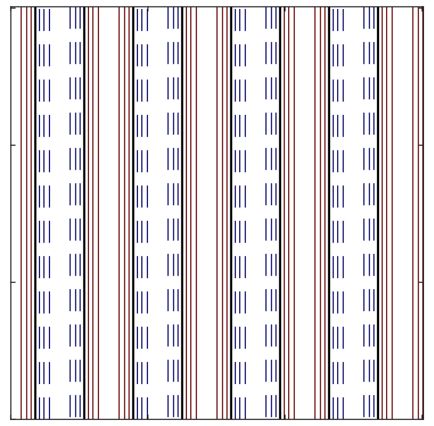
(we use monoscale topography)

for these values a typical wind stress forcing is: $\tau = 0.2\,\mathrm{N\,m}^{-2} \Leftrightarrow \dfrac{F}{\ell_\eta \eta_{\mathrm{rms}}^2} \approx 0.02$



three flow regimes

	weak wind stress	intermediate (“realistic”) wind stress	strong wind stress
$\dfrac{F}{\ell_\eta \eta_{\mathrm{rms}}^2} \approx$	0.001	0.02	0.05
	steady flow	time-dependent flow	strong *U* (flow does not see topography)



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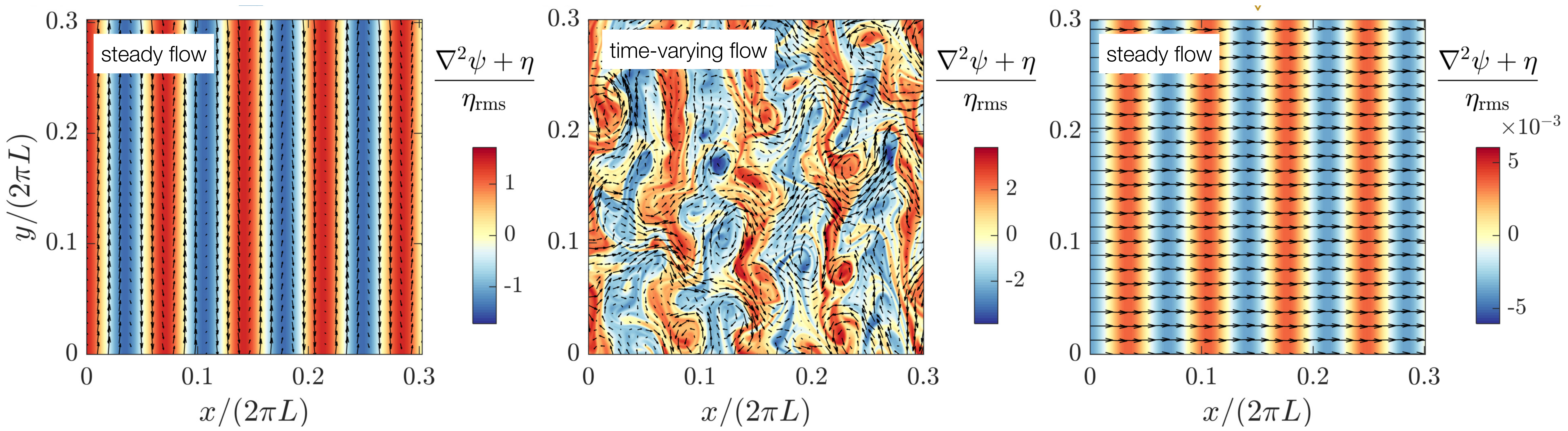


Constantinou 2018

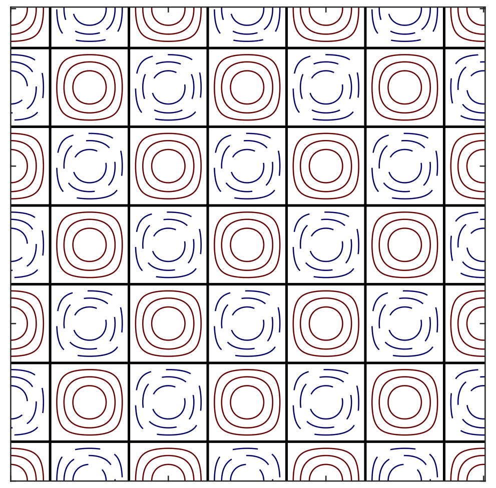
Question:

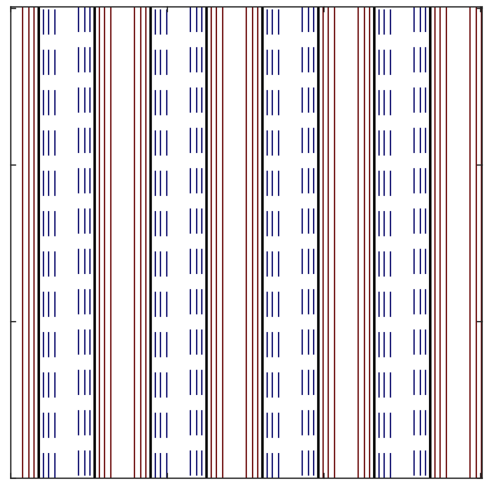
Does this barotropic QG model show eddy saturation?

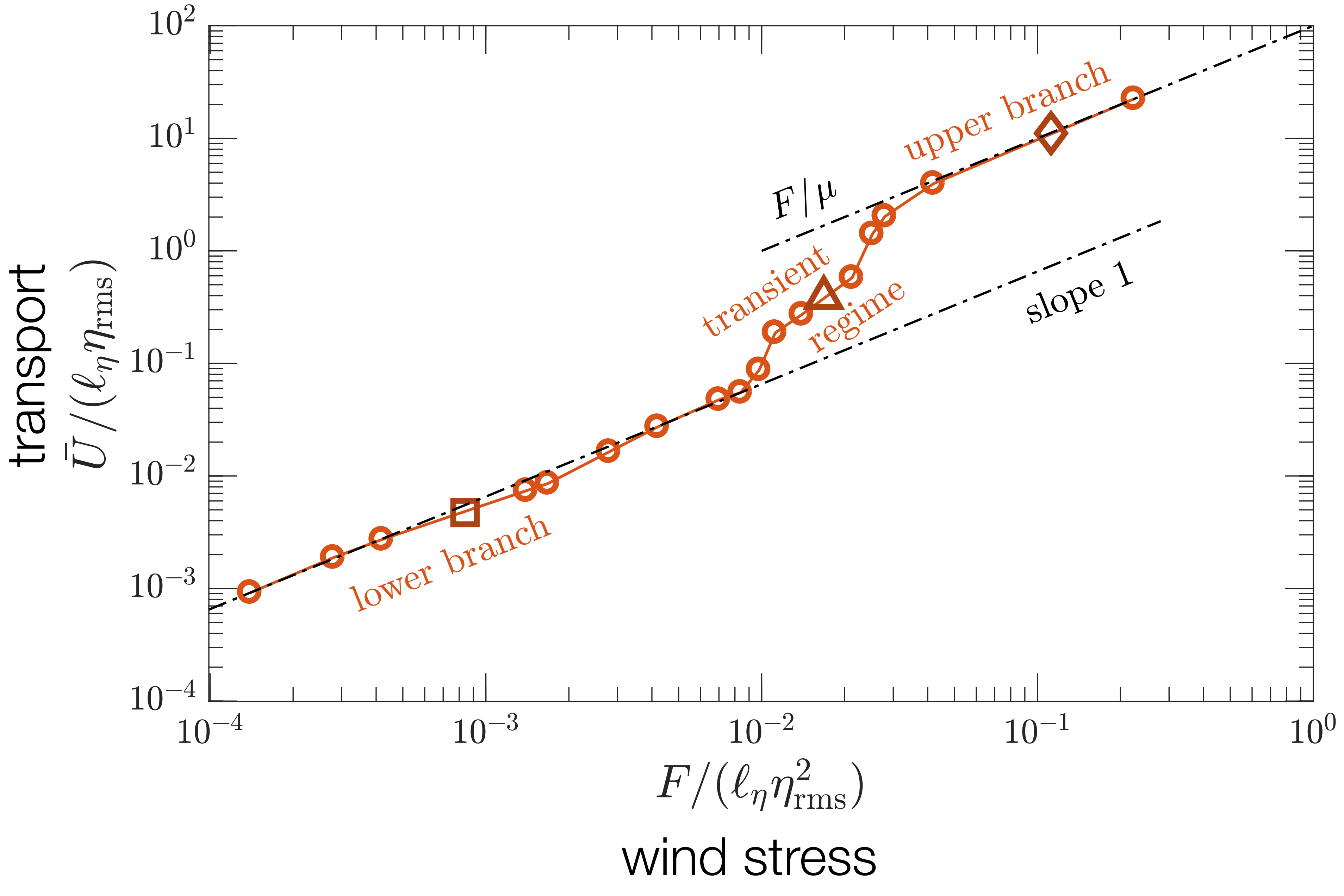
Do we need baroclinicity?

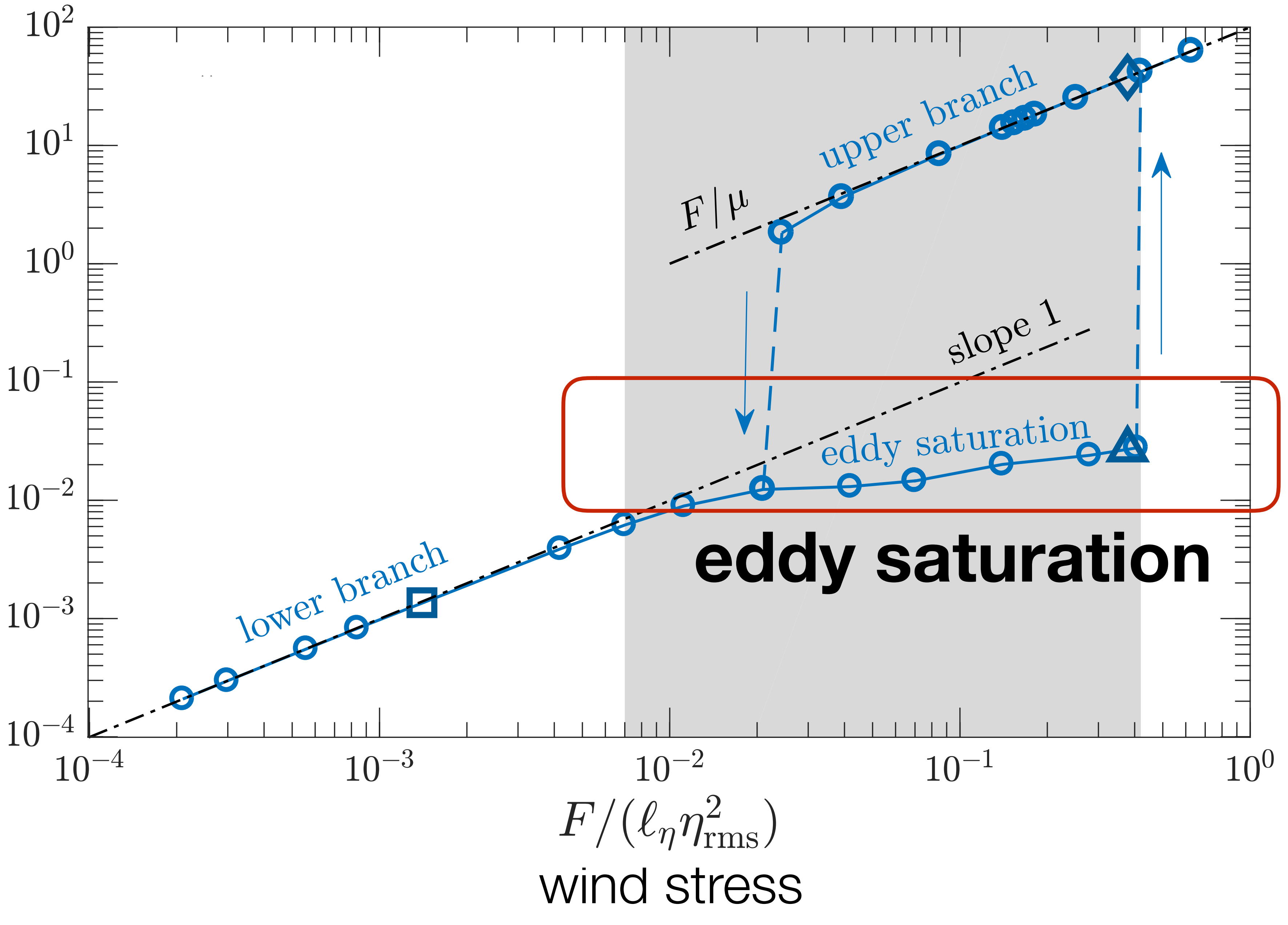
how does the transport vary with wind stress
in this barotropic QG model?

all parameters same, different topography



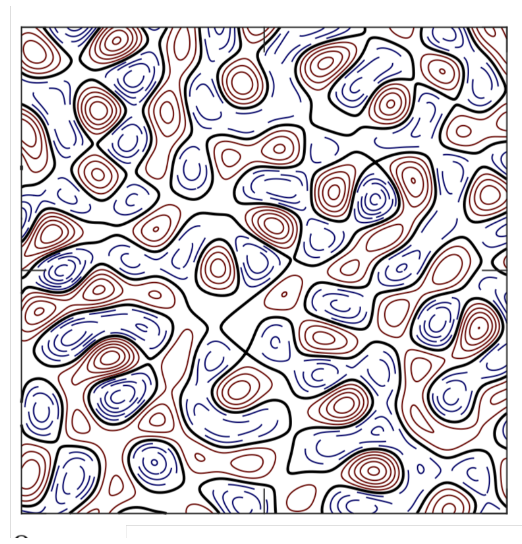


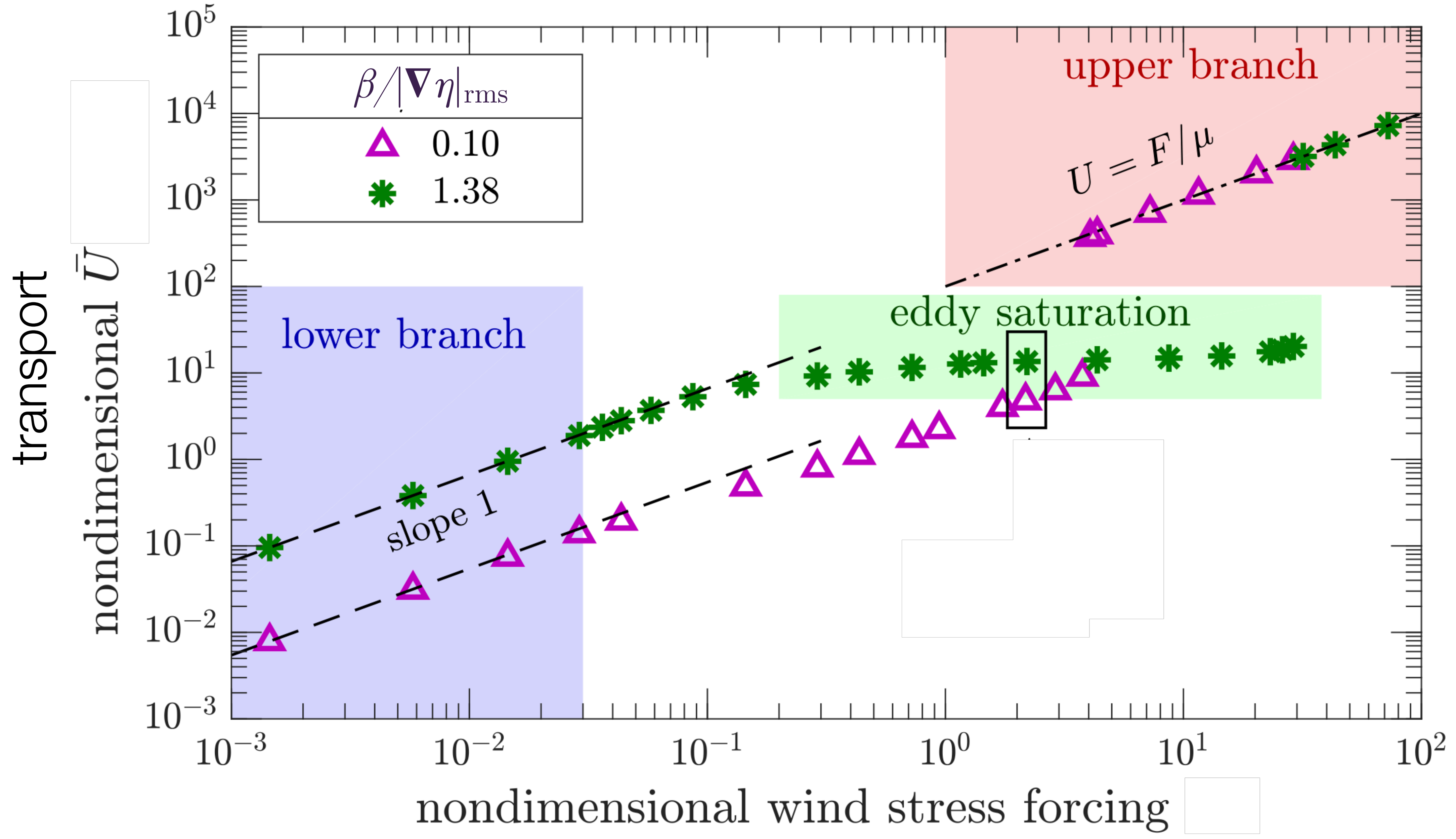




Constantinou 2018

all parameters same, different value of $\beta/|\nabla\eta|_{\mathrm{rms}}$



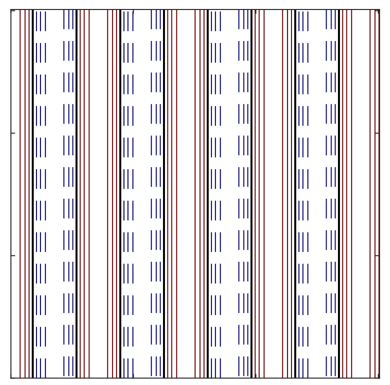


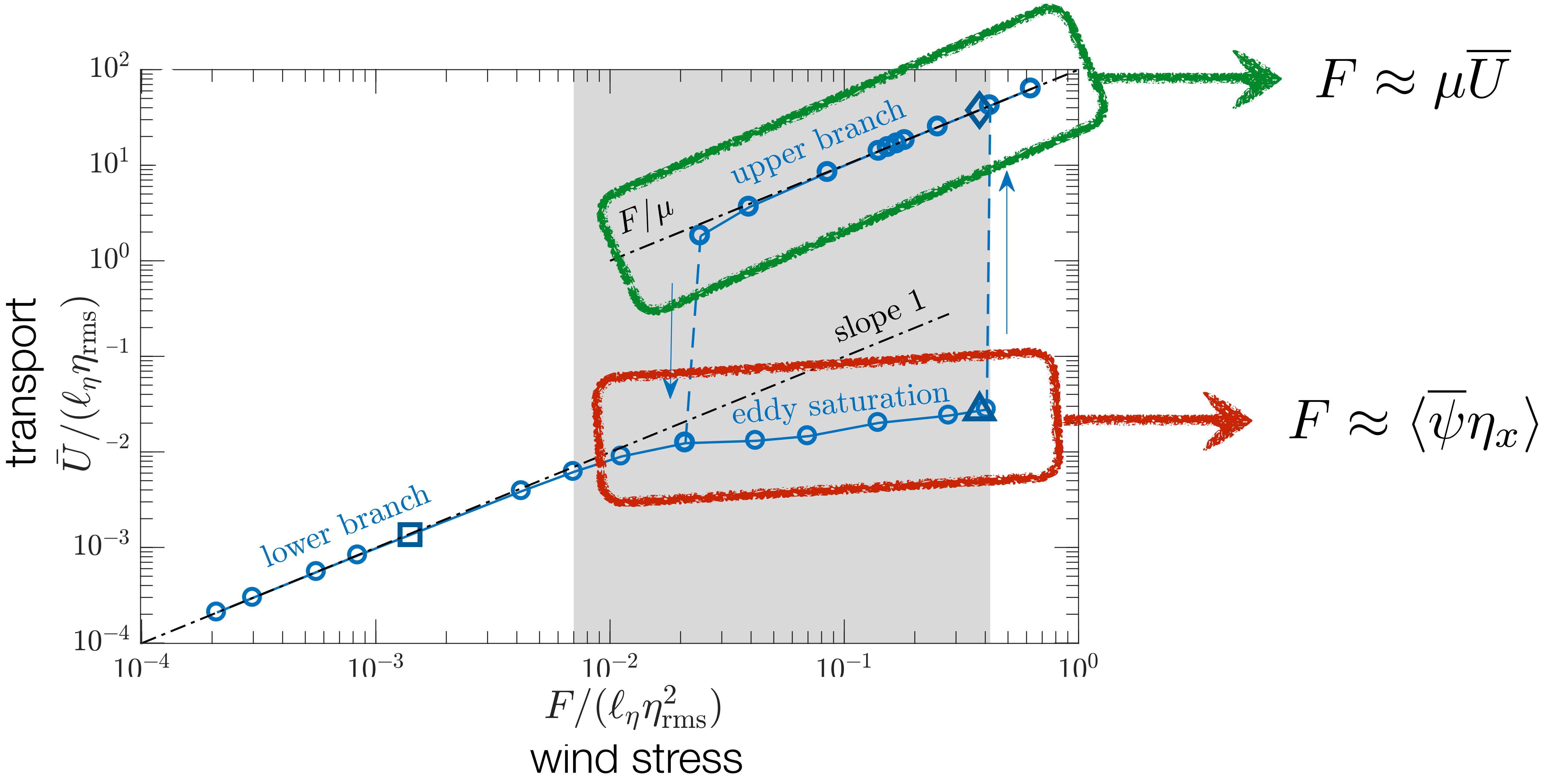
Constantinou & Young 2017

momentum balance

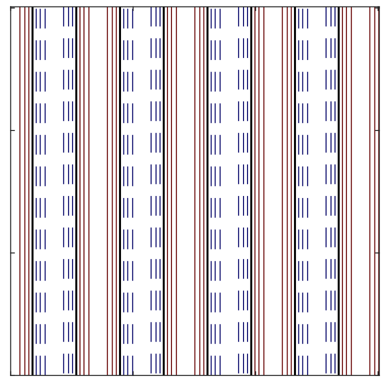
at equilibrium: $$F = \mu \overline{U} + \langle \overline{\psi} \eta_x \rangle$$

wind stress — bottom drag — form stress

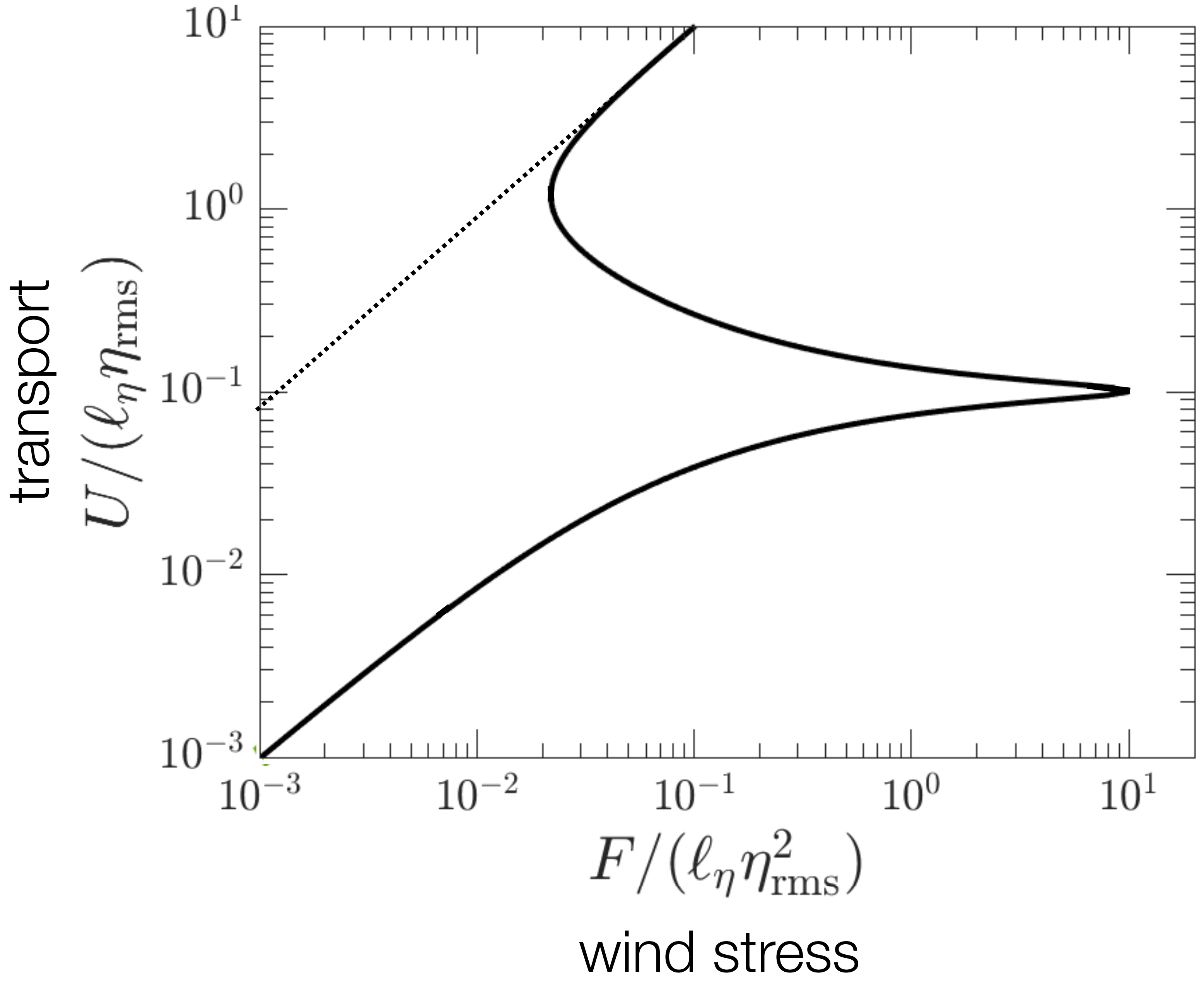




what produces eddy saturated states
in this barotropic QG model?



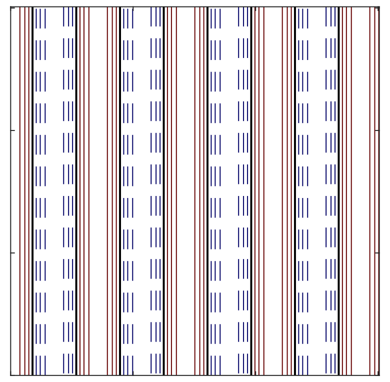
stability analysis for topography $\propto \cos(mx)$



topography induces
multiple equilibria
à la Charney & DeVore 1979

(Hart 1979, Charney & Flierl 1980, Pedlosky 1981,
Källén 1982, Rambaldi & Flierl 1983, Yoden 1985)

Constantinou 2018



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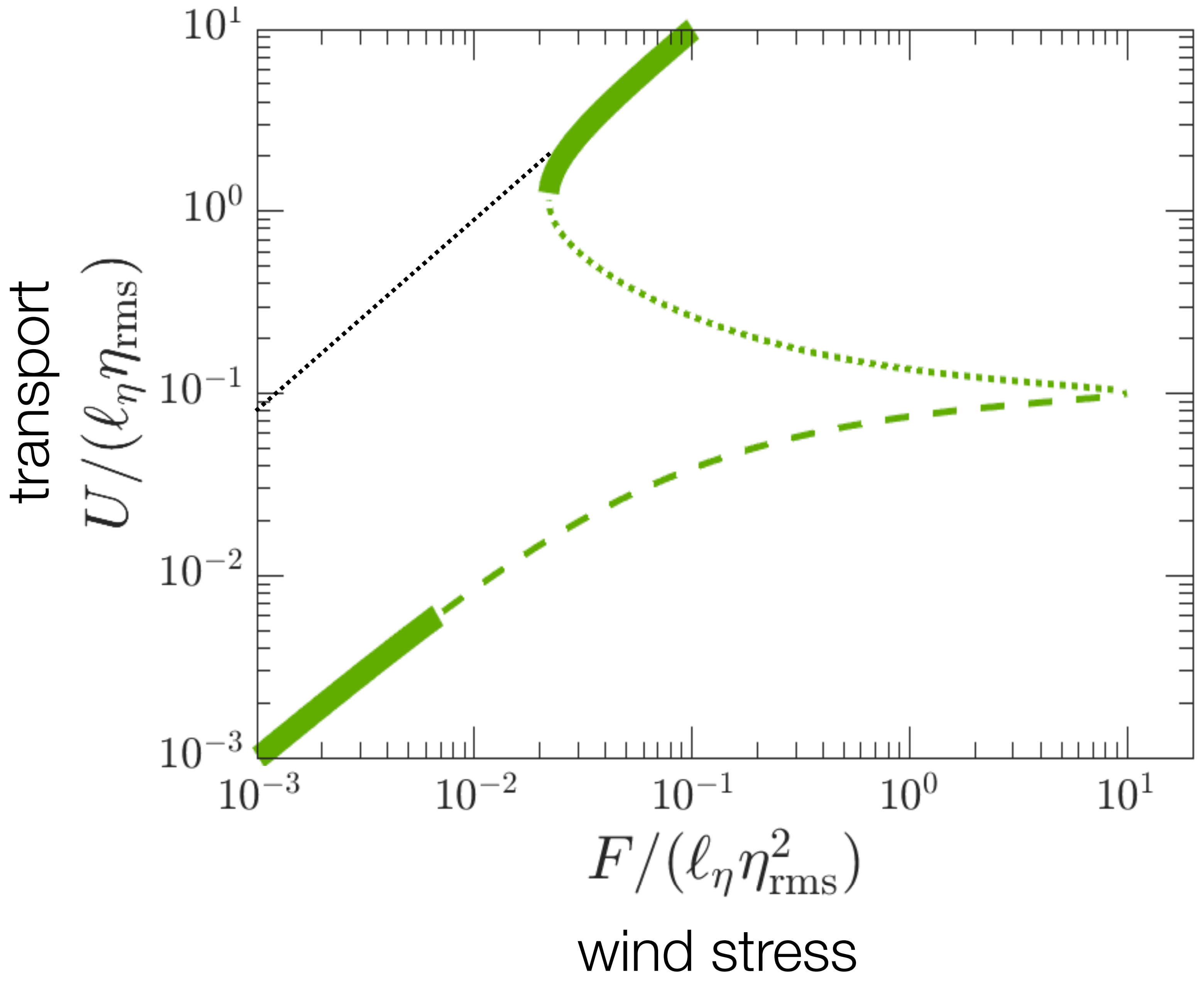
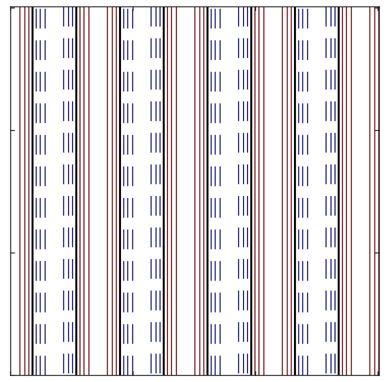


unstable

stable *only* within low-dim manifold (Constantinou 2018)

stable

(Hart 1979, Charney & Flierl 1980, Pedlosky 1981,
Källén 1982, Rambaldi & Flierl 1983, Yoden 1985)



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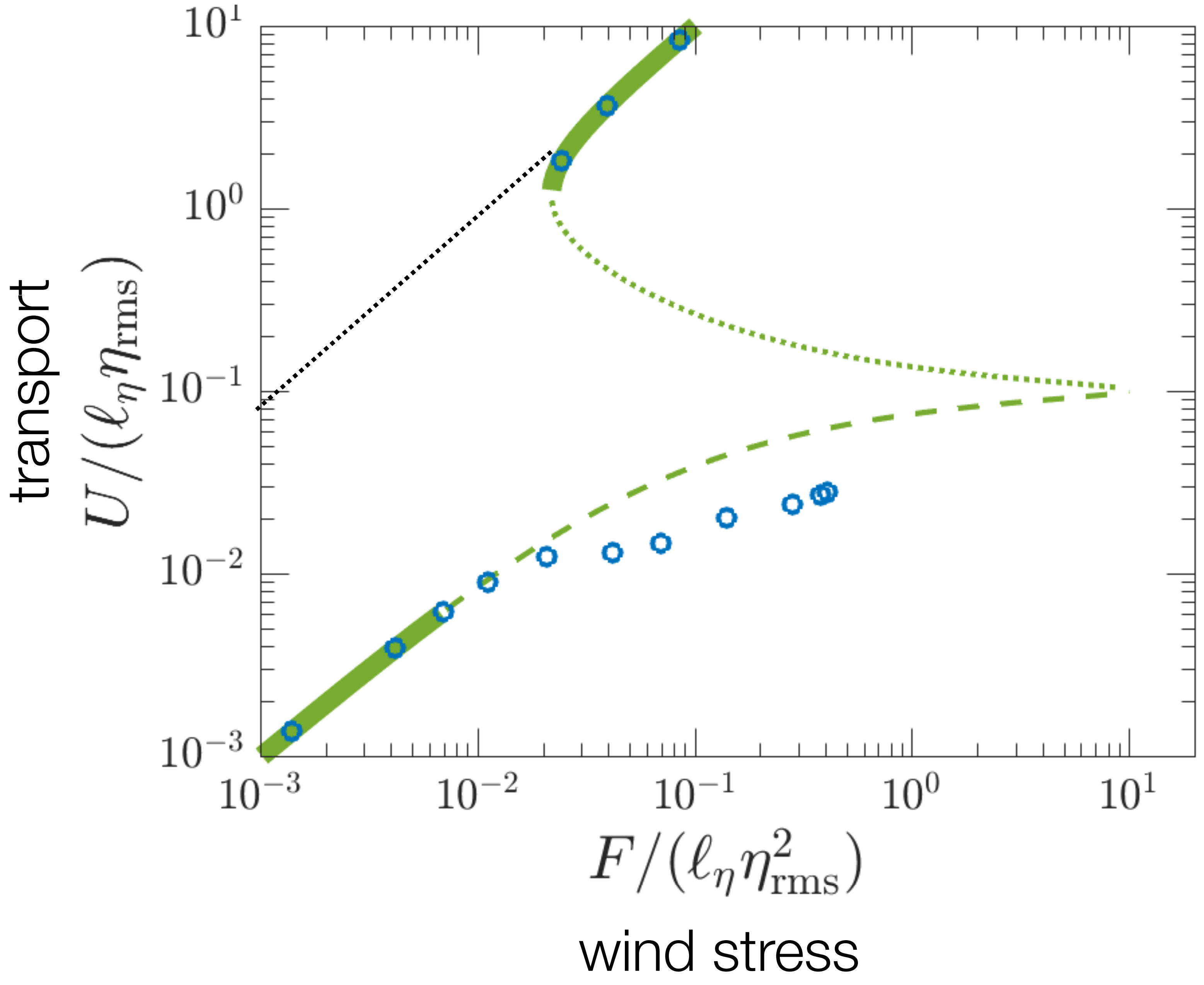

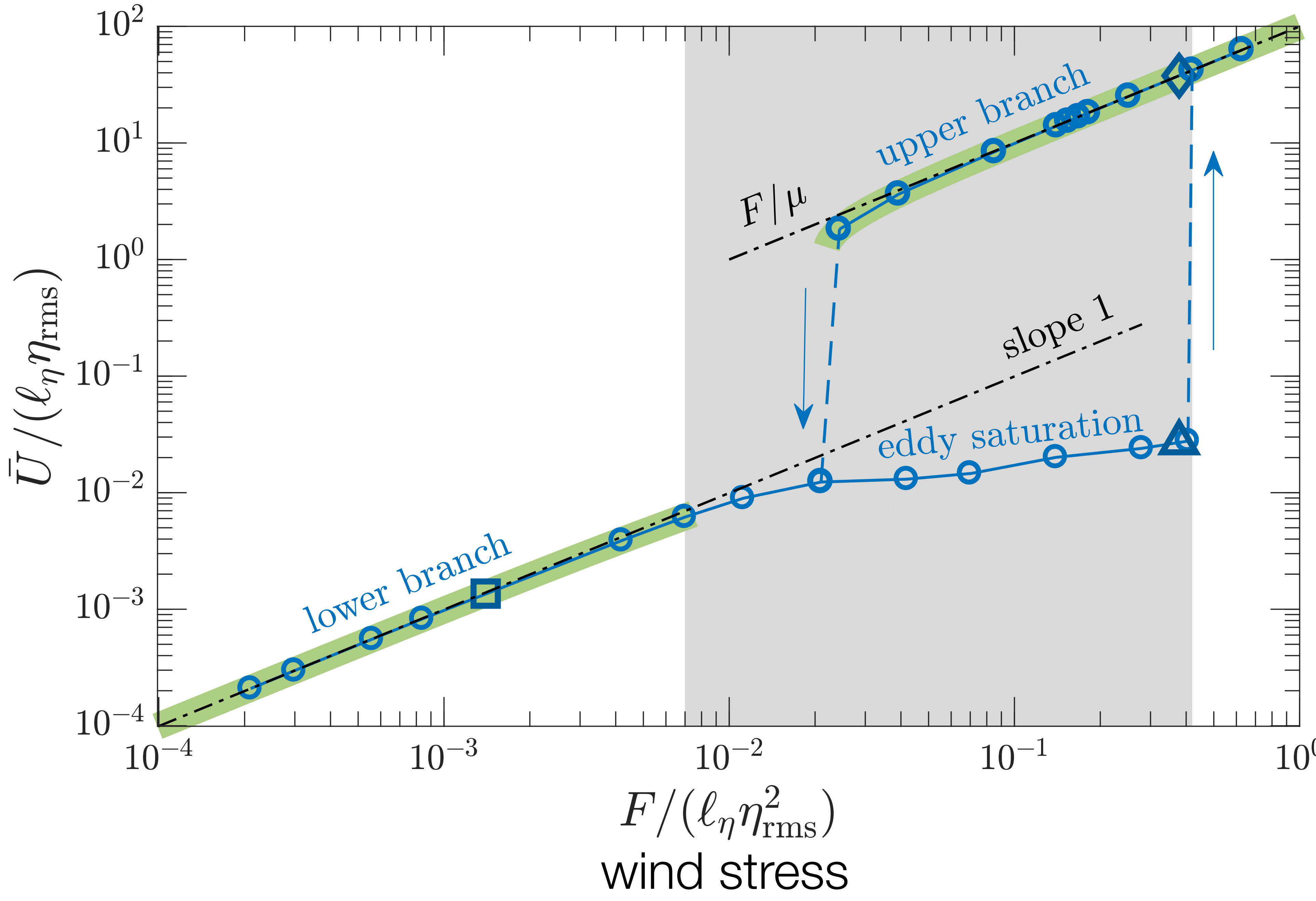

unstable

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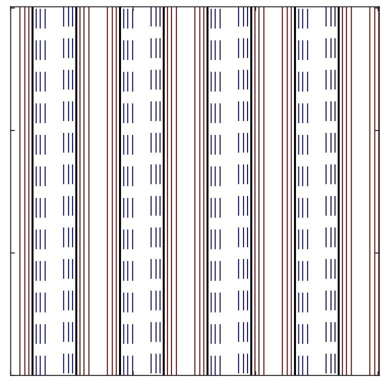
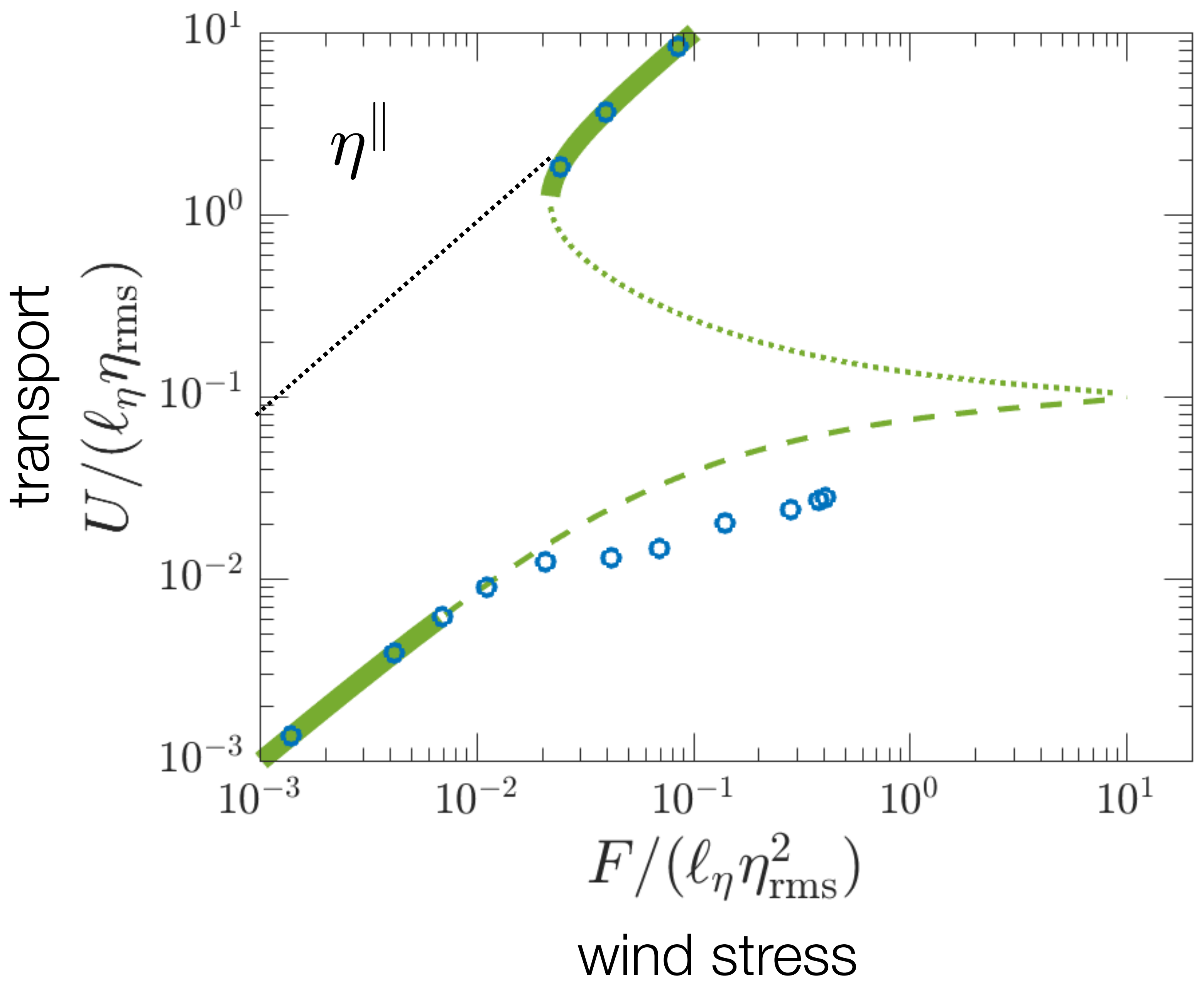
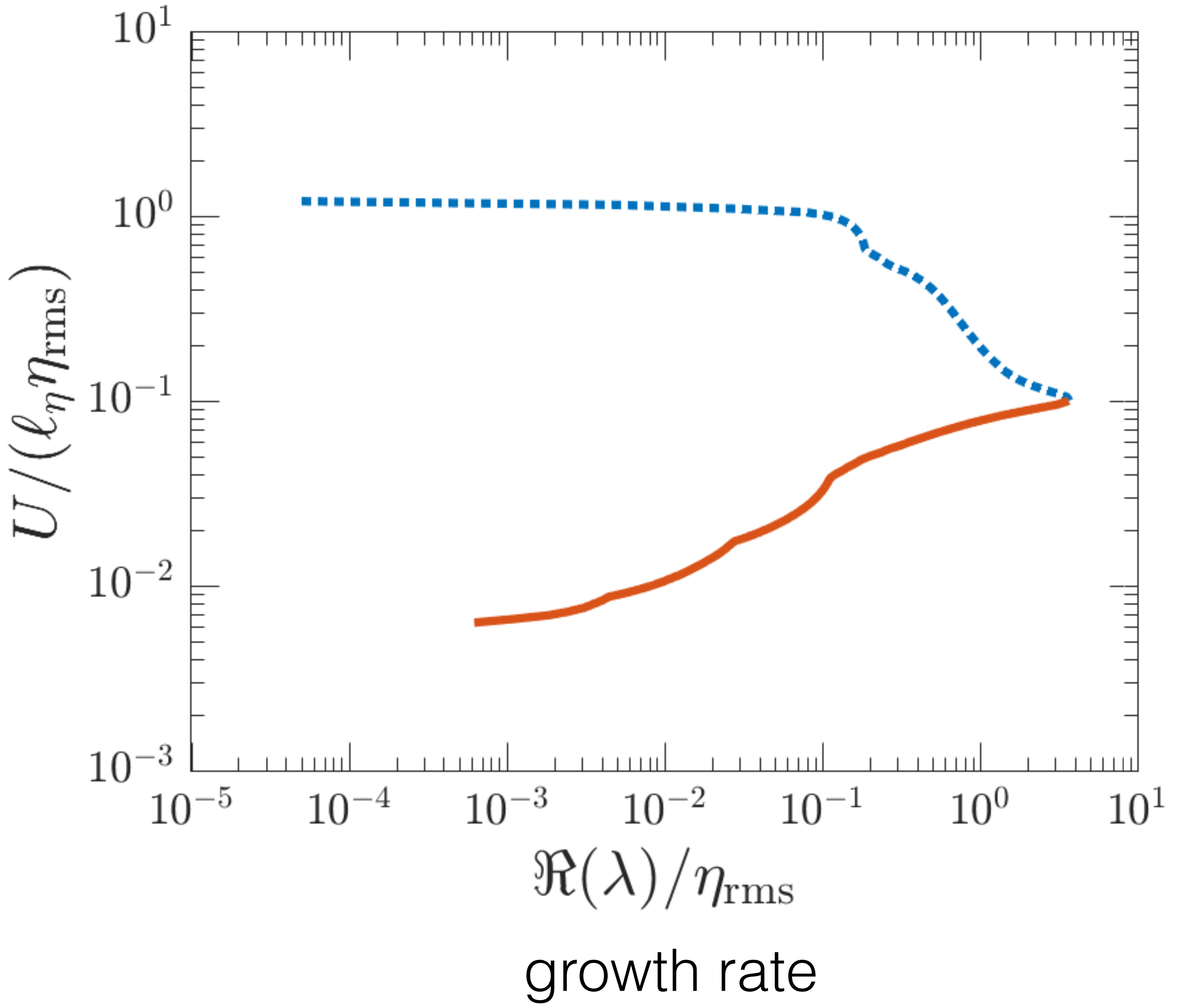
stable

numerical solutions

stability analysis for topography $\propto \cos(mx)$







unstable
stable *only* within low-dim manifold (Constantinou 2018)
stable
numerical solutions

Max instability growth rate increases ~10^4 times with a 10-fold increase in U!

Minor changes in $U \longrightarrow$ large transient energy production

Transient eddies balance *most* of the momentum imparted by $F \longrightarrow$ eddy saturation

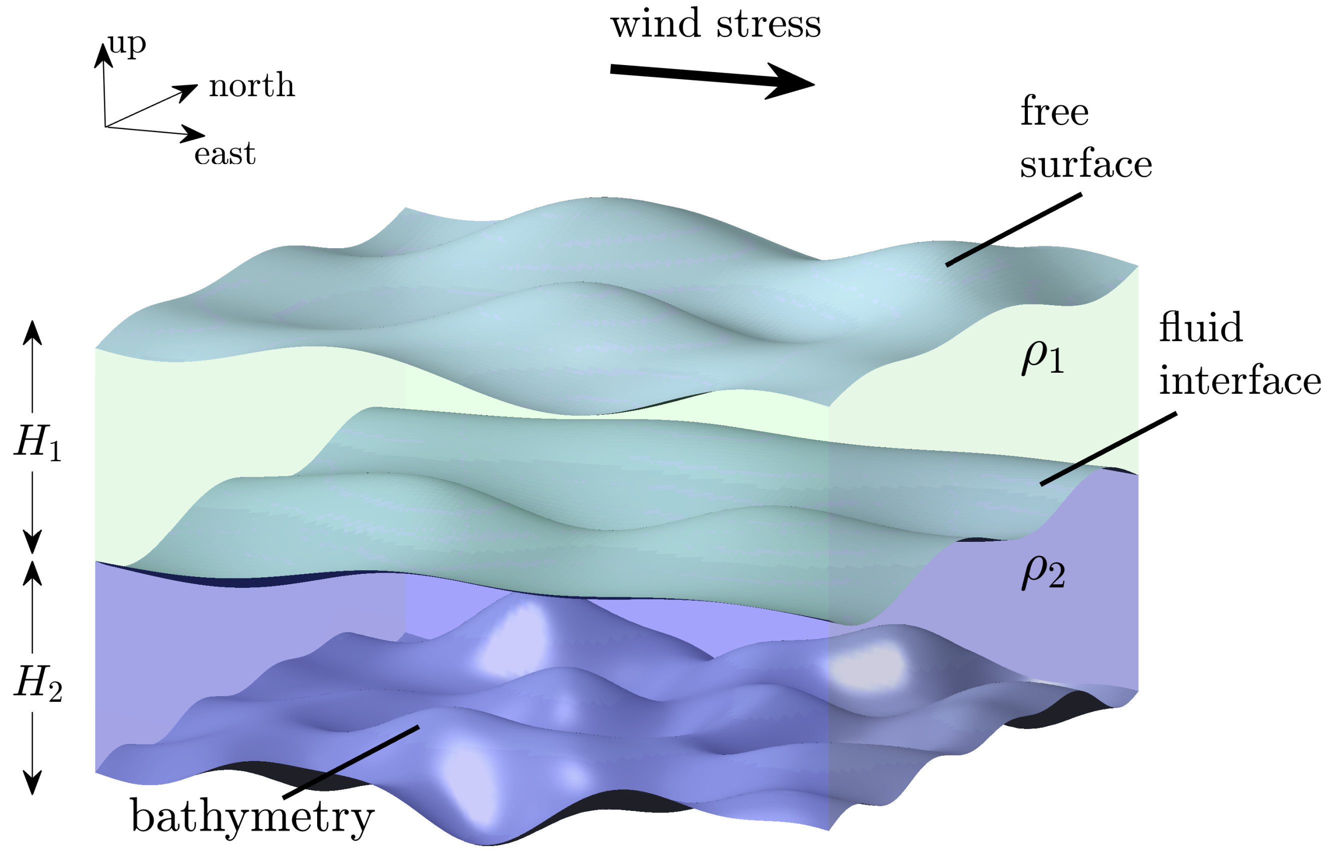
(Similarly as in the baroclinic scenario.)

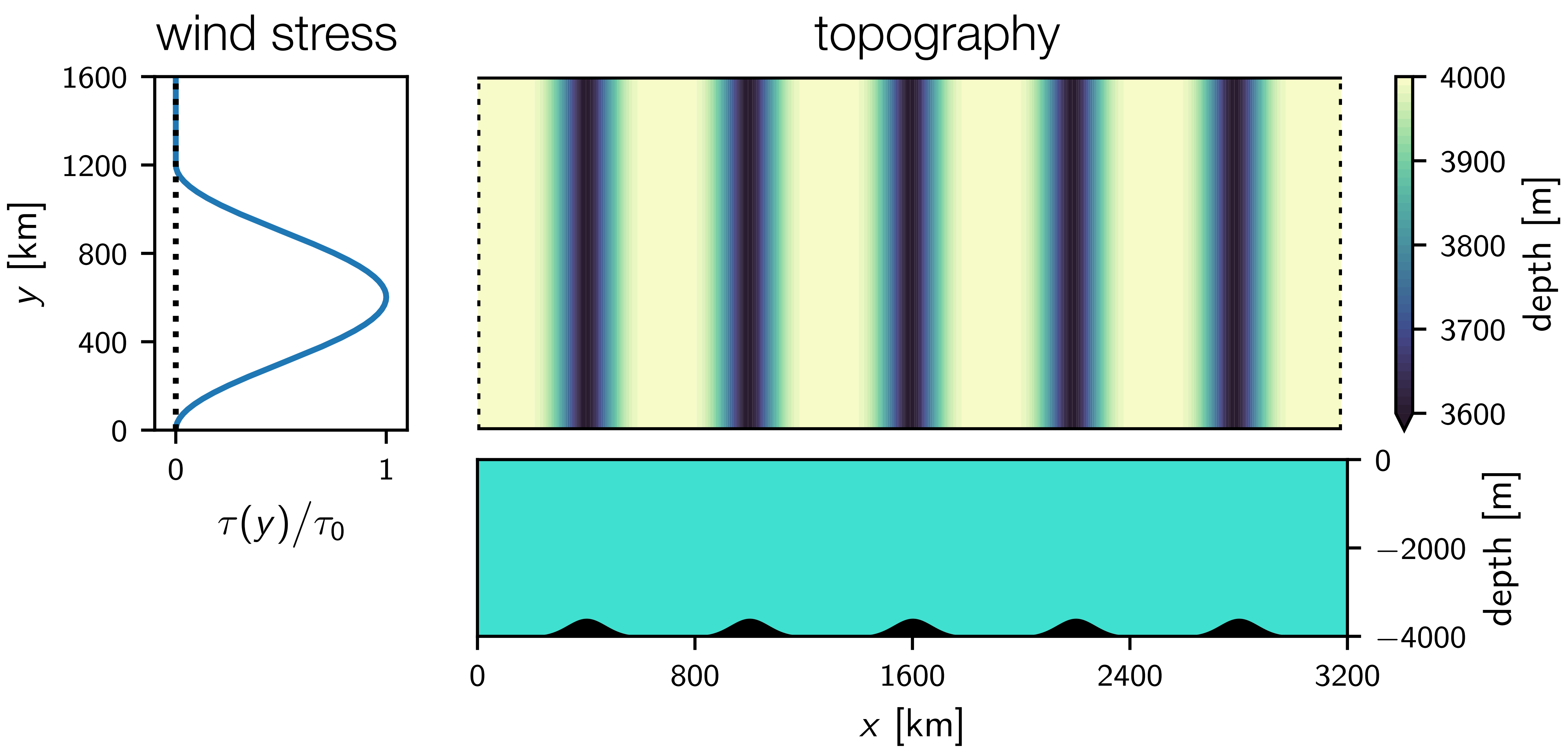
let's change page now

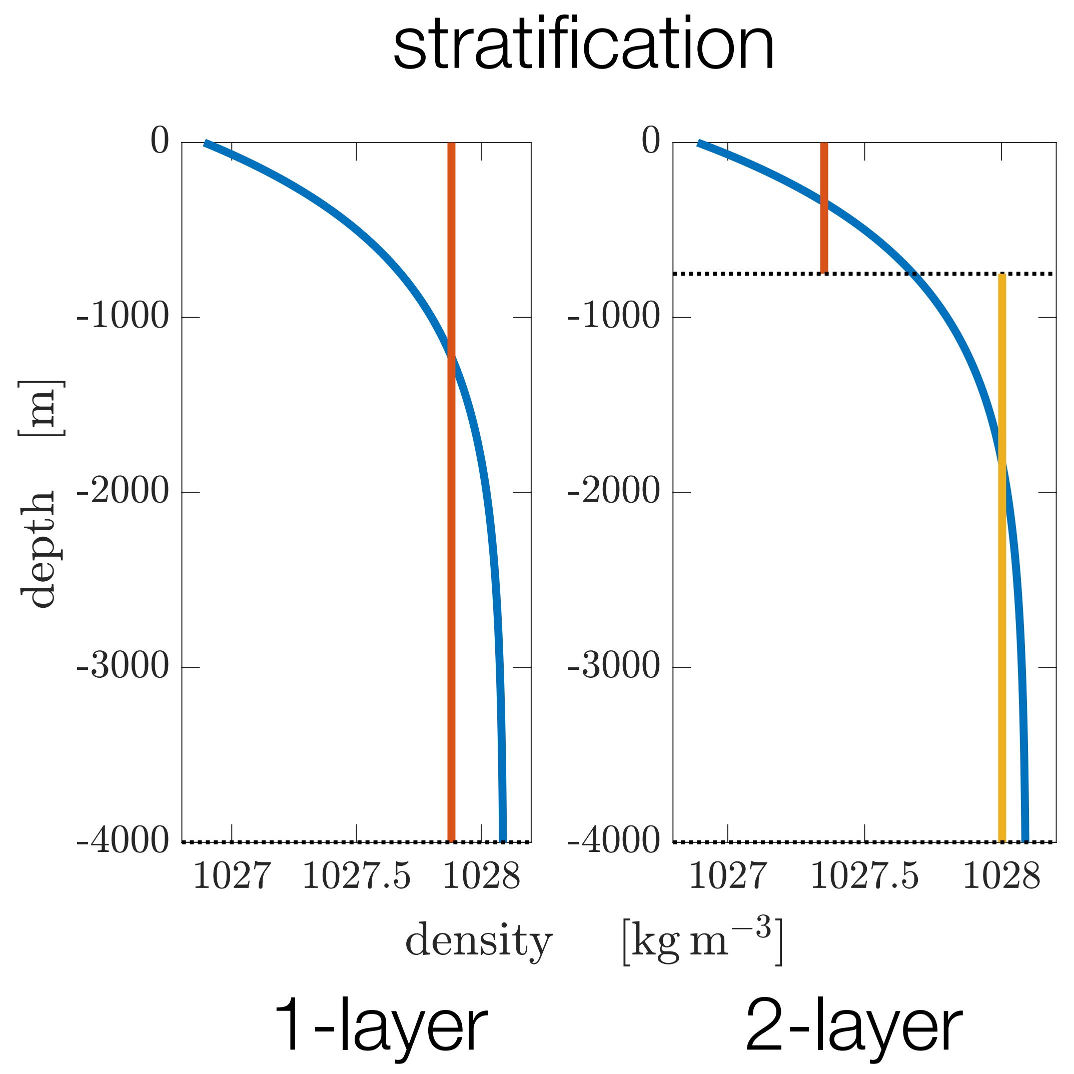
(what's the time?)

a setup with both BT and BC "eddy saturations"

- re-entrant channel with "bumpy" bottom
- $L_x = 3200$ km, $L_y = 1600$ km, and $H = 4$ km
- β-plane with Southern Ocean parameters
- modest stratification (few fluid layers of constant ϱ)
- 1st Rossby radius of deformation: 15.7 km (for >1 layers)
- Modular Ocean Model v6 (MOM6) in isopycnal mode



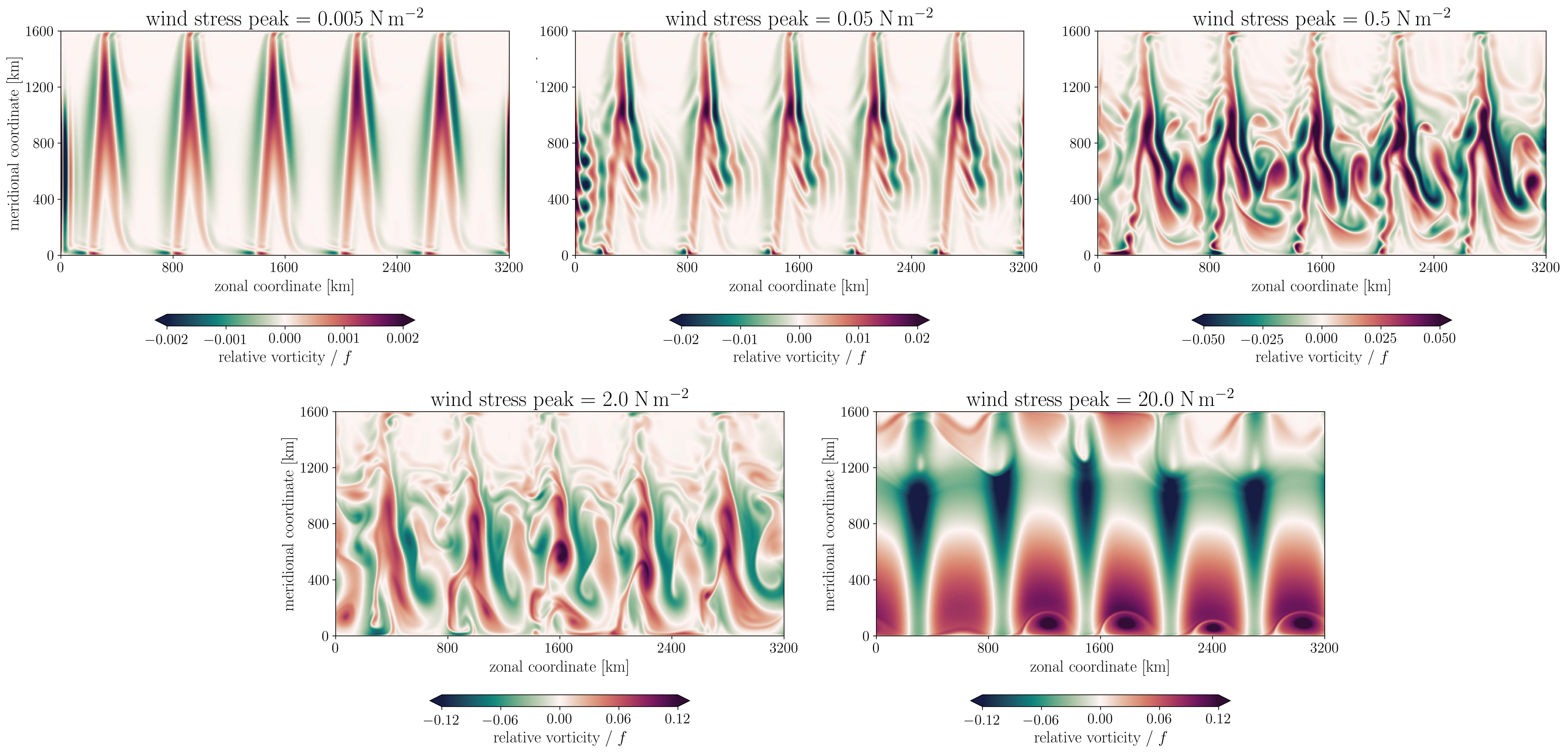




flow structure for 1-layer configuration

relative vorticity

$v_x - u_y$



flow structure for 1-layer configuration

upper branch

(flow barely "sees" the topography)

relative

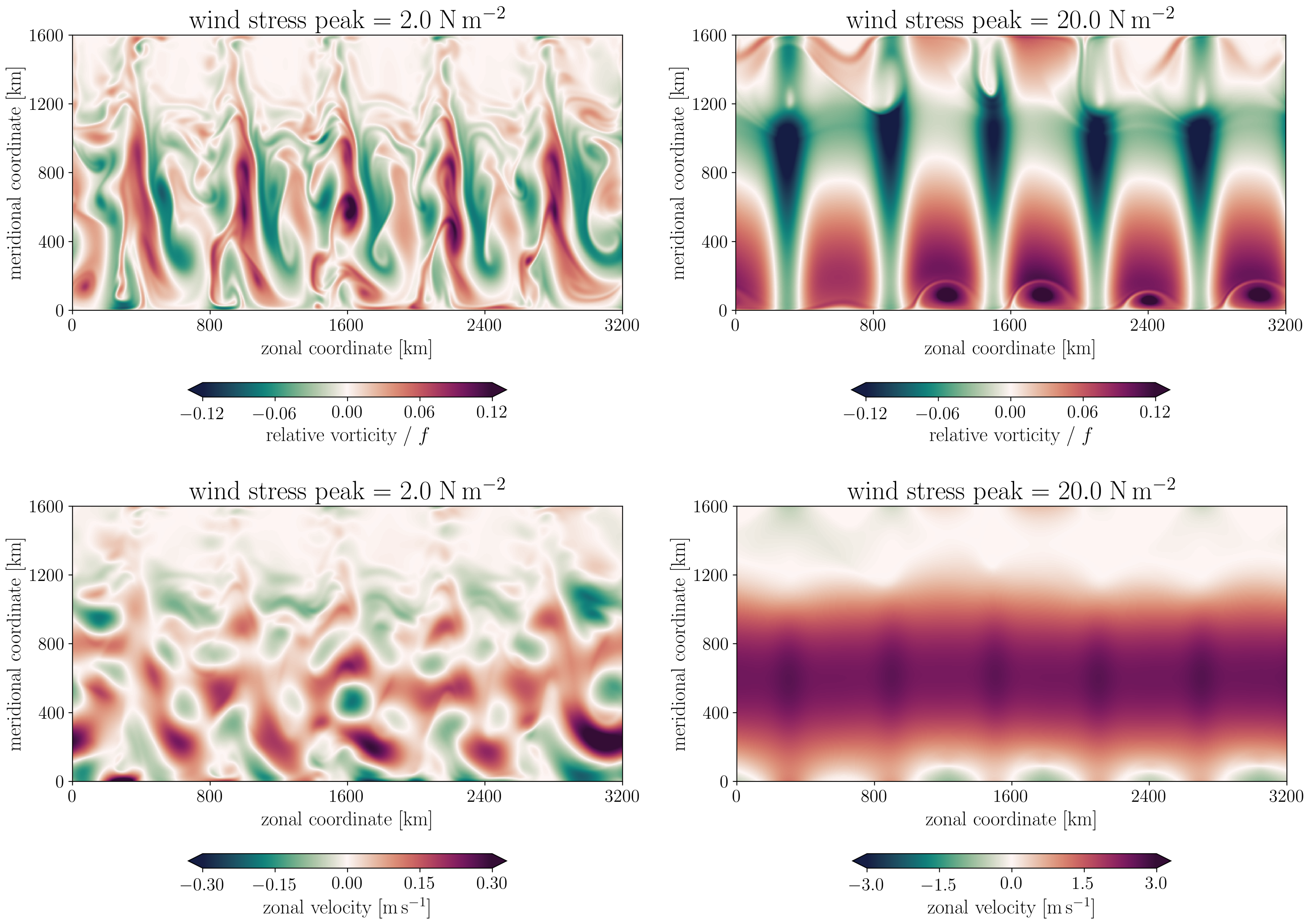
vorticity

$v_x - u_y$

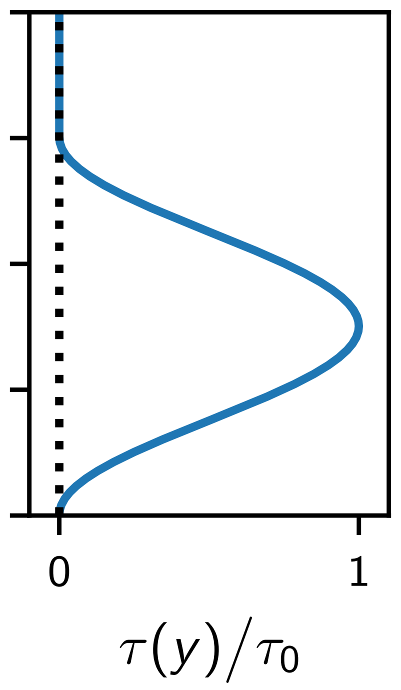
zonal

flow

u



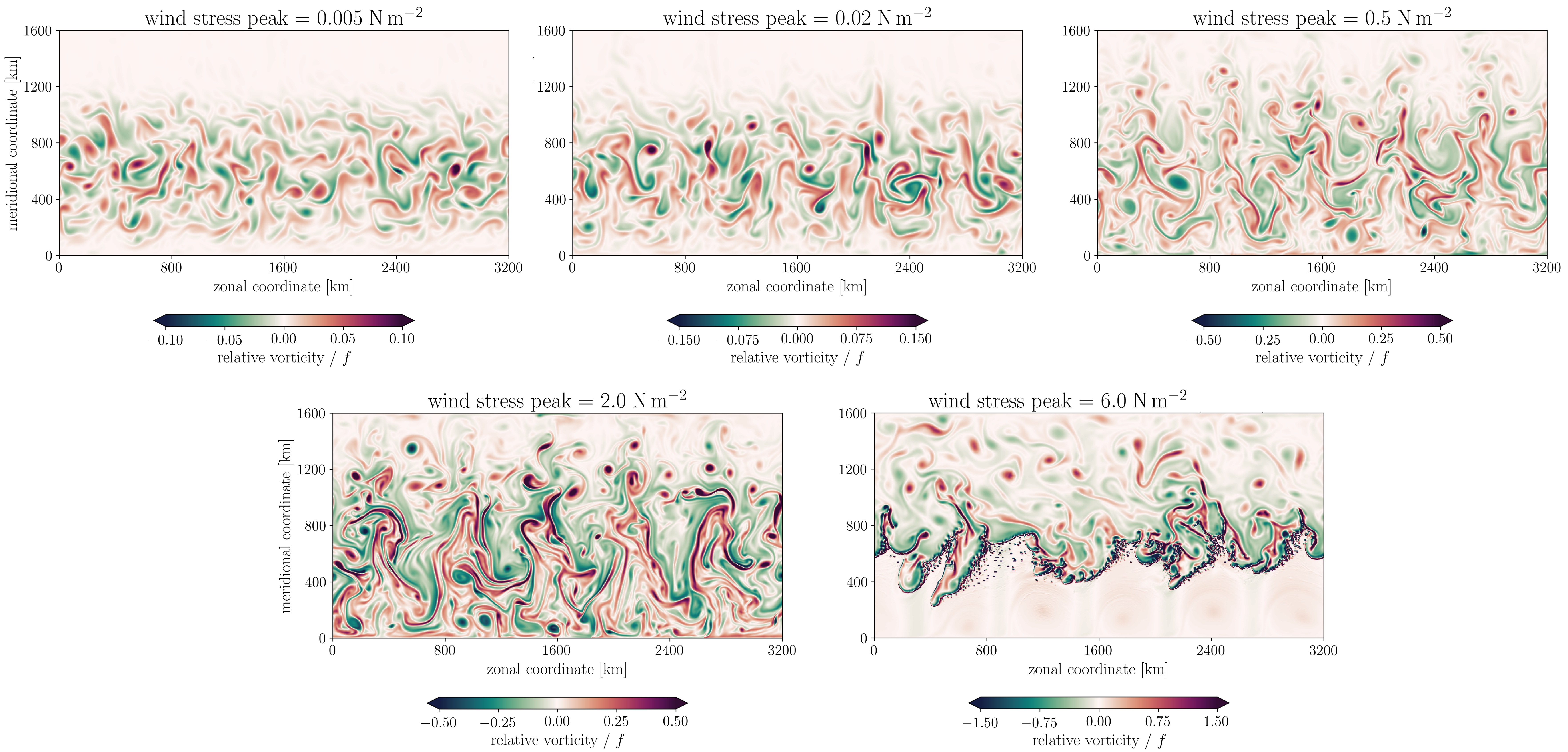
wind stress



flow structure for 2-layer configuration

top-layer relative vorticity

$v_x - u_y$

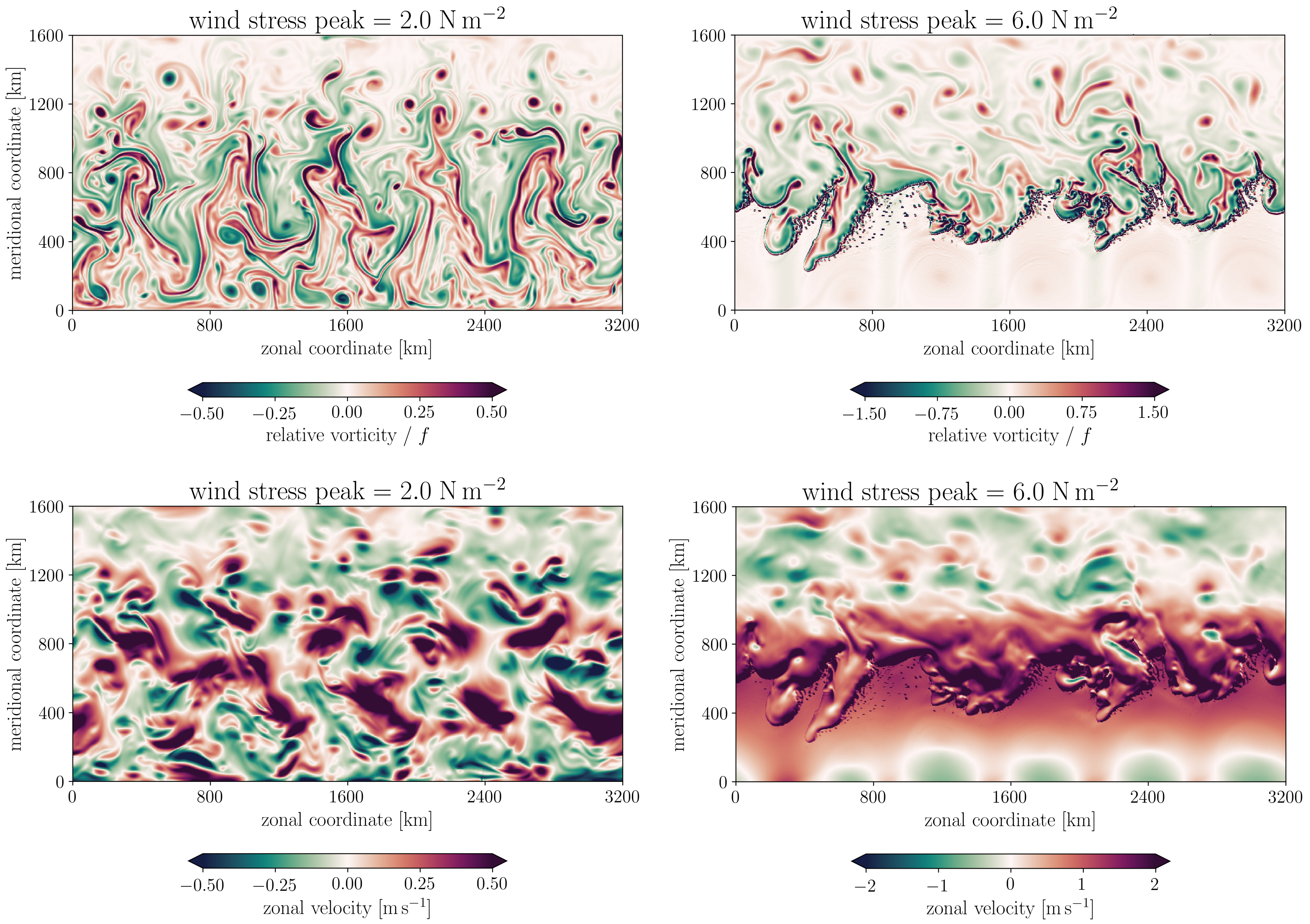


flow structure for 2-layer configuration

upper branch??

top-layer
relative
vorticity
$v_x - u_y$

top-layer
zonal
flow
u

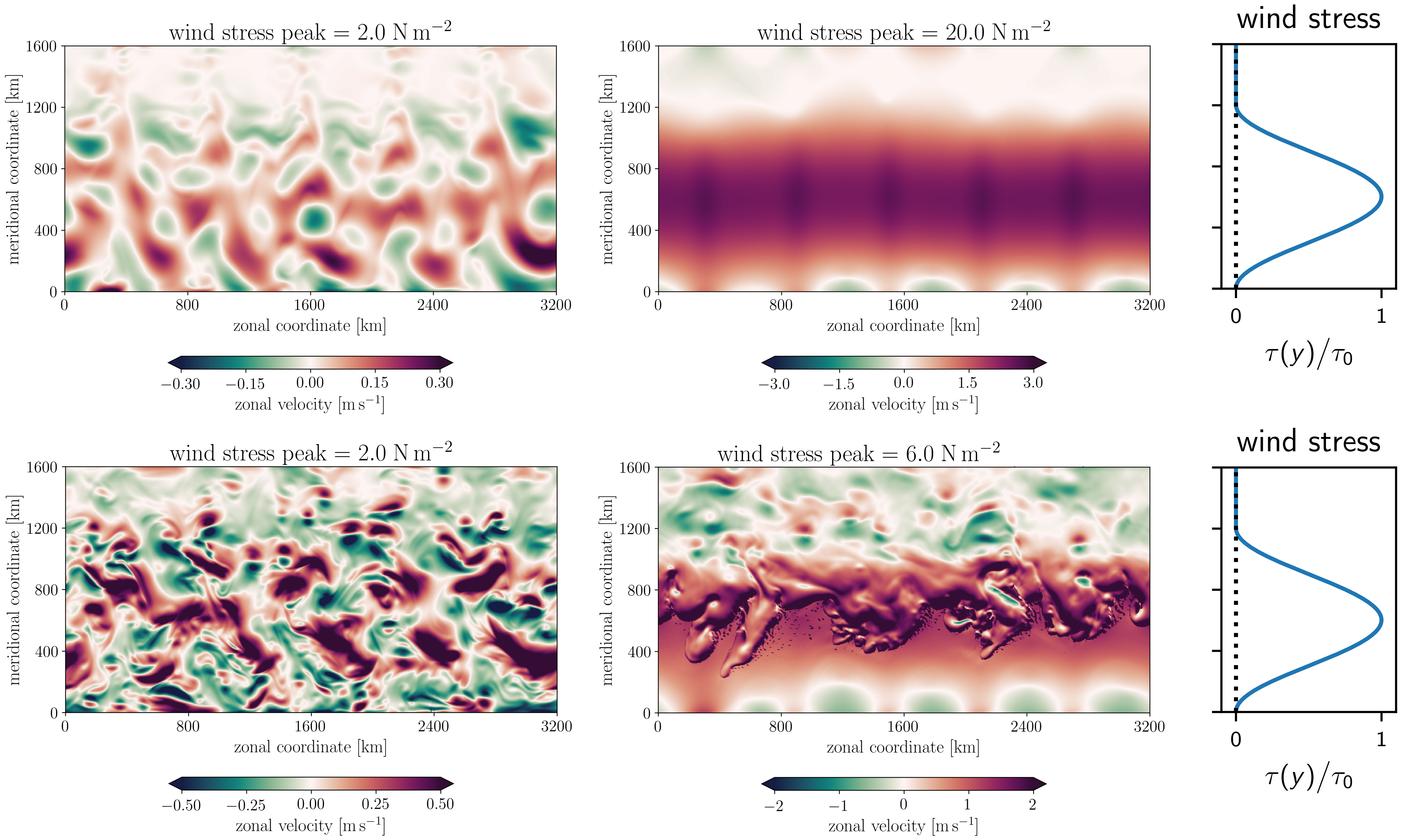


comparison of flow structure for 1-layer and 2-layer configurations

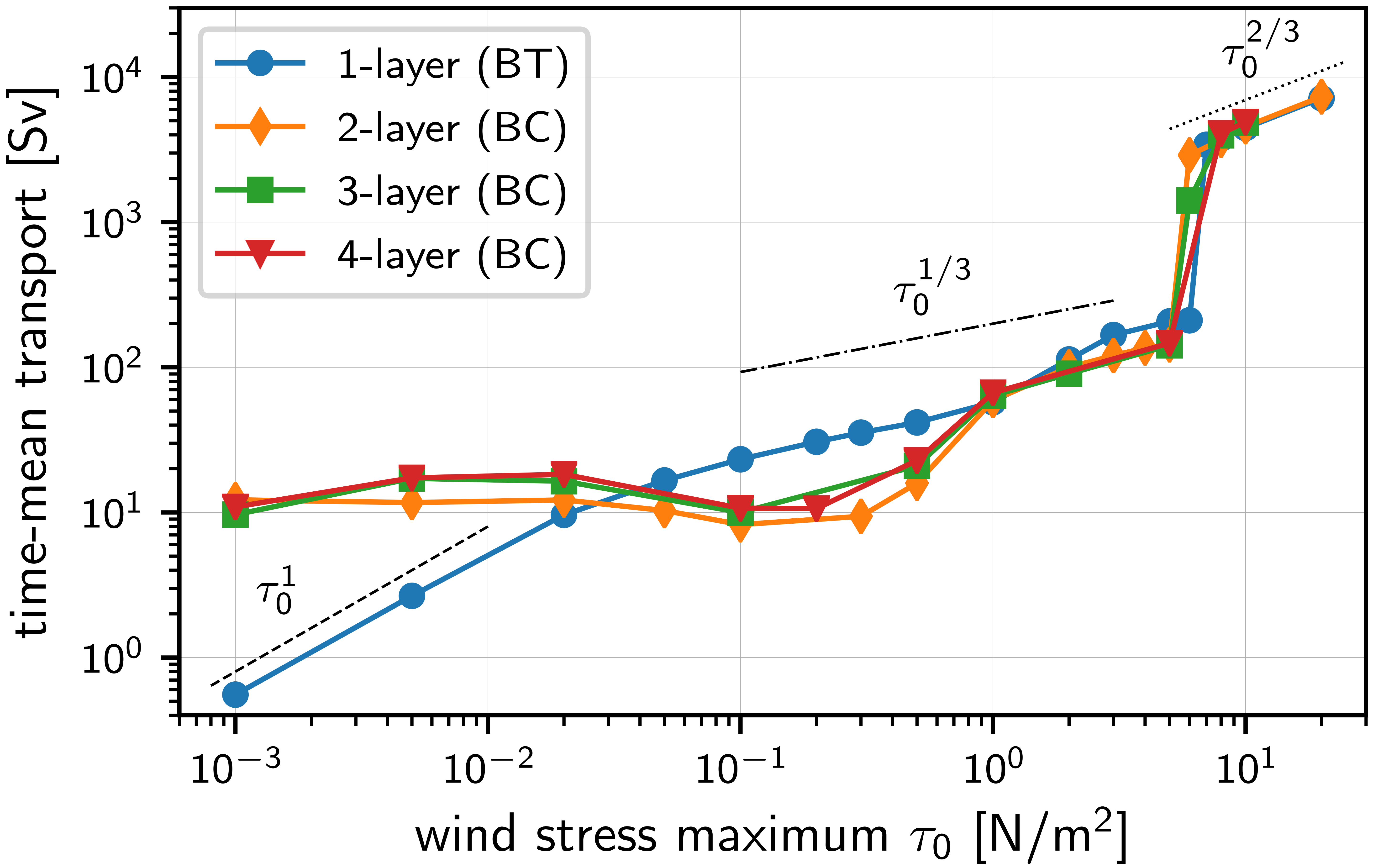
upper branch
(flow barely "sees" the topography)

1-layer

2-layer



transport versus wind stress

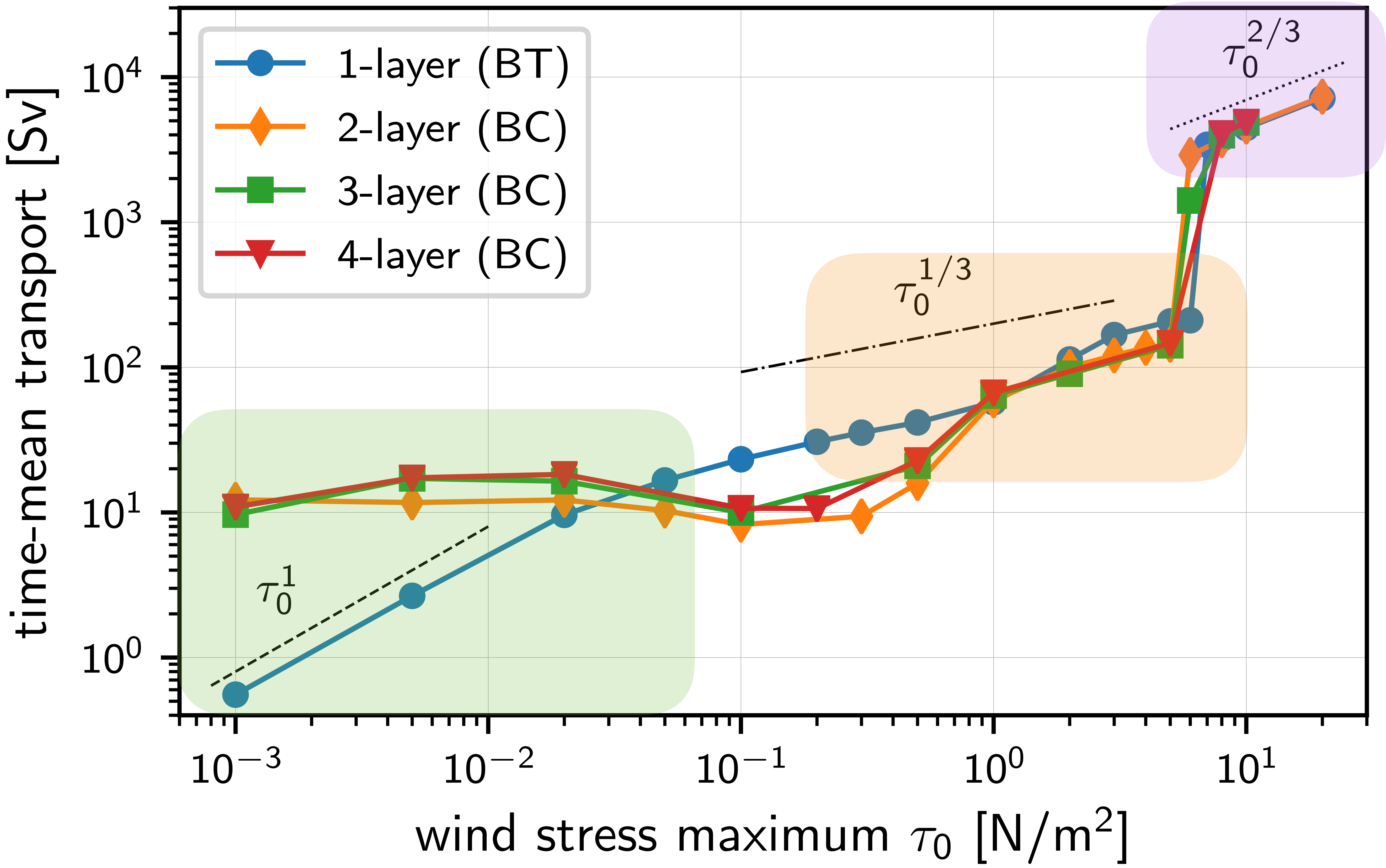


Baroclinic cases show strong eddy saturation.

The single-layer case **also** shows insensitivity to wind stress
(transport grows only about 10-fold over 100-fold wind stress increase)

Constantinou & Hogg (in progress)

transport versus wind stress



most momentum balances from bottom drag

most momentum balances from form stress

Baroclinic cases show strong eddy saturation.

The single-layer case **also** shows insensitivity to wind stress
(transport grows only about 10-fold over 100-fold wind stress increase)

Constantinou & Hogg (in progress)

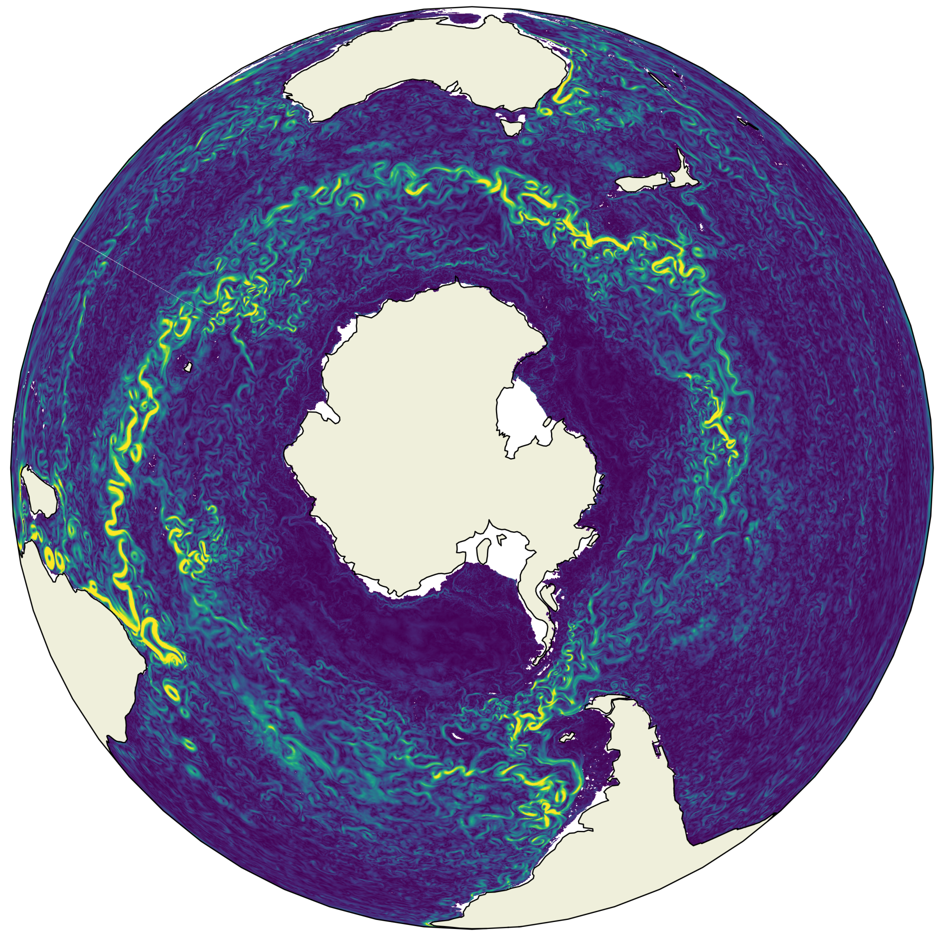
conclusions

This barotropic QG model shows eddy saturation.
This is surprising! All previous arguments were based on baroclinicity.

The barotropic—topographic instability is able to produce transient eddies in this model in a similar manner as baroclinic instability.

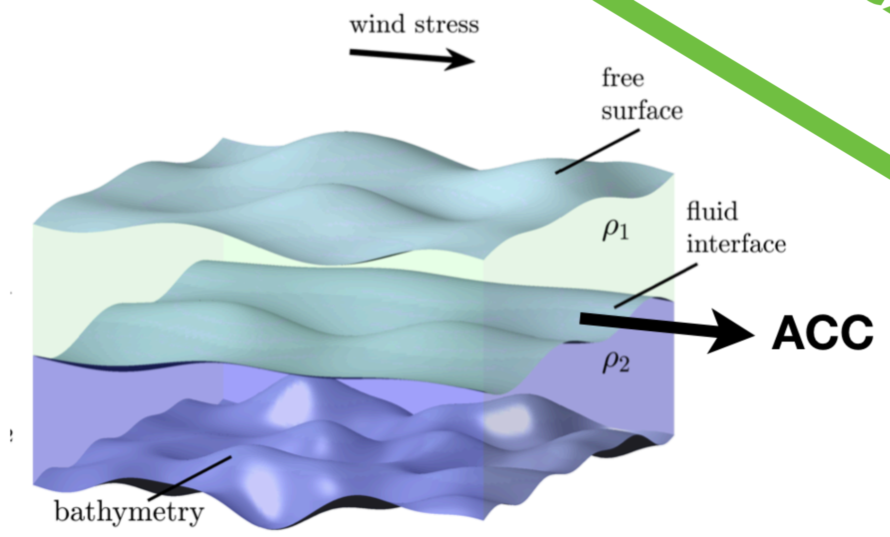
Barotropic eddy saturation "survives" in a primitive-equations multilayer channel model.

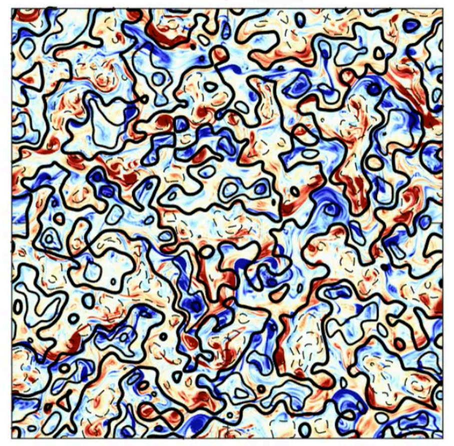
The flow-transition bifurcation to the upper branch survives with baroclinic dynamics.



?

start from simple conceptual models
that are tractable
and build up complexity





Discovery of **barotropic eddy saturation** changes a paradigm and highlights the role of topographically-induced eddies in setting up the large-scale oceanic circulation.

Constantinou and Young (2017). Beta-plane turbulence above monoscale topography. *J. Fluid Mech.*, 827, 415-447.

Constantinou (2018). A barotropic model of eddy saturation. *J. Phys. Oceanogr.* 48 (2), 397-411.

Constantinou & Hogg (2019?). Baroclinic versus barotropic eddy saturation. (being written up; for now just contact me)

thank you